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SPHERICAL TRIGONOMETRY  
WITH  
NAVAL AND MILITARY APPLICATIONS







(Frontispiece.)

(Courtesy, Lockheed Aircraft Corporation.)

# Spherical Trigonometry

*with*

## Naval and Military Applications

BY

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SPHERICAL TRIGONOMETRY  
WITH NAVAL AND MILITARY APPLICATIONS

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## PREFACE

This text is written with a view to the needs of men who expect to become officers in the navy, army, or air corps. While treating the subject of spherical trigonometry in detail, it takes up the most important applications of trigonometry and logarithms to navigation and related topics. Each topic is explained carefully, illustrated by examples, and followed by a list of problems designed to give familiarity with the topic and to call attention to important features. These are supplemented by numerous pictures that are interesting in themselves and serve the purpose of visually calling the student's attention to the direct nature of the applications. They suggest to his mind the actual situation and the reality of the problem.

Logarithms are treated completely because they are used in all branches of the service and their use acquaints the learner with the essential process of using tables generally. Small angle methods are employed to find heights and distances and to explain the underlying principle of the range finder. Maps are very important in military work. This text treats, among others, Mercator charts, stereographic projections, and mooring board graphs, showing the laws involved in the relative motions of ships, airplanes, and torpedoes. The topics of plane sailing, middle latitude sailing, course and distance for cruises, the location of position for ships and airplanes, all are considered in their turn. Finally the fundamental process of navigation, namely, that of determining a "fix," is considered in detail.

Forms are suggested for most computations. They are compact and simple; they save time and induce habits of forethought and orderliness. Also, since the same type of forms is used in the navy, this feature prepares directly for naval computation.

The authors' "Five-place Logarithmic and Trigonometric Tables," used at the U. S. Military Academy, are available for use with this text. They embody a principle that makes interpolation short and easy. The use of these tables prepares directly for the use of the U. S. Navy's logarithmic and trigonometric tables, compiled by the authors.

*In this book the emphasis is on fundamental ideas stripped of confusing details that tend to obscure the underlying principles.*

Outlines of suggested material for a short course, a medium course, and a complete course follow:

## Short Course

- ### Medium Course

- ## Complete Course

LYMAN M. KELLS,  
WILLIS F. KERN,  
JAMES R. BLAND.

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## GREEK ALPHABET

Letters	Names	Letters	Names	Letters	Names
$\alpha$	Alpha	$\iota$	Iota	$\rho$	Rho
$\beta$	Beta	$\kappa$	Kappa	$\sigma$ $\varsigma$	Sigma
$\gamma$	Gamma	$\lambda$	Lambda	$\tau$	Tau
$\delta$	Delta	$\mu$	Mu	$\upsilon$	Upsilon
$\epsilon$	Epsilon	$\nu$	Nu	$\varphi$	Phi
$\zeta$	Zeta	$\xi$	Xi	$\chi$	Chi
$\eta$	Eta	$\omicron$	Omieron	$\psi$	Psi
$\theta$	Theta	$\pi$	Pi	$\omega$	Omega

## LIST OF SYMBOLS

$\equiv$ , read *is identical with*.

$\neq$ , read *is not equal to*.

$<$ , read *is less than*.

$>$ , read *is greater than*.

$\leq$ , read *is less than or equal to*.

$\geq$ , read *is greater than or equal to*.

$(x, y)$ , read *point whose coordinates are  $x$  and  $y$* .





## CHAPTER I

### LOGARITHMS

**1. Introduction.** The labor involved in many numerical computations is considerably lessened by the use of logarithms. In the following articles we shall discover that in a sense the use of logarithms reduces multiplication to addition, division to subtraction, raising to a power to multiplication, and extracting a root to division. For this reason logarithms constitute a remarkable labor-saving device in computation.

We shall learn presently that logarithms are exponents and that the laws that govern the use of exponents are the ones that govern the use of logarithms. Hence, before discussing logarithms, we shall recall from algebra the laws of exponents.

**2. Laws of exponents.** It is proved in algebra that, when the exponents  $m$  and  $n$  are any numbers, the following laws hold:

$$\begin{array}{ll}
 \text{(I)} & a^m a^n = a^{m+n}. \\
 \text{(II)} & \frac{a^m}{a^n} = a^{m-n}. \\
 \text{(III)} & (a^m)^n = a^{mn}. \\
 \text{(IV)} & (ab)^m = a^m b^m. \\
 \text{(V)} & \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.
 \end{array}$$

#### EXERCISES

1. Evaluate the following:

$$\begin{array}{lll}
 (a) & 3^2 3^{-3}. & (d) & 3^{-\frac{2}{3}} 3^{\frac{7}{3}}. & (g) & (25 \times 49)^{-\frac{1}{2}}. \\
 (b) & 7^{-\frac{3}{4}} \sqrt[7]{7^{10}}. & (e) & \frac{5^{-\frac{3}{2}}}{\sqrt{5}}. & (h) & \left(\frac{3}{2}\right)^{-3}. \\
 (c) & 3^{-\frac{1}{2}} 3^0. & (f) & (3^{-1})^{\frac{2}{3} 7}. & (i) & \left(\frac{8}{27}\right)^{-\frac{2}{3}}.
 \end{array}$$

2. Find, in each case, the value of  $x$  which satisfies the equation:

$$\begin{array}{lll}
 (a) & 10^x = 1000. & (f) & x^{-2} = 100. & (k) & 7^x = 1. \\
 (b) & 3^{-3} = x. & (g) & 10^0 = x. & (l) & x^{-1} = 0.01. \\
 (c) & x^4 = 10,000. & (h) & x^{-2} = 10^0. & (m) & 7^x = 343. \\
 (d) & x^{-\frac{1}{2}} = 3. & (i) & (36)^x = \frac{1}{6}. & (n) & \left(\frac{1}{x}\right)^{-2} = 16. \\
 (e) & 4^x = \frac{1}{2}. & (j) & x^{-\frac{1}{3}} = \sqrt{7}. & (o) & 2^{\frac{1}{x}} = 4^3.
 \end{array}$$

3. Find  $x$  if

(a)  $10^x = \frac{1}{10}.$

(b)  $10^x = 0.001.$

(c)  $10^x = 0.0001.$

(d)  $10^x = 1000.$

(e)  $10^x = 1.$

(f)  $10^x = 100,000.$

4. Solve each of the following equations for  $x$ :

(a)  $(3)(2)^x + 4 = 100.$

(b)  $5^{x+3} - 5^{2x} = 0.$

(c)  $(8)(2)^x - 2^{4x} = 0.$

(d)  $(8)(3^x) = (27)(2^x).$

(e)  $(x-2)^0 = x^2 + 1.$

(f)  $27^x = 81.$

(g)  $(3^{\frac{1}{2}})(9)^{2x} = 3^{-\frac{2}{3}}.$

(h)  $(\frac{1}{2}\frac{6}{5})^{-\frac{1}{2}} = 5\sqrt{x}.$

(i)  $(\frac{8}{27})^{-\frac{1}{3}} = 2x^{-1}.$

(j)  $(7^{x^2-1})(49^{1-x}) = \sqrt{7}.$

(k)  $(\frac{9x}{4})^{-\frac{1}{2}} - 3^{-2} = 3^{-3}.$

(l)  $\frac{1}{2}\sqrt{x}\sqrt[3]{x} = 64.$

**3. Definition of a logarithm.** If  $b$ ,  $L$ , and  $N$  are numbers such that  $b$  raised to the power  $L$  is equal to  $N$ , then  $L$  is called the logarithm of  $N$  to the base  $b$ . In symbols, if

$$b^L = N, \quad \text{then} \quad L = \log_b N. \quad (1)$$

Stated differently, *the logarithm of a number to a given base is the power to which the base must be raised to produce the number.*

The two equations in (1) express the same relation between the base  $b$ , the number  $N$ , and the logarithm  $L$ . The second one is read:  $L$  is the logarithm of  $N$  to the base  $b$ . Also  $N$  is called the antilogarithm of  $L$  (or the number whose logarithm is  $L$ ) to the base  $b$ . Since  $5^2 = 25$ , 2 is the logarithm of 25 to the base 5, and 25 is the antilogarithm of 2 to the base 5. Similarly, we have

$$\begin{aligned} 10^3 &= 1000, & \therefore & 3 = \log_{10} 1000; \\ 10^{-2} &= 0.01, & \therefore & -2 = \log_{10} 0.01; \\ 3^{\frac{1}{2}} &= \sqrt{3}, & \therefore & \frac{1}{2} = \log_3 \sqrt{3}. \end{aligned}$$

Since  $1^x = 1$  for all values of  $x$ , 1 cannot be used as a base for logarithms. Also a negative number is not used as base; for many real numbers would have imaginary logarithms to a negative base. For example, if  $(-3)^x = 27$ ,  $x$  is imaginary. Although any positive number different from 1 might be used as a base, 10 is often chosen for reasons that will appear as our study continues.

## EXERCISES

Write each of the following exponential equations as a logarithmic equation:

1.  $2^4 = 16.$

4.  $(\frac{1}{2})^{-2} = 4.$

7.  $25^{-\frac{1}{2}} = \frac{1}{5}.$

2.  $10^2 = 100.$

5.  $8^{\frac{2}{3}} = 4.$

8.  $10^0 = 1.$

3.  $\sqrt{100} = 10.$

6.  $10^{-2} = 0.01.$

9.  $10^{-3} = 0.001.$

Write each of the following equations as an exponential equation:

10.  $\log_2 8 = 3.$

12.  $\log_7 49 = 2.$

14.  $\log_9 \frac{1}{3} = -\frac{1}{2}.$

11.  $\log_5 1 = 0.$

13.  $\log_{10} 0.1 = -1.$

15.  $\log_9 1 = 0.$

In each of the following exercises, find the value of  $x$ :

16.  $\log_6 x = 2.$

23.  $\log_{10} 100 = x.$

30.  $\log_x 49 = 2.$

17.  $\log_x \frac{1}{4} = 2.$

24.  $\log_2 32 = x.$

31.  $\log_{27} 3 = x.$

18.  $\log_5 25 = x.$

25.  $\log_5 (\frac{1}{5^{\frac{1}{25}}}) = x.$

32.  $\log_2 \left( \frac{1}{\sqrt[3]{16}} \right) = x.$

19.  $\log_x 15 = 1.$

26.  $\log_{10} x = 2.$

33.  $\log_5 x = 1.$

20.  $\log_2 x = 3.$

27.  $\log_{10} x = -2.$

34.  $\log_b x = 1.$

21.  $\log_2 x = -2.$

28.  $\log_x 3 = -\frac{1}{2}.$

35.  $\log_x (\frac{1}{9}) = 2.$

22.  $\log_4 x = -\frac{1}{2}.$

29.  $\log_x 49 = -2.$

36.  $\log_b x = 0.$

Show that

37.  $(\log_b a)(\log_a b) = 1.$

38.  $(\log_b a)(\log_c b)(\log_a c) = 1.$

39.  $\log_b \left( \frac{1}{b} \right) = -1.$

40. Why cannot unity be used as a base for a system of logarithms?

41. Why cannot a negative number be used as a base for a system of logarithms?

**4. Laws of logarithms.** There are three fundamental laws of logarithms with which the student must be thoroughly familiar. These laws are easily derived from the laws of exponents.

*I. The logarithm of the product of two numbers is equal to the sum of the logarithms of the factors.*

*Proof.* Let  $M$  and  $N$  be any two positive numbers, and let

$$x = \log_b N, \quad \text{and} \quad y = \log_b M. \quad (2)$$

Then we may write

$$b^x = N, \quad \text{and} \quad b^y = M. \quad (3)$$

Multiplying member by member the first of equations (3) by the second, we get

$$b^x b^y = b^{x+y} = MN, \quad \text{or} \quad \log_b MN = x + y. \quad (4)$$

Substituting the values of  $x$  and  $y$  from (2) in (4), we get

$$\log_b MN = \log_b M + \log_b N.$$

By repeated application of the first law it is readily proved that *the logarithm of the product of any finite number of factors is equal to the sum of the logarithms of the factors.*

II. *The logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.*

*Proof.* Dividing member by member the first of equations (3) by the second, we get

$$\frac{N}{M} = \frac{b^x}{b^y} = b^{x-y}, \quad \text{or} \quad \log_b \frac{N}{M} = x - y. \quad (5)$$

Substituting the values of  $x$  and  $y$  from (2) in (5), we get

$$\log_b \frac{N}{M} = \log_b N - \log_b M.$$

III. *The logarithm of a number affected by an exponent is the product of the exponent and the logarithm of the number.*

*Proof.* Let

$$x = \log_b N, \quad \text{or} \quad N = b^x. \quad (6)$$

Raising both members of  $N = b^x$  to the  $p$ th power, we obtain

$$N^p = b^{px},$$

Therefore, in accordance with (1)

$$\log_b N^p = px. \quad (7)$$

Substitution of the value of  $x$  from (6) in (7) gives

$$\log_b N^p = p \log_b N.$$

**Example 1.** Find the value of  $\log_{10} \sqrt{0.001}$ .

*Solution.*  $\log_{10} \sqrt{0.001} = \log_{10} (0.001)^{\frac{1}{2}} = \frac{1}{2} \log_{10} 0.001$   
 $= \frac{1}{2} \log_{10} \frac{1}{1000} = \frac{1}{2}(-3) = -\frac{3}{2}.$

**Example 2.** Write  $\log_b \sqrt[3]{\frac{a^2(c+d)^{\frac{1}{2}}}{c^5}}$  in expanded form.

$$\begin{aligned} \text{Solution. } \log_b \sqrt[3]{\frac{a^2(c+d)^{\frac{1}{2}}}{c^5}} &= \frac{1}{3} \log_b \frac{a^2(c+d)^{\frac{1}{2}}}{c^5} \\ &= \frac{1}{3} [\log_b a^2 + \log_b (c+d)^{\frac{1}{2}} - \log_b c^5] \\ &= \frac{1}{3} [2 \log_b a + \frac{1}{2} \log_b (c+d) - 5 \log_b c]. \end{aligned}$$

**Example 3.** Write  $\frac{3}{2} \log_b (x+1) + \frac{1}{3} \log_b x - 2 \log_b (x^2+1)$  in contracted form.

$$\begin{aligned} \text{Solution. } \frac{3}{2} \log_b (x+1) + \frac{1}{3} \log_b x - 2 \log_b (x^2+1) \\ &= \log_b (x+1)^{\frac{3}{2}} + \log_b x^{\frac{1}{3}} - \log_b (x^2+1)^2 \\ &= \log_b \frac{(x+1)^{\frac{3}{2}} x^{\frac{1}{3}}}{(x^2+1)^2}. \end{aligned}$$

Another form of the answer is found as follows:

$$\log_b \frac{(x+1)^{\frac{3}{2}} x^{\frac{1}{3}}}{(x^2+1)^2} = \log_b \left[ \frac{(x+1)^9 x^2}{(x^2+1)^{12}} \right]^{\frac{1}{6}} = \frac{1}{6} \log_b \frac{(x+1)^9 x^2}{(x^2+1)^{12}}.$$

### EXERCISES

1. Verify the following:

- $\log_{10} \sqrt{1000} + \log_{10} \sqrt{0.1} = 1.$
- $\log_2 \sqrt{8} + \log_2 \sqrt{2} = 2.$
- $\log_8 (2)^5 + \log_7 (\frac{1}{49})^{\frac{1}{2}} = 1.$
- $\log_2 \sqrt{8} + \log_3 (\frac{1}{3})^2 = -\frac{1}{2}.$
- $\log_5 \sqrt{125} + \log_{13} \sqrt[3]{169} = \frac{13}{6}.$
- $\log_{11} \frac{1}{11} + 2 \log_{11} \sqrt{11} = 0.$
- $\log_2 (0.5)^3 - \log_4 \sqrt[6]{64} = -\frac{7}{2}.$
- $\log_5 1 - \log_7 6^0 = 0.$
- $\log_{10} 10^5 - \log_{10} 10^2 + \log_{10} 10^{-2} + \log_{10} 1 = 1.$

2. Write the following logarithmic expressions in expanded form:

- $\log_b \frac{a^2 b^{\frac{1}{2}}}{c^3}.$
- $\log_b \left( \frac{a^3 b^6}{c^2} \right)^{\frac{1}{2}}.$
- $\log_b \sqrt[5]{\frac{a^{\frac{1}{2}} c^{\frac{5}{2}}}{d^7}}.$
- $\log_b P(1+r)^n.$
- $\log_b \frac{a^3 c d^5}{7 \sqrt[4]{e}}.$
- $\log_b \frac{\sqrt[3]{x(x-y)}}{z(x+y)}.$
- $\log_b \frac{\sqrt[3]{p^2(1-q)}}{p^{\frac{1}{2}}(1+q)}.$
- $\log_b \frac{[\sqrt{p-1}]^3}{q^2}.$
- $\log_b \left[ \frac{(p^0 - 5)^{\frac{1}{2}}}{(p-7)^2} \right]^5.$
- $\log_b \frac{(x+g)x^2}{\sqrt{x-y(z+y)}}.$
- $\log_b \frac{a(c-d)^2}{6(a+f)}.$
- $\log_b \sqrt[5]{\left[ \frac{a^2(c-d)^3}{c\sqrt{a-d}} \right]^2}.$

3. Write the following expressions in contracted form.

- (a)  $\log_b a + 2 \log_b c - \frac{1}{2} \log_b d$ .
- (b)  $\frac{1}{2} \log_b a - 3 \log_b c - 4 \log_b (a + c)$ .
- (c)  $\frac{1}{2} \log_b (a + c) + \frac{1}{2} \log_b (a - c)$ .
- (d)  $\log_b 3c - \frac{4}{3} \log_b d + \log_b e$ .
- (e)  $\frac{1}{3} [\log_b a + 2 \log_b (c - d) - 4 \log_b c - \frac{1}{3} \log_b (2 - a)]$ .
- (f)  $5[\frac{1}{2} \log_b (a - c) + \log_b (a + d) - 6 \log_b d - 2 \log_b a]$ .

4. Take from a five-place table the following logarithms:

$$\log_{10} 2 = 0.30103, \log_{10} 3 = 0.47712, \log_{10} 7 = 0.84510.$$

From these numbers find  $\log_{10} 4$ ,  $\log_{10} 9$ ,  $\log_{10} 28$ ,  $\log_{10} 32$ ,  $\log_{10} \frac{4}{3}$ ,  $\log_{10} \frac{3}{4}$ .

5. Using the logarithms in Exercise 4, find  $\log_{10} \frac{2}{3}$ ,  $\log_{10} \frac{3}{2}$ ,  $\log_{10} 343$ ,  $\log_{10} \sqrt{2}$ ,  $\log_{10} \sqrt[3]{7}$ ,  $\log_{10} 5$ .

6. Using the logarithms in Exercise 4, find the value of the logarithm of each of the following expressions:

- |                              |   |
|------------------------------|---|
| (a) $\frac{(2)(5)}{3}$ .     | (d) $\sqrt{\frac{(30)(21)}{8}}$ .                         |
| (b) $\frac{(10)(6)}{7}$ .    | (e) $\sqrt[5]{\frac{(6)(4)(7)^{\frac{1}{2}}}{28}}$ .      |
| (c) $\frac{(3)(9)(5)}{14}$ . | (f) $\frac{(9)^{\frac{1}{2}}(12)(4)^{\frac{1}{3}}}{35}$ . |

**5. Common logarithms. Characteristic.** In computation, it is convenient and customary to employ logarithms to the base 10. Logarithms to this base are called *common logarithms*. Throughout this text we shall use common logarithms only, and we shall write  $\log N$  as an abbreviation of  $\log_{10} N$ . Thus when the base is omitted it will be understood that the base is 10.

In this system of logarithms, the logarithm of any integral power of 10 is an integer, while the logarithm of any positive number not an integral power of 10 may be written as an integer plus a decimal. In general, the logarithm of a number consists of two parts, an integer called the *characteristic*, and a decimal called the *mantissa*. The characteristic is found by inspection; the mantissa is found from a table. We shall now deduce rules for finding the characteristic.

Consider the following table:

$10^5 = 100,000$	or	$\log 100,000 = 5,$
$10^4 = 10,000$	or	$\log 10,000 = 4,$
$10^3 = 1000$	or	$\log 1000 = 3,$
$10^2 = 100$	or	$\log 100 = 2,$
$10^1 = 10$	or	$\log 10 = 1,$
$10^0 = 1$	or	$\log 1 = 0,$
$10^{-1} = 0.1$	or	$\log 0.1 = -1,$
$10^{-2} = 0.01$	or	$\log 0.01 = -2,$
$10^{-3} = 0.001$	or	$\log 0.001 = -3,$
$10^{-4} = 0.0001$	or	$\log 0.0001 = -4,$
$10^{-5} = 0.00001$	or	$\log 0.00001 = -5.$

From the foregoing table, we get by inspection the following information:

Number	Number of digits to left of decimal point	Logarithm	Characteristic
$1 < N < 10$	1	0 + a decimal	0
$10 < N < 100$	2	1 + a decimal	1
$100 < N < 1000$	3	2 + a decimal	2
$1000 < N < 10,000$	4	3 + a decimal	3
$10^n < N < 10^{n+1}$	$n + 1$	$n + \text{a decimal}$	$n$

From the data just tabulated, we infer the following rule:

**Rule 1.** *The characteristic of the common logarithm of a number greater than 1 is positive and is one less than the number of digits to the left of the decimal point.*

Similarly, we get

Number	Number of zeros to right of decimal point	Logarithm	Characteristic
$0.1 < N < 1$	0	$-1 + \text{a decimal}$	$-1 \text{ or } 9 - 10$
$0.01 < N < 0.1$	1	$-2 + \text{a decimal}$	$-2 \text{ or } 8 - 10$
$0.001 < N < 0.01$	2	$-3 + \text{a decimal}$	$-3 \text{ or } 7 - 10$
$10^{-n} < N < 10^{-(n-1)}$	$n - 1$	$-n + \text{a decimal}$	$-n \text{ or } (10 - n) - 10$

From the tabulated data, we infer the following rule:

**Rule 2.** *The characteristic of the common logarithm of a positive number less than 1 is negative and is numerically one greater than the number of zeros immediately following the decimal point.*

When the characteristic is negative, it is convenient to add 10 to the characteristic and subtract 10 at the right of the mantissa. Thus  $\log 0.02545 = -2 + \text{a decimal} = 8 + \text{a decimal} - 10$ . In general, if the characteristic  $-n$  of  $\log N$  is negative, change it to the equivalent value  $(10 - n) - 10$ , or  $(20 - n) - 20$ , etc. *To obtain directly the characteristic of the logarithm of a number less than 1, subtract from 9 the number of zeros immediately following the decimal point; write the result before the mantissa and  $-10$  after it.*

*Illustrations:*

Number	Characteristic	Rule
4261	3	1
3.6121	0	1
0.1210	-1 or 9 - 10	2
0.0025	-3 or 7 - 10	2
0.00000345	-6 or 4 - 10	2

### EXERCISES

Write the characteristic of the logarithm of each number:

- |             |             |                |                |
|-------------|-------------|----------------|----------------|
| 1. 7.613.   | 5. 761.3.   | 9. 89,261.     | 13. 3101.      |
| 2. 467,916. | 6. 31.12.   | 10. 412.16.    | 14. 14,481.10. |
| 3. 20.02.   | 7. 0.0371.  | 11. 0.0000309. | 15. 0.30001.   |
| 4. 3.00008. | 8. 0.81219. | 12. 0.003872.  | 16. 0.000810.  |

**6. Effect of changing the decimal point in a number.** Any number may be written in the form  $N \times 10^k$ , where  $N$  is a number between 1 and 10 and  $k$  is an integer. Thus we may write  $1,782,500 = 1.7825 \times 10^6$ ,  $17825 = 1.7825 \times 10^4$ . Evidently a shift of the decimal point appears in this notation as a change in  $k$ . Now  $\log [N \times 10^k] = \log N + k \times 1$ . Since a shift of the decimal point changes  $k$ , but not  $\log N$ , it appears that the mantissa of  $\log N$  is not affected by the position of the decimal point. In other words, a change in the position of the decimal



point in a given sequence of figures has no effect on the mantissa; its sole effect is to change the characteristic. Because of this fact, 10 affords a particularly convenient base for a system of logarithms to be used for purposes of computation.

**7. The mantissa.** Mantissas can be computed by use of advanced mathematics and, except in special cases, are unending decimal fractions. Computed mantissas are tabulated in tables of logarithms, also called tables of mantissas. These tables are called "three-place," "four-place," "five-place," etc., according as the mantissas tabulated contain 3, 4, 5, etc., significant figures. The choice of a table of logarithms should depend upon the degree of accuracy required and the accuracy of the data. In this text we shall discuss and use a five-place table, thus obtaining results accurate to five significant figures.

**8. To find the logarithm of a number.** In general, a five-place table of logarithms gives the mantissas of all integral numbers lying between 999 and 10,000. The first three digits of the numbers are found in the left-hand column headed *N*, and the fourth digit is in the row at the top of the page. Therefore the mantissa of a number with four significant figures is in the row with the first three significant figures of the number and in the column headed by the fourth.

**Example 1.** Find  $\log 42.43$ .

*Solution.* By the rule in §5, the characteristic is found to be 1. To find the mantissa, first find 424 in the left-hand column headed *N*, then follow the row containing 424 until the column headed by 3 is reached. Here we find 62767. Therefore the mantissa is 0.62767. Hence

$$\log 42.43 = 1.62767.$$

**Example 2.** Find  $\log 0.0416$ .

*Solution.* By the rule in §5, the characteristic is found to be 8. -10. Using 4160, we find the mantissa to be 0.61909. Therefore

$$\log 0.0416 = 8.61909 - 10.$$

## EXERCISES

Verify the following:

- |                             |                                      |
|-----------------------------|--------------------------------------|
| 1. $\log 2934 = 3.46746$ .  | 6. $\log 0.3132 = 9.49582 - 10$ .    |
| 2. $\log 3.478 = 0.54133$ . | 7. $\log 0.0003146 = 6.49776 - 10$ . |
| 3. $\log 28.7 = 1.45788$ .  | 8. $\log 0.03426 = 8.53479 - 10$ .   |
| 4. $\log 1.817 = 0.25935$ . | 9. $\log 0.272 = 9.43457 - 10$ .     |
| 5. $\log 981.7 = 2.99198$ . | 10. $\log 0.005075 = 7.70544 - 10$ . |

**9. Interpolation.** From the five-place table of logarithms we cannot obtain directly the logarithm of a number with five significant figures. However, by a process known as interpolation, we can find the mantissa of a number having a fifth significant figure. In this process we use the principle of proportional parts, which states that, for small changes in  $N$ , the corresponding changes in  $\log N$  are proportional to the changes in  $N$ . Although this principle is not strictly true, it is sufficiently accurate to lead to results correct to the number of figures given in the table.

The process of interpolation is illustrated by means of the following example:

**Example.** Find  $\log 235.47$ .

*Solution.* From the table of logarithms we find the logarithms in the following form and then compute the differences exhibited.

$$\left. \begin{array}{l} \log 235.40 \\ \log 235.47 \\ \log 235.50 \end{array} \right\} 7 \left\} \begin{array}{l} = 2.37181 \\ 10 = ? \\ = 2.37199 \end{array} \right\} d \left\} 18 \text{ (tabular difference)}$$

By the principle of proportional parts, we have

$$\frac{7}{10} = \frac{d}{18} \quad \text{or} \quad d = \left( \frac{7}{10} \right) (18) = 13 \text{ (nearly).}$$

We add  $d = 13$  to the last two figures of 2.37181 to obtain

$$\log 235.47 = \mathbf{2.37194}.$$

Notice that the value used for  $d$  was 13 instead of 12.6 because the table of logarithms is accurate only to five decimal places.

In order to save work in interpolating when finding the mantissas of five-place numbers, each tabular difference occurring in the table has been multiplied by 0.1, 0.2, . . . 0.9, and the results placed on the right-hand sides of the pages where these tabular differences occur. These tabulated results, called tables of proportional parts (P.P.), are headed by the tabular difference for which they have been formed, and the decimal points have been omitted. To interpolate in the example just solved, we locate the proportional parts table headed 18, and opposite 7 in the left-hand column we find  $d = 13$ .

### EXERCISES

Find the logarithm of each of the following:

- |             |                 |
|-------------|-----------------|
| 1. 40.488.  | 6. 0.0038345.   |
| 2. 3.0473.  | 7. 0.086452.    |
| 3. 10,201.  | 8. 0.000076123. |
| 4. 108.17.  | 9. 0.027038.    |
| 5. 0.21544. | 10. 0.18253.    |

**10. To find the number corresponding to a given logarithm.** Generally in every problem involving logarithms, it is necessary not only to find the logarithms of numbers but also to perform the inverse process, that of finding a number corresponding to a given logarithm.

If  $\log N = L$ , then  $N$  is the number corresponding to the logarithm  $L$ . The number  $N$  is called the *antilogarithm* of  $L$ . To find the antilogarithm  $N$  of the logarithm  $L$ , first use the given mantissa to find the sequence of figures in  $N$ , and then use the given characteristic to place the decimal point so as to agree with the rule of §5.

**Example.** Given  $\log N = 1.60334$ , find  $N$ .

*Solution.* The mantissa .60334 is not found exactly in the table, but we find the two successive mantissas .60325 and .60336, between which the given mantissa lies. From the table we find the numbers in the following form and then compute the differences exhibited.

$$\begin{array}{rcl} 1.60325 & \left. \vphantom{\begin{array}{l} 1.60325 \\ 1.60334 \\ 1.60336 \end{array}} \right\} 9 & \left. \vphantom{\begin{array}{l} = \log 40.110 \\ = \log N \\ = \log 40.120 \end{array}} \right\} x \\ 1.60334 & \left. \vphantom{\begin{array}{l} 1.60325 \\ 1.60334 \\ 1.60336 \end{array}} \right\} 11 & \left. \vphantom{\begin{array}{l} = \log 40.110 \\ = \log N \\ = \log 40.120 \end{array}} \right\} 10 \\ 1.60336 & \left. \vphantom{\begin{array}{l} 1.60325 \\ 1.60334 \\ 1.60336 \end{array}} \right\} & \left. \vphantom{\begin{array}{l} = \log 40.110 \\ = \log N \\ = \log 40.120 \end{array}} \right\} \end{array}$$

By the principle of proportional parts, we have

$$\frac{x}{10} = \frac{9}{11}, \quad \text{or} \quad x = \frac{(9)(10)}{11} = 8 \text{ (nearly).}$$

We add  $x = 8$  to the last figure of 40.110 to obtain

$$N = 40.118.$$

This interpolation should be performed by means of the table of proportional parts. In the P.P. column under the block corresponding to the tabular difference 11, we find the difference 9; immediately to the left of this we find 8, the fifth significant figure in the number  $N$ .

### EXERCISES

Find  $x$  in each of the following:

- |                             |                              |
|-----------------------------|------------------------------|
| 1. $\log x = 8.66200 - 10.$ | 6. $\log x = 2.99876.$       |
| 2. $\log x = 3.89779.$      | 7. $\log x = 0.87484.$       |
| 3. $\log x = 5.31664.$      | 8. $\log x = 0.42239.$       |
| 4. $\log x = 9.70000 - 10.$ | 9. $\log x = 1.11240.$       |
| 5. $\log x = 7.97295 - 10.$ | 10. $\log x = 6.54782 - 10.$ |

11. Find  $x$  in each of the following:

- |                          |                          |
|--------------------------|--------------------------|
| (a) $\log x = -0.34345.$ | (c) $\log x = -3.12864.$ |
| (b) $\log x = -2.41325.$ | (d) $\log x = -0.16132.$ |

**11. The use of logarithms in computations.** The following examples will illustrate how logarithms are used.

**Example 1.** Evaluate  $(461)(4.321).$

*Solution.* Denoting the product by  $x$ , we may write

$$x = (461)(4.321).$$

Equating the logarithms of the two members of this equation, we get

$$\log x = \log 461 + \log 4.321.$$

Looking up the logarithms of the numbers, we obtain

$$\begin{aligned} \log 461 &= 2.66370 \\ \log 4.321 &= \underline{0.63558} \end{aligned}$$

Adding, we have

$$\log x = 3.29928.$$

The antilogarithm of 3.29928, is

$$x = 1992.0.$$

**Example 2.** Evaluate  $\frac{(217)(3.18)}{62.142}$ .

*Solution.* Let  $x = \frac{(217)(3.18)}{62.142}$ .

Then  $\log x = \log 217 + \log 3.18 - \log 62.142$ .

$$\log 217 = 2.33646$$

$$\log 3.18 = 0.50243$$

$$\text{Sum} = 2.83889$$

$$\log 62.142 = 1.79338$$

Subtracting, we obtain  $\log x = 1.04551$

The antilogarithm of 1.04551 is

$$x = 11.105.$$

**Example 3.** Evaluate  $(2.713)^3$ .

*Solution.* Let  $x = (2.713)^3$ . Then

$$\log x = 3 \log 2.713 = 3(0.43345) = 1.30035.$$

$$\therefore x = 19.969.$$

**Example 4.** Evaluate  $\sqrt[3]{0.7214}$ .

*Solution.* Let  $x = \sqrt[3]{0.7214} = (0.7214)^{\frac{1}{3}}$ . Then

$$\log x = \frac{1}{3} \log 0.7214 = \frac{1}{3}(9.85818 - 10).$$

If we should divide this logarithm by 3, the characteristic of the resulting logarithm would not be in the standard form. Hence we first add 20 and then subtract 20, writing the logarithm in the form  $29.85818 - 30$ . Then we write

$$\begin{array}{r} 3)29.85818 - 30 \\ \hline \end{array}$$

Dividing, we get  $\log x = 9.95273 - 10$

or  $x = 0.89688.$

## EXERCISES

Evaluate the following:

1.  $52,564 \times 0.0082546$ .
2.  $\frac{0.0031593 \times 684.82}{0.0096548}$ .
3.  $(1.045)^{24}$ .
4.  $7\frac{1}{2}$ .
5.  $(0.03628)^{\frac{1}{5}}$ .
6.  $\sqrt[11]{(442.84)^3}$ .
7.  $(33.982)^{-\frac{1}{2}}$ .
8.  $\frac{75,859 \times 0.0028242}{37,568 \times 0.09185}$ .

**12. Cologarithms.** Subtracting a first number from a second is equivalent to adding the negative of the first to the second. Hence, to avoid subtraction in dealing with logarithms, we introduce cologarithms.

*The cologarithm of a number is the negative of its logarithm.* Therefore adding the cologarithm of a number is equivalent to subtracting its logarithm.

To avoid negative mantissas, the cologarithm of a number  $n$ , written  $\text{colog } n$ , is found by using the form  $\text{colog } n = 10 - \log n - 10$ . Thus  $\text{colog } 2 = 10 - \log 2 - 10 = 10 - 0.30103 - 10 = 9.69897 - 10$ , and  $\text{colog } 0.3 = 10 - (9.47712 - 10) - 10 = 0.52288$ . The student will find it convenient in getting  $\text{colog } n$  to *begin at the left of  $\log n$ , subtract each of its digits from 9 except the last significant one, and subtract that from 10.*

The following example will illustrate the use of cologarithms.

**Example.** Find  $x$  if  $x = \frac{342.10}{(6710)(0.31820)}$ .

*Solution.*  $\log x = \log 342.10 - \log 6710 - \log 0.31820$   
 $= \log 342.10 + \text{colog } 6710 + \text{colog } 0.31820$

	$\log 342.10 = 2.53415$
$\log 6710 = 3.82672,$	$\text{colog } 6710 = 6.17328 - 10$
$\log 0.31820 = 9.50270 - 10,$	$\text{colog } 0.31820 = 0.49730$
	$\log x = 9.20473 - 10$

and  $x = 0.16023$ .

## EXERCISES

1. Verify the following:

- (a)  $\text{colog } 179.82 = 7.74516 - 10$ .
- (b)  $\text{colog } 0.63273 = 0.19878$ .
- (c)  $\text{colog } 7.5328 = 9.12304 - 10$ .
- (d)  $\text{colog } 23.975 = 8.62024 - 10$ .

2. Using cologarithms, find the value of

$$(a) \frac{36.21}{7.215} \quad (b) \frac{42.21}{0.2861} \quad (c) \frac{41.262}{(61.84)(1612)} \quad (d) \frac{142.3}{0.02813}$$

**13. Computation by logarithms.** In solving complicated problems, the computer is helped materially by a good form. The one discussed below has the advantages of simplicity, completeness of record, and brevity. It is practically self-explanatory since the main feature consists in reference of every function on a line to the first number in the line; a complete record of logarithms and operations is tabulated, and little writing is required. Since the outline of the form can always be made in advance, the student should first make this outline and then perform the computation without interruption. Speed and accuracy are gained by this method.

The form will be used in the following solution.

**Example 1.** Find  $x$  if  $x = \frac{a^{\frac{1}{3}} \sqrt[5]{bc^2}}{dc^4}$  and  $a = 8.1632$ ,  $b = 729.77$ ,  $c = 0.46330$ ,  $d = 5.2133$ ,  $e = 0.32411$ .

*Solution.* First write the formula

$$\log x = \frac{1}{3} \log a + \frac{1}{5} \log b + 2 \log c + \text{colog } d + 4 \text{ colog } e.$$

The following form contains the solution:

$a = 8.1632$	$\log a = 0.91186$	$\frac{1}{3} \log a = 0.30395$
$b = 729.77$	$\log b = 2.86318$	$\frac{1}{5} \log b = 0.57264$
$c = 0.04633$	$\log c = 8.66586 - 10$	$2 \log c = 7.33172 - 10$
$d = 5.2133$	$\log d = 0.71711$	$\text{colog } d = 9.28289 - 10$
$e = 0.32411$	$\log e = 9.51069 - 10$	$4 \text{ colog } e = 1.95724$
$x = 0.28083$		$\log x = 9.44844 - 10$

Note that each number in any line relates to the first number in the line, and the relation is indicated that the record of operations is complete, that little writing is required, and that an examiner could easily perceive and follow the steps taken.

In the following solution a form is indicated, but the computation is left as in exercise to the student.

**Example 2.** Find  $x$  if  $x = \left[ \frac{\sqrt{c} \times a^2}{a + \sqrt{e}} \right]^{\frac{1}{3}}$  where  $a = 61.214$ ,

$c = 12.112$ , and  $e = 139.02$ .

*Solution.* First we write the formula

$$\log x = \frac{1}{3} \left[ \frac{1}{2} \log c + 2 \log a + \operatorname{colog} (a + \sqrt{e}) \right]$$

and then make the following form:

$e = 139.02$	$\log e =$	$\frac{1}{2} \log e =$	
$\sqrt{e} =$		$\log \sqrt{e} =$	
$a = 61.214$	$\log a =$		
$a + \sqrt{e} =$	$\log (a + \sqrt{e}) =$		$\frac{2 \log a}{\operatorname{colog} (a + \sqrt{e})} =$
$c = 12.112$	$\log c =$		$\frac{\frac{1}{2} \log c}{3) \quad \quad \quad} =$
$x =$			$\log x =$

The student should perform the computation to obtain  $x = 5.6319$ .

### EXERCISES

Make a form or outline for computing each of the following:

- |   |   |
|---|---|
| <p>1. <math>\frac{(32.861)^2(3.1416)^{\frac{1}{3}}}{(62.181)^3}</math>.</p> | <p>3. <math>\left[ \frac{a^2 b^3 c^{\frac{1}{2}}}{d^5 e} \right]^2</math>.</p>  |
| <p>2. <math>\sqrt[3]{\frac{(31.64)^2(62.12)}{(9.31)^5}}</math>.</p>         | <p>4. <math>\sqrt[5]{\frac{a^2 \sqrt{b} \sqrt[3]{c}}{d^3 \sqrt{e}}}</math>.</p> |

### 14. Remarks on computation by logarithms.

(a) When interpolating, do not carry logarithms beyond the number of decimal places given in the table used.

(b) When evaluating an expression containing negative numbers, use logarithms to compute desired positive components, and then combine the results with appropriate signs. In this text a symbol  $(-)$  before a logarithm will indicate that a negative number is under consideration; thus if  $\log x = (-)9.87123 - 10$ ,  $x = -0.74342$ .\*

(c) Make a form like that of Example 1, §13, before beginning computation.

(d) Strive for accuracy in computation. Speed comes with practice.

\* This does not mean that a negative number has a real logarithm. The minus symbols serve merely to keep a record of the signs involved in the given expression.



**Example.** Find the value of  $x$  if  $x = \sqrt[5]{\frac{(-47.123)^2(-36.184)^{\frac{1}{2}}}{\sqrt{31.118}}}$ .

*Solution.*

$$\log (-x) = \frac{1}{5}[2 \log 47.123 + \frac{1}{2} \log 36.184 + \frac{1}{2} \operatorname{colog} 31.118].$$

$a = -47.123$	$\log a = (-)1.67324$	$2 \log a = 3.34648$
$b = -36.184$	$\log b = (-)1.55852$	$\frac{1}{2} \log b = (-)0.51951$
$c = 31.118$	$\log c = 1.49301$	$\frac{1}{2} \operatorname{colog} c = 9.25350 - 10$
		<div style="text-align: right;">5)    (-)3.11949</div>
$x = -4.2063$		<div style="text-align: right;">log <math>x = (-)0.62390</math></div>

### EXERCISES

Find by use of logarithms the results of the following exercises. In each case make a complete outline or form before using the tables.

- |  |   |
|--|---|
| <p>1. <math>3.1416 \times 2.7183</math>.</p> <p>2. <math>29.572 \times 0.00036841</math>.</p> <p>3. <math>335,000,000 \times 0.000099854</math>.</p> <p>4. <math>2727.5 \times 0.37375</math>.</p> <p>5. <math>1487 \times 3.139 \times 42.96</math>.</p> <p>6. <math>\frac{2.9275 \times 34.278}{505.92}</math>.</p> <p>7. <math>\frac{48.962 \times 39.595}{78.545}</math>.</p> <p>8. <math>\frac{2964.5 \times 38.423}{75.65 \times 84.384}</math>.</p> <p>9. <math>\frac{2954.5 \times 64.532}{911.36 \times 318.5}</math>.</p> <p>10. <math>\frac{26.893 \times 0.0000545}{319.62 \times 0.00068432}</math>.</p> <p>11. <math>(1.5)^{15}</math>.</p> <p>12. <math>\sqrt[3]{31}</math>.</p> <p>24. <math>[(-8.90172)(732.95)^{\frac{1}{2}}(0.0954)^{\frac{2}{3}}]^2</math>.</p> <p>25. <math>\sqrt{(27.5)^2 - (3.483)^2}</math>.</p> | <p>13. <math>\sqrt{347.3}</math>.</p> <p>14. <math>\sqrt[3]{0.17638 \times 2.1279}</math>.</p> <p>15. <math>\left[\frac{19.876}{38.345}\right]^2</math>.</p> <p>16. <math>(0.00062584)^{\frac{1}{3}}</math>.</p> <p>17. <math>(665.35)^{-\frac{1}{2}}</math>.</p> <p>18. <math>\sqrt{\frac{(57.45)(423.34)}{(178)(89)}}</math>.</p> <p>19. <math>\frac{(-80,941)\sqrt[5]{-0.031}}{(54,082)\sqrt[6]{0.0712}}</math>.</p> <p>20. <math>\frac{4 \times 28.7 \times \sqrt{345}}{29 \times 137}</math>.</p> <p>21. <math>\sqrt{(67.811)^2 + (83.314)^2}</math>.</p> <p>22. <math>\sqrt{(7631.25)^2 - (6712.15)^2}</math>.</p> <p>23. <math>\sqrt[3]{\frac{(23.975)(5.793)^2}{179.82}}</math>.</p> <p>26. <math>\frac{5086(-0.0008769)^3}{(9802)(0.001984)^4}</math>.</p> |
|--|---|

\* *Hint.* First factor the radicand.

$$27. \frac{1954.7 \times \sqrt[5]{0.0030121}}{\sqrt[4]{17,959 \times (0.84132)^8 (560.63)}}.$$

$$28. \frac{(0.04)^{\frac{2}{3}}(0.057897)^{\frac{4}{3}}}{(87.67)^{0.9}}.$$

$$29. \sqrt[4]{\frac{(348.7)^2(-2.685)^3(3.08212)}{(2.678)^{\frac{1}{2}}(0.08216)^4(-800,013)}}.$$

$$30. \sqrt[3]{\frac{(0.002452)^{\frac{1}{2}}(86.47)^3(-128.721)}{(-5280)(-0.07115)^2(-62.472)}}.$$

$$31. \sqrt{\frac{a^{\frac{1}{2}}b}{a^2 - b}}, a = 7.5328, b = 6384.$$

$$32. \sqrt[5]{\frac{b}{a^3}} - \sqrt{a^2c}; a = 735.9, b = 0.198, c = 27.$$

$$33. \frac{a^2c^{\frac{1}{2}}}{bD}; D = a + c^2, a = 23.722, b = 571.17, c = 0.03218.$$

34. Given  $a = 3.7124$ ,  $b = 32.617$ , find  $\log(a + b)$ ,  $\log(a - b)$ ,  $\log \frac{a}{b}$ ,  $\log ab$ .

35. Find  $K$ , given  $s = \frac{1}{2}(a + b + c + d)$ ,

$$K = \sqrt{(s - a)(s - b)(s - c)(s - d)},$$

$a = 6.3246$ ,  $b = 7.7459$ ,  $c = 8.5441$ ,  $d = 5.1961$ .

36.  $\frac{a^3b^2c}{d^{\frac{2}{3}}}$ , given  $a = 0.00275$ ,  $b = 100.5$ ,  $c = 5075.5$ ,  $d = 0.001875$ .

37.  $\left[ \frac{a^5b^3c^2d^{\frac{1}{2}}}{e^2fg^4} \right]^{\frac{1}{3}}$ , given  $a = 301.03$ ,  $b = 0.00036954$ ,  $c = 0.0028182$ ,  
 $d = 35,890,000$ ,  $e = 0.000002814$ ,  $f = 561.29$ ,  $g = 2718.3$ .

38. Find the weight of a steel sphere 1.0127 ft. in diameter if steel weighs 490 lb. per cu. ft.

39. Find the weight of a cube of metal weighing 530 lb. per cu. ft. if the edge of the cube is 1.6271 ft.

40. A conical piece of wood weighs 92 lb. If the area of the base of the solid is 1.3341 sq. ft., find the altitude. (The wood weighs 33 lb. per cu. ft.)

41. During a rain 0.521 in. of water fell. Find how many gallons of water fell on a level 10.7-acre park. (Take 1 cu. ft. = 7.48 gal., 1 acre = 43,560 sq. ft.)

42. The time  $t$  of oscillation of a simple pendulum of length  $l$  ft. is given in seconds by the formula

$$t = \pi \sqrt{\frac{l}{32.16}}.$$

Find the time of oscillation of a pendulum 3.326 ft. long. (Take  $\pi = 3.142$ .)

43. What is the weight in tons of a solid cast-iron sphere whose radius is 5.343 ft. if the weight of 1 cu. ft. of water is 62.355 lb. and the specific gravity of cast iron is 7.154?

44. Find the volume and surface of a sphere of radius 14.71.

45. The stretch of a brass wire when a weight is hung at its free end is given by the relation

$$S = \frac{mgl}{\pi r^2 k},$$

where  $m$  is the weight applied,  $g = 980$ ,  $l$  is the length of the wire,  $r$  is its radius, and  $k$  is a constant. Find  $k$  for the following values:  $m = 944.2$  g.,  $l = 219.2$  cm.,  $r = 0.32$  cm., and  $S = 0.060$  cm.

46. Find the length  $l$  of a wire that stretches 5.9 cm. for a weight of 1826.5 g. hanging at its free end, when the diameter of the wire is 0.064 cm. and  $k = 1.1 \times 10^{12}$ .

47. The weight  $P$  in pounds that will crush a solid cylindrical cast-iron column is given by the formula

$$P = 98,920 \frac{d^{3.55}}{l^{1.7}},$$

where  $d$  is the diameter in inches and  $l$  the length in feet. What weight will crush a cast-iron column 6 ft. long and 4.3 in. in diameter?

48. For wrought-iron columns the crushing weight is given by

$$P = 299,600 \frac{d^{3.55}}{l^2}.$$

What weight will crush a wrought-iron column of the same dimensions as that in Problem 47?

49. The weight  $W$  of 1 cu. ft. of saturated steam depends upon the pressure in the boiler according to the formula

$$W = \frac{P^{0.941}}{330.36},$$

where  $P$  is the pressure in pounds per square inch. What is  $W$  if the pressure is 280 lb. per sq. in.?

**15. Change of base in logarithms.** Occasionally it is necessary to find the logarithm of a number  $N$  to a base  $b$  other than 10. To do this we let

$$\log_b N = x, \quad \text{or} \quad b^x = N.$$

Equating the logarithms to the base 10 of the two members of this equation, we get

$$x \log_{10} b = \log_{10} N, \quad \text{or} \quad x = \frac{\log_{10} N}{\log_{10} b}.$$

Since the divisor and dividend of this fraction are logarithms, they will generally be numbers of several digits. Therefore it is advisable to perform the indicated division by means of logarithms.

**Example.** Find the value of  $\log_3 0.092118$ .

*Solution.* Let  $x = \log_3 0.092118$ . Then  $3^x = 0.092118$ . Equating the logarithms to the base 10 of the two members of this equation, we obtain

$$x \log_{10} 3 = \log_{10} 0.092118$$

or

$$x = \frac{\log_{10} 0.092118}{\log_{10} 3} = \frac{8.96434 - 10}{0.47712} = \frac{-1.03566}{0.47712}.$$

This quotient is evaluated as follows:

$a = -1.0357$	$\log a = (-)0.01523$ $\text{colog } b = 0.32137$ <hr style="border: none; border-top: 1px solid black;"/> $\log x = (-)0.33660$
$b = 0.47712$	
$x = -2.1707$	

**16. Solution of equations of the form  $x = a^b$ ,  $a = x^b$ .** We shall now illustrate the method of solving equations of the form  $x = a^b$ , and  $a = x^b$ , in which  $a$  and  $b$  are given numbers.

**Example 1.** Find  $x$  if  $x = (3.21)^{8.27}$ .

*Solution.*  $\log x = 8.27 \log 3.21 = (8.27)(0.50651)$ .

The solution is displayed below.

$a = 8.27$	$\log a = 0.91751$ $\log b = 9.70459 - 10$ <hr style="border: none; border-top: 1px solid black;"/> $\log (\log x) = 0.62210$
$b = 0.50651$	
$\log x = 4.1889$	

Therefore  $\log x = 4.1889$ , from which we get  $x = \mathbf{15,449}$ .

**Example 2.** Find  $x$  if  $x^{7.2143} = 0.080133$ .

*Solution.* Equate the logarithms of the two members of the given equation and solve for  $\log x$  to obtain

$$7.2143 \log x = \log 0.080133$$

or

$$\log x = \frac{\log 0.080133}{7.2143} = \frac{8.90381 - 10}{7.2143} = \frac{-1.09619}{7.2143}$$

The evaluation of the quotient for  $\log x$  follows:

$a = -1.0962$	$\log a$	$= (-)0.03989$
$b = 7.2143$	$\log b = 0.85820$	$= 9.14180 - 10$
$\log x = -0.15195$	$\log (\log x)$	$= (-)9.18169 - 10$

To make the mantissa of  $\log x$  positive add it to  $10 - 10$  to obtain

$$\log x = 10 - 0.15195 - 10 = 9.84805 - 10.$$

Therefore

$$x = \mathbf{0.70477}.$$

### EXERCISES

- |  |  |
|--|--|
| <p>1. <math>x = \log_7 100</math>.</p> <p>2. <math>x = \log_{0.88} 99,324</math>.</p> <p>3. <math>x = \log_{27} 0.00328</math>.</p> <p>4. <math>x = \log_{0.0954} 87,543</math>.</p> <p>5. <math>x = \log_{20} 100</math>.</p> <p>6. <math>x = \log_8 27,569</math>.</p> <p>7. <math>x = \log_{3.7} 0.8173</math>.</p> <p>8. <math>x = \log_{21} 0.09827</math>.</p> <p>17. Given <math>3^{x+y} = 2(5^x)</math>, <math>x - y = 1</math>, find <math>x</math> and <math>y</math>.</p> <p>18. How long will it take a sum of money to double itself if put at 4 per cent compound interest? This is represented by <math>(1.04)^x = 2</math> where <math>x</math> is the number of years. Solve for <math>x</math>.</p> <p>19. Solve the equation <math>e^x + e^{-x} = y</math>, for <math>x</math> (a) when <math>y = 2</math>, (b) when <math>y = 4</math>. <math>e = 2.7183</math>.</p> | <p>9. <math>5^{\frac{1}{x}} = 1.307</math>.</p> <p>10. <math>5^{2x} = 317.46</math>.</p> <p>11. <math>\log_x 8 = 0.35678</math>.</p> <p>12. <math>\log_x 2 = 0.69315</math>.</p> <p>13. <math>\log_x 0.07936 = 2.983</math>.</p> <p>14. <math>x^{2.892} = 0.07936</math>.</p> <p>15. <math>(1.5)^{\frac{1}{x}} = 32</math>.</p> <p>16. <math>4.02 = (2.37)^{\frac{1}{x+1}}</math>.</p> |
|--|--|

**20.** If fluid friction is used to retard the motion of a flywheel making  $V_0$  revolutions per min., the formula  $V = V_0 e^{-kt}$  gives the number of revolutions per minute after the friction has been applied  $t$  seconds. If the constant  $k = 0.35$ , how long must the friction be applied to reduce the number of revolutions from 200 to 50 per min.?  $e = 2.7183$ .

**21.** The pressure,  $P$ , of the atmosphere in pounds per square inch, at a height of  $z$  ft. is given approximately by the relation

$$P = P_0 e^{-kz},$$

where  $P_0$  is the pressure at sea level and  $k$  is a constant. Observations at sea level give  $P_0 = 14.72$ , and at a height of 1122 ft.,  $P = 14.11$ . What is the value of  $k$ ?

**22.** Assuming the law in Exercise 21 to hold, at what height will the pressure be half as great as at sea level?

**23.** If a body of temperature  $T_1^\circ$  is surrounded by cooler air of temperature  $T_0^\circ$ , the body will gradually become cooler, and its temperature,  $T^\circ$ , after a certain time, say  $t$  min., is given by Newton's law of cooling, that is,

$$T = T_0 + (T_1 - T_0)e^{-kt},$$

where  $k$  is a constant. In an experiment a body of temperature  $55^\circ\text{C}$ . was left to itself in air whose temperature was  $15^\circ\text{C}$ . After 11 min. the temperature was found to be  $25^\circ$ . What is the value of  $k$ ?

**24.** Assuming the value of  $k$  found in Exercise 23, what time will elapse before the temperature of the body drops from  $25^\circ$  to  $20^\circ$ ?

**25.** Solve the equation  $\log_3 (3x + 1) = 2$  for  $x$ .

**26.** Solve the equation  $\log_{10} (x^2 - 21x) = 2$  for  $x$ .

**17. Graph of  $y = \log_{10} x$ .** If we assign values to  $x$  in the equation  $y = \log_{10} x$  and find the corresponding values of  $y$ , we shall obtain the coordinates of points on the curve  $y = \log_{10} x$ . A few of these values are tabulated in the accompanying table. Plotting these points and drawing a smooth curve through

$x$	0.5	1	3	5	8	10	15	20	25	30	35	40
$y$	-0.3	0	0.48	0.70	0.9	1	1.17	1.3	1.4	1.48	1.54	1.6

them, we obtain the graph shown in Fig. 1. For convenience, the unit on the  $y$ -axis has been taken ten times as large as the unit on the  $x$ -axis.

If the student retains a mental picture of this graph, he will find it easy to recall the following facts:

- (a) A negative number has no real number for its logarithm.
- (b) The logarithm of a positive number is negative or positive according as the number is less than or greater than 1.
- (c) If the number  $x$  approaches zero,  $\log x$  decreases without limit.
- (d) If the number  $x$  increases indefinitely,  $\log x$  increases without limit.

In the process of interpolation in logarithms, values are inserted as if the change in the logarithm between the nearest

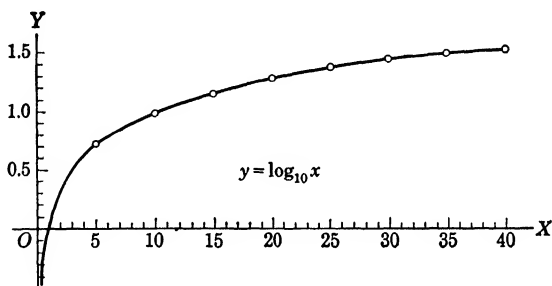


FIG. 1.

tabulated values were directly proportional to the change in the number. This assumes that the graph of  $y = \log x$  for the interval concerned is a straight line. From the graph it is apparent this would be approximately true. In other words, when a number is changed by an amount that is very small in comparison with the number itself, the change in the value of the logarithm of the number is very nearly proportional to the change in the number.

### EXERCISES

1. Plot the graph of  $y = \log_5 x$ .

*Hint.*  $\log_5 x = \frac{\log_{10} x}{\log_{10} 5}$ .

2. Plot the graph of  $x = \log_5 y$ .
3. Plot the graph of  $x = \log_2 y$ .

### 18. MISCELLANEOUS EXERCISES

Find by use of logarithms the results of the following exercises. In each case make a complete outline or form before using the tables.

1.  $3.87 \times 57.6$ .
2.  $7.0928 \times 0.0052683$ .
3.  $22.9 \times 4.95 \times 0.643$ .
4.  $0.0063982 \times 23.473 \times 0.062547$ .
5.  $\frac{76.9}{3.14}$ .
6.  $\frac{1}{0.8236}$ .
7.  $\frac{8.211}{0.6634}$ .
8.  $\frac{49.36 \times 0.7657}{8.439}$ .
9.  $\frac{6.47 \times 12.93 \times 0.2462}{896 \times 0.0074939}$ .
10.  $(0.09245)^3$ .
11.  $\sqrt[6]{0.002855}$ .
12.  $\sqrt[4]{0.0070008}$ .
13.  $(0.935)^{\frac{2}{5}}$ .
14.  $(4.267)^{0.4}$ .
15.  $(19.26)^{1.2}$ .
16.  $\frac{(41.911)^{\frac{5}{4}}}{\sqrt[5]{(3.215)^3 \times 0.78356}}$ .
17.  $\frac{(89.1)^{\frac{2}{3}} \times (0.764)^{0.2}}{\sqrt[4]{0.0387}}$ .
18.  $\frac{(7.9036)^{1.1} \times \sqrt[5]{(0.50267)^3}}{(0.0014123)^{0.9}}$ .
19.  $(-0.091111)^{-\frac{3}{2}}$ .
20.  $\frac{45.86 \times (0.7288)^{\frac{3}{2}}}{(-9.423)^{\frac{5}{3}}}$ .
21.  $\frac{(-0.49173)^{\frac{2}{3}}}{\sqrt[5]{-207.99}}$ .
22.  $\frac{1}{\sqrt[4]{(170.5)^3 - 15}}$ .
23.  $\frac{\sqrt{0.7285} + (2.706)^{\frac{3}{2}}}{318.2 \times (0.06004)^2}$ .
24.  $\frac{(0.8195)^{-0.3} + (0.9713)^{0.4}}{(5.004)^{-\frac{1}{3}}}$ .
25.  $\frac{\log 9.5}{\log 4.27}$ .
26.  $\frac{\log 0.87189}{\log 0.022223}$ .

27. The radius  $r$  of the inscribed circle of a triangle in terms of its sides  $a$ ,  $b$ , and  $c$  is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

where  $s = \frac{1}{2}(a + b + c)$ . Compute  $r$  when (a)  $a = 0.525$ ,  $b = 0.261$ ,  $c = 0.438$ ; (b)  $a = 698.2$ ,  $b = 476.3$ ,  $c = 744.9$ ; (c)  $a = 3.0023$ ,  $b = 2.1128$ ,  $c = 1.5007$ .

28. The number  $n$  of revolutions per minute of a certain water turbine is given by

$$n = \frac{400}{61.3} h^{1.3} P^{-0.4},$$



where  $h$  is the height of fall in feet, and  $P$  is the horsepower developed. Compute  $n$  when  $h = 15$  ft. and  $P = 98$  hp.

29. The formula  $D = \sqrt[3]{\frac{W}{0.5236(A - G)}}$  gives the diameter of a spherical balloon which is to lift a cable of weight  $W$ . Find  $D$  if  $A = 0.0807$ ,  $G = 0.0050$ ,  $W = 1250$ .

30. The amount  $S$  of a principal of  $P$  dollars, interest compounded annually for  $n$  years at the rate  $i$ , is

$$S = P(1 + i)^n.$$

If a war bond sells today for \$75 and will be redeemed in 10 years for \$100, what rate of interest compounded annually will be paid?

*Hint.*  $S = 100$ ,  $P = 75$ ,  $n = 10$ .

31. The range  $R$  on a horizontal plane of a projectile fired at an angle  $\theta$ , with velocity  $v_0$ , is

$$R = \frac{v_0^2 \sin 2\theta}{g}.$$

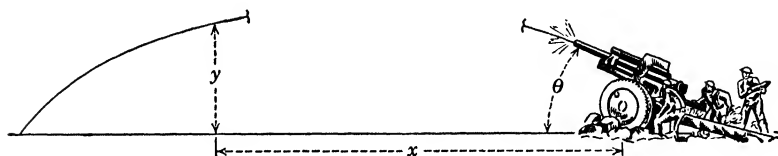
Find the muzzle velocity of a projectile fired at sea whose maximum range is 22.7 miles.

*Hint.*  $R = 22.7 \times 6080$  ft.,  $g = 32.17$  ft. per sec. per sec.,  $\theta = 45^\circ$ .

32. If the height  $y$  in feet of a projectile above a horizontal plane at time  $t$  in seconds is given by the equation

$$y = -16t^2 + 600t,$$

show that its height at  $t = 18.75$  sec. is 5625 ft.



33. If the height  $y$  (see Fig. 2) of a projectile in terms of the horizontal distance  $x$  from the gun is given by

$$y = x \tan \theta - \frac{\frac{1}{2}gx^2}{v_0^2 \cos^2 \theta},$$

where  $\theta$  is the angle of elevation of the gun,  $v_0$  is the initial velocity, and  $g = 32$  ft. per sec. per sec. (approx.), find  $y$  when  $x = 38,970$  ft.,  $\theta = 30^\circ$ ,  $v_0 = 2400$  ft. per sec.

**34.** The expressions

$$x = 104.6t$$

$$y = 6070(1 - e^{-0.0322t}) + 1000t$$

give the horizontal distance  $x$  and the vertical distance  $y$  at time  $t$  of a shell projected from an airplane at an angle of  $85^\circ$  below the horizontal, with an initial velocity of 1200 ft. per sec. Find the position of the shell at the end of 5 sec. (see Fig. 3).

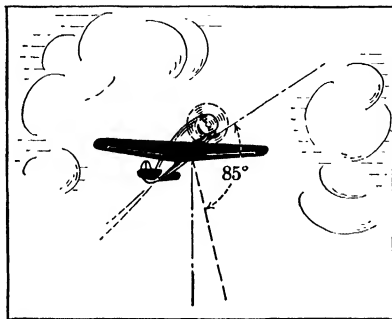


FIG. 3.

**35.** If the air pressure on the ground is 14.7 lb. per sq. in., the pressure  $P$  at height  $h$  ft. is given approximately by

$$P = 14.7e^{-0.0000377h}.$$

Find the air pressure at the height of (a) 10,000 ft., (b) 15,000 ft.

**36.** If the force  $F$  exerted by a parachute on a man of weight  $W$  lb. falling  $v$  ft. per sec. is given by

$$F = \frac{Wv}{15},$$

find the force exerted on a 160-lb. man by a parachute just as it opens if he is then falling at 98 ft. per sec. (see Fig. 4).



FIG. 4.

**37.** When a ship is displaced from its vertical position it makes a complete oscillation by rolling from port to starboard and back in a time  $t$  sec. given by

$$t = 2\sqrt{\frac{r^2}{gm}},$$

where  $g = 32.17$ ,  $r$  is a constant depending on the weight and shape of the ship, and  $m$  is the metacentric height. If  $r = 38.06$  ft.,  $m = 7.874$  ft.,  $g = 32.17$  ft. per sec. per sec., find the time of an oscillation of the ship.

**38.** A plane descending with a speed of 120 miles per hour at an angle of  $20^\circ$  with the horizontal drops a bomb when 700 ft. high (see

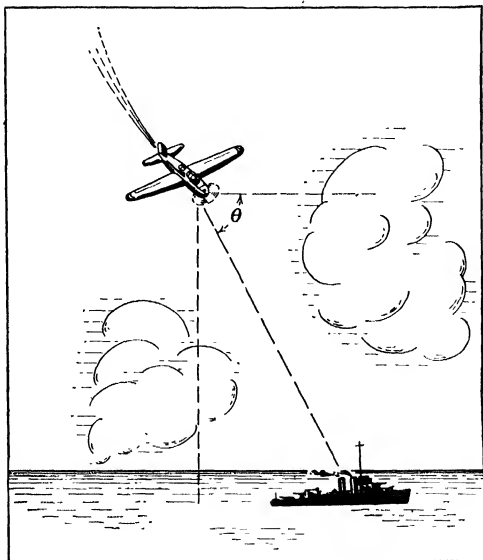


FIG. 5.

Fig. 5). The vertical distance  $y$  and the horizontal distance  $x$  of the bomb from the point of release are given by the equations

$$\begin{aligned} y &= 60.2t + 16.1t^2, \\ x &= 165.4t. \end{aligned}$$

(a) Find the distance the bomb moves horizontally if it strikes the warship shown in the figure in 4.98 sec. (b) Find the angle of depression  $\theta$  of the target as observed by the pilot when releasing the bomb. (c) Find the vertical distance the bomb falls during the first 2.5 sec.

**39.** Find the total time required for a 23.8-knot torpedo to make its maximum run of 12,640 yd. Take 2027 yd. = 1 nautical mile and assume the speed as constant.

**40.** In a certain situation the captain of a warship desired to come as close to an enemy scout as possible. The time in hours required to attain this position is given by the formula

$$\text{Time} = \frac{bc}{a(a^2 - b^2)^{\frac{1}{2}}}$$

where  $c$  = initial distance of the scout from the warship,  $a$  = speed in knots of the scout,  $b$  = speed in knots of warship. Find the time required if  $b = 28.4$  knots,  $a = 32.7$  knots,  $c = 20.9$  nautical miles.

## CHAPTER II

### REVIEW OF PLANE TRIGONOMETRY

**19. Review of definitions and fundamental relations.** Before entering into a study of spherical trigonometry it is well to review briefly the definitions of the trigonometric functions and their fundamental properties.

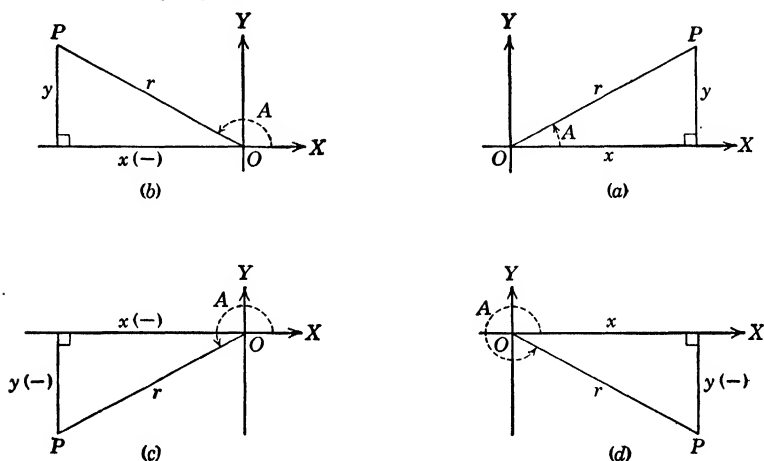


FIG. 1.

*Definitions of the trigonometric functions.* If  $A$  is any angle (see Fig. 1) the trigonometric functions of  $A$  are defined as follows:

$$\left. \begin{aligned} \sin A &= \frac{\text{ordinate}}{\text{distance}} = \frac{y}{r}, & \csc A &= \frac{\text{distance}}{\text{ordinate}} = \frac{r}{y}, \\ \cos A &= \frac{\text{abscissa}}{\text{distance}} = \frac{x}{r}, & \sec A &= \frac{\text{distance}}{\text{abscissa}} = \frac{r}{x}, \\ \tan A &= \frac{\text{ordinate}}{\text{abscissa}} = \frac{y}{x}, & \cot A &= \frac{\text{abscissa}}{\text{ordinate}} = \frac{x}{y}. \end{aligned} \right\} \quad (1)$$

*Signs of the functions.* Observing that  $x$  is negative and that  $y$  and  $r$  are positive in the second quadrant, we see that the  $\sin \theta$  ( $y/r$ ) and  $\csc \theta$  ( $r/y$ ) are positive and the other four trigonometric functions are negative for second-quadrant angles. Similarly,  $x$  and  $y$  are both negative in the third quadrant, so that the tangent ( $y/x$ ) and the cotangent ( $x/y$ ) are both positive, and

the other functions are negative for third-quadrant angles. Finally, in the fourth quadrant,  $x$  and  $r$  are positive, so that the cosine ( $x/r$ ) and the secant ( $r/x$ ) are positive and the other functions are negative for fourth-quadrant angles.

*The fundamental identities.* From the way in which the trigonometric functions of  $A$  are defined it is evident that they are not independent of each other. The student should be familiar with the following fundamental relations:

$$\left. \begin{aligned} \csc A &= \frac{1}{\sin A}, \\ \sec A &= \frac{1}{\cos A}, \\ \cot A &= \frac{1}{\tan A}. \end{aligned} \right\} \quad (2)$$

$$\tan A = \frac{\sin A}{\cos A}, \quad \cot A = \frac{\cos A}{\sin A}. \quad (3)$$

$$\left. \begin{aligned} \sin^2 A + \cos^2 A &= 1, \\ \tan^2 A + 1 &= \sec^2 A, \\ 1 + \cot^2 A &= \csc^2 A. \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \cos (90^\circ - A) &= \sin A, & \sin (90^\circ - A) &= \cos A, \\ \cot (90^\circ - A) &= \tan A, & \tan (90^\circ - A) &= \cot A, \\ \csc (90^\circ - A) &= \sec A, & \sec (90^\circ - A) &= \csc A, \end{aligned} \right\} \quad (5)$$

or, stated in words, any trigonometric function of an acute angle is equal to the co-function of its complement.

$$\left. \begin{aligned} \sin (-A) &= -\sin A, & \csc (-A) &= -\csc A, \\ \cos (-A) &= \cos A, & \sec (-A) &= \sec A, \\ \tan (-A) &= -\tan A, & \cot (-A) &= -\cot A. \end{aligned} \right\} \quad (6)$$

*The addition and subtraction formulas.*

$$\begin{aligned} \sin (A + B) &= \sin A \cos B + \cos A \sin B. \\ \cos (A + B) &= \cos A \cos B - \sin A \sin B. \end{aligned} \quad (7)$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}. \quad (8)$$

$$\begin{aligned} \sin (A - B) &= \sin A \cos B - \cos A \sin B, \\ \cos (A - B) &= \cos A \cos B + \sin A \sin B. \end{aligned} \quad (9)$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}. \quad (10)$$

*Double- and half-angle formulas.*

$$\left. \begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta, \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta, \\ \cos 2\theta &= 2 \cos^2 \theta - 1, \\ \cos 2\theta &= 1 - 2 \sin^2 \theta. \end{aligned} \right\} \quad (11)$$

$$\left. \begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta}, \\ \sin \frac{1}{2}\varphi &= \pm \sqrt{\frac{1 - \cos \varphi}{2}}, \\ \cos \frac{1}{2}\varphi &= \pm \sqrt{\frac{1 + \cos \varphi}{2}}, \\ \tan \frac{1}{2}\varphi &= \pm \sqrt{\frac{1 - \cos \varphi}{1 + \cos \varphi}} = \frac{1 - \cos \varphi}{\sin \varphi}. \end{aligned} \right\} \quad (12)$$

*Conversion formulas.*

$$\left. \begin{aligned} \sin (\theta + \varphi) + \sin (\theta - \varphi) &= 2 \sin \theta \cos \varphi, \\ \sin (\theta + \varphi) - \sin (\theta - \varphi) &= 2 \cos \theta \sin \varphi, \\ \cos (\theta + \varphi) + \cos (\theta - \varphi) &= 2 \cos \theta \cos \varphi, \\ \cos (\theta + \varphi) - \cos (\theta - \varphi) &= -2 \sin \theta \sin \varphi. \end{aligned} \right\} \quad (13)$$

$$\left. \begin{aligned} \sin \alpha + \sin \beta &= 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta), \\ \sin \alpha - \sin \beta &= 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta), \\ \cos \alpha + \cos \beta &= 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta), \\ \cos \alpha - \cos \beta &= -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta). \end{aligned} \right\} \quad (14)$$

*Reduction to acute angles.* Some tables give the values of the trigonometric functions for angles only up to  $90^\circ$ . For an angle greater than  $90^\circ$  the value of any function can be found by using these tables and resorting to the following formula:

$$\text{Any function of } (n \cdot 90^\circ \pm A) = \pm \left\{ \begin{array}{l} \text{same function of } A \text{ if} \\ n \text{ is even.} \\ \text{co-function of } A \text{ if } n \\ \text{is odd.} \end{array} \right. \quad (15)$$

The sign to be placed before the resulting function of  $A$  is the same as the sign of the original function in the quadrant of  $n \cdot 90^\circ \pm A$ , where  $A$  is thought of as an acute angle.

*Sine law.*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}. \quad (16)$$

The equations (16) are referred to as *the law of sines*. This law may be stated as follows: *The sides of a triangle are proportional to the sines of the opposite angles.*

*Cosine law.*

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A. \\ b^2 &= a^2 + c^2 - 2ac \cos B, \\ c^2 &= a^2 + b^2 - 2ab \cos C. \end{aligned} \quad (17)$$

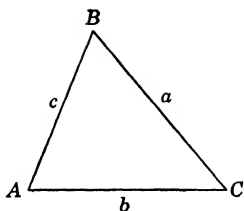


FIG. 2.

The law of cosines embodied in equations (17) may be stated as follows: *The square of any side of a plane triangle is equal to the sum of the squares of the other two sides diminished by twice the product of those two sides and the cosine of their included angle.*

*Law of tangents.*

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}. \quad (18)$$

**20. Length of circular arc.** Figure 3 shows a central angle of 1 radian and a central angle of  $\theta$  radians in a circle of radius  $r$ . Since two central angles in a circle have the same ratio as their intercepted arcs, we have

$$\frac{\theta}{1} = \frac{s}{r}$$

or

$$s = r\theta \text{ units.} \quad (19)$$

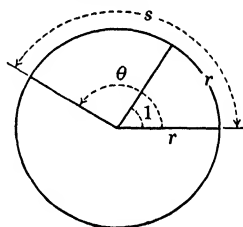


FIG. 3.

**Example 1.** A target in the form of a circular arc having its center at a gun is 3000 yd. from the gun and subtends at the gun an angle of 0.015 radian. Find the length of the target.

*Solution.* Here  $r = 3000$  yd., and  $\theta = 0.015$  radian. Substituting these numbers in (19), we obtain

$$s = r\theta = 3000(0.015) = 45 \text{ yd.}$$

**Example 2.** The nautical mile, or sea mile, used in the United States is the arc length subtended on a circle of diameter 7917.59 miles by a central angle of  $1'$  (7918 miles is approximately the diameter of a sphere having a volume equal to that of the earth). Find the length of the nautical mile.

*Solution.* Using formula (19) with

$$r = \frac{1}{2}(7917.6)(5280) \quad \text{and} \quad \theta = \frac{1}{60} \times \frac{\pi}{180},$$

we obtain

$$S = \frac{1}{2}(7917.6)(5280) \frac{\pi}{60 \times 180} = 6080.4 \text{ ft.}$$

This is approximately the length of the nautical mile. A more accurate value is 6080.27 ft.

### EXERCISES

1. For a circle of radius 720 ft., find the length of arc subtended by a central angle of (a)  $18^\circ$ ; (b)  $28^\circ 30'$ ; (c)  $17^\circ 20' 30''$ ; (d)  $20' 30''$ ; (e)  $38''$ ; (f)  $(a/\pi)^\circ$ .

2. For a circle having a circumference 3000 ft. in length, find in degrees, minutes, and seconds the central angle subtended by an arc of length (a) 300 ft.; (b) 10 ft.; (c) 1 ft.; (d) 12 ft.; (e) 2807 ft.

3. Show that a central angle of  $\theta$  degrees subtends on the circumference of a circle of radius  $r$  a length  $s$  given by

$$\frac{\theta}{180} = \frac{s}{\pi r}.$$

4. If a circular arc of 30 ft. subtends 4 radians at the center of its circle, find the radius of the circle.

5. If two angles of a plane triangle are respectively equal to 1 radian and  $\frac{1}{2}$  radian, express the third angle in degrees.

6. An enemy battery 6000 yd. distant from an observation post subtends at the post an angle of  $\frac{1}{80}$  radian. How many yards of front does the battery occupy if the post is directly in front of it?

7. Find approximately the angle in radians subtended by a church spire 160 ft. high at a point in the horizontal plane through the base of the spire and distant 1 mile from it.

8. An automobile whose wheels are 34 in. in diameter travels at the rate of 25 miles per hour. How many revolutions per minute does a wheel make? What is its angular velocity in radians per second?

9. A mil\* is  $\frac{1}{1800}$  of a right angle. Find the fraction of a radian in 1 mil and the number of mils in 1 radian.

10. A mil is approximately the angle subtended at the center of a circle having a radius of 1000 yd. by an arc length of 1 yd. on the circle. If for a circle  $r$  and  $s$  are expressed in yards and  $\theta$  in mils, prove that

$$s = \frac{r\theta}{1000} \text{ (approx.)}.$$

\* For a discussion of the mil, see Appendix A.



11. An enemy battery, range 6000 yd., subtends an angle of 12 mils. How many yards of front does it occupy (see Exercise 10)?

12. A grade is the hundredth part of a right angle. Express an angle of 1 grade in radians. Also show that a mil is  $\frac{1}{16}$  of a grade.

13. Assuming the earth to be a perfect sphere 7917 miles in diameter, find the length of an arc on the equator that subtends an angle of  $1^\circ$  at the center of the earth. Also find the distance between two points on the same meridian if one is  $8^\circ$  north of the equator and the other  $5^\circ 30'$  south of the equator.

14. When the moon is 239,000 miles from the earth, its diameter subtends about  $31'$  of angle at a point on the earth. Using this fact, compute the diameter of the moon by assuming that the diameter is the arc of a circle having its center at a point on the earth.

15. The larger of two wheels about which a belt is drawn taut has a 3-ft. radius. If the centers of the wheels are 6 ft. apart and if the arc of the larger wheel in contact with the belt subtends at its center an angle of 3.4 radians, find the radius of the smaller wheel.

16. An automobile has tires 28 in. in diameter. Find the angular velocity in radians per second of the wheel of the automobile when going 50 miles per hour.

17. The drive wheel of a locomotive is 6 ft. in diameter. Find its angular velocity in radians per minute when the train is moving 60 miles per hour.

18. The drive wheel of a locomotive is 6 ft. in diameter. If it makes 500 radians per minute, find the speed of the train in miles per hour.

19. Find the average speed of a man who runs two laps in 30 sec. on a circular track that is 35 ft. in diameter.

*In exercises 20 to 25, give approximate answers based on formula (19).*

20. On approaching the shore, the captain of the ship shown in Fig. 4 measured the angle of elevation of the top of a flagstaff and found it to be  $2^\circ 10'$ . If he knew the height of the staff was 32 ft. and

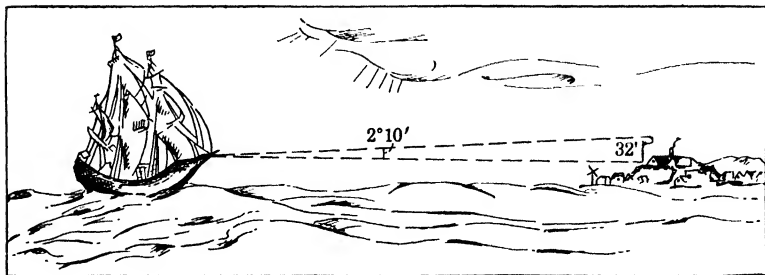


FIG. 4.

if the foot of the staff was on the same level with the captain's eye, find his distance from the flagstaff.

**21.** A lighthouse 100 ft. high stands on a rock. From the bottom of the lighthouse the angle of depression of a ship is  $2^{\circ}47'$ , and from the top of the lighthouse its angle of depression is  $4^{\circ}2'$ . What is the height of the rock? What is the horizontal distance from the lighthouse to the ship?

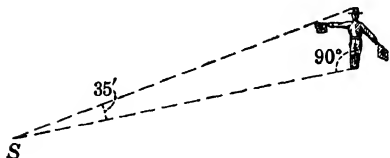


FIG. 5.

**22.** The signal-corp man shown in Fig. 5 subtends an angle of  $35'$  at station  $S$ . If he is 6 ft. tall, find his distance from the station.

**23.** On approaching a fort situated on a plain, a reconnoitering party finds at one place that the fort subtends an angle of  $3^{\circ}$  and at a place 200 ft. nearer the fort that it subtends an angle of  $6^{\circ}$ . How high is the

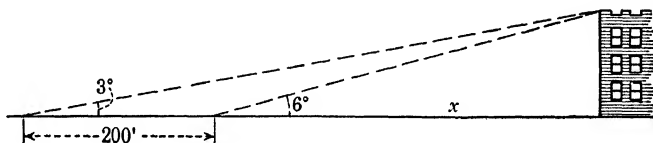


FIG. 6.

fort, and what is the distance to it from the second place of observation (see Fig. 6)?

**24.** The line of sight of a gun passes through a target 10,000 yd. away. Through an error in the sighting mechanism of the gun the plane of fire makes an angle of 10 mils with the vertical plane through the line of sight. How far from the target will the shell burst occur if the gun is correctly elevated?

**25.** Statistics show that when a shell bursts within 50 ft. of an airplane it registers an effective hit. Find, for effective shooting, the maximum deviation from the direction that would give a central hit on an airplane distant 10,000 yd. Assume the airplane extends through a circle of diameter 75 ft.

**26.** An error of  $1^{\circ}$  in the course of an airplane causes an error of approximately 1 mile on a 60-mile trip. Show that this is true. Use this fact to find the displacement from destination for (a) an error in course of  $2^{\circ}$  for a 300-mile trip, (b) an error in course of  $3^{\circ}$  for a 180-mile trip, (c) an error in course of  $1.5^{\circ}$  on a 250-mile trip.

### CHAPTER III

## THE RIGHT SPHERICAL TRIANGLE

**21. Introduction.** Just as plane trigonometry has for its object the study of the relations existing among the sides and angles of a plane triangle, so spherical trigonometry has for its



*(Courtesy, John Hancock Mutual Life Insurance Company)*  
Chart your course right

object the study of the relations connecting the sides and angles of a spherical triangle. Since the earth is approximately a sphere, this theory will apply when distances and directions on the earth are in question. Hence the subject of spherical trigonometry is basic in navigation, geodesy, and astronomy.

**22. The spherical triangle.** The circle in which a plane through the center of a sphere intersects the sphere is called a

great circle. As in plane geometry, an arc on a great circle is measured by the angle that it subtends at the center and will be expressed in degrees, minutes, and seconds.

A spherical triangle consists of three arcs of great circles that form the boundaries of a portion of a spherical surface. As in plane geometry, the vertices of the spherical triangle will be denoted by capital letters  $A$ ,  $B$ , and  $C$  and the sides opposite by  $a$ ,  $b$ , and  $c$ , respectively. The magnitude of an angle of a spherical triangle is that of the plane angle formed by tangents to the sides of the angle at its vertex. *In general, we shall consider only spherical triangles, each of whose sides and each of whose angles is less than  $180^\circ$ .*

The planes of the great circles belonging to a spherical triangle form a trihedral angle at the center of the sphere (see Fig. 1).

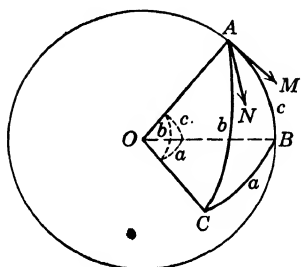


FIG. 1.

The face angles of this trihedral angle, being measured by their intercepted arcs, are designated by the same letters as the corresponding sides of the spherical triangle. The tangents to the arcs  $AB$  and  $AC$  at point  $A$ , being perpendicular to the radius  $OA$ , are the sides of the plane angle of dihedral angle  $M-AO-N$ . These tangents measure angle  $A$  of the spherical triangle  $ABC$ . Hence *an angle of the*

*spherical triangle is measured by the dihedral angles made by the planes of its sides.*

### Important propositions from solid geometry :

1. *The sum of the angles of a spherical triangle is greater than  $180^\circ$  and less than  $540^\circ$ ; that is,  $180^\circ < A + B + C < 540^\circ$ .*
2. *If two angles of a spherical triangle are equal, the sides opposite are equal; and conversely.*
3. *If two angles of a spherical triangle are unequal, the sides opposite are unequal, and the greater side lies opposite the greater angle; and conversely.*
4. *The sum of two sides of a spherical triangle is greater than the third side.*

### EXERCISES

1. If each angle of a spherical triangle is a right angle, what is the value of each side?

2. Show that if a spherical triangle has two right angles, the sides opposite these angles are quadrants and the third angle has the same measure as the opposite side.

3. The face angles of the trihedral angle associated with a spherical triangle are each  $90^\circ$  and the radius of the sphere is 10 in. Find the angles of the triangle in degrees, and find the sides both in degrees and in inches.

4. Find the magnitude of the face angles and of the dihedral angles of the trihedral angle associated with a spherical triangle whose sides are  $90^\circ$ ,  $90^\circ$ , and  $60^\circ$ .

5. The face angles of a trihedral angle at the center of the earth are  $50^\circ$ ,  $60^\circ 38'$ ,  $45^\circ 50' 20''$ . Find in nautical miles\* the lengths of the sides of the associated spherical triangle on the surface of the earth.

6. Two great circles on a sphere intersect at an angle of  $23^\circ 30'$ . Find the least great-circle distance from the pole of one to a point on the other.

7. What can be said regarding the size and shape of a spherical equiangular triangle if the sum of its angles is (a) nearly equal to  $180^\circ$ ; (b) nearly equal to  $540^\circ$ ?

8. Find all sides and angles of a spherical triangle having as angles  $A = 90^\circ$ ,  $B = 90^\circ$ , and

- |                      |                       |                       |
|----------------------|-----------------------|-----------------------|
| (a) $C = 30^\circ$ . | (c) $C = 120^\circ$ . | (e) $C = 110^\circ$ . |
| (b) $C = 45^\circ$ . | (d) $C = 70^\circ$ .  | (f) $C = 160^\circ$ . |

9. Show that the sum of the angles of a right spherical triangle is greater than  $180^\circ$  and less than  $360^\circ$ .

**23. Formulas relating to the right spherical triangle.** Since spherical triangles having more than one right angle can be solved by inspection, we shall be concerned with right spherical triangles having only one right angle.

In this article, ten formulas relating to the right spherical triangle are derived, and in the next article simple rules for writing these formulas are given.

The solution of all cases of spherical triangles generally considered in spherical trigonometry can be solved by means of these formulas.

In Fig. 2 is represented a spherical pyramid that is part of a sphere having unit radius and center  $O$ . In the right spherical triangle  $ABC$  bounding the base of the pyramid,  $C$  is a right angle,

\* A nautical mile is the length of an arc of a great circle on a sphere the size of the earth subtended by an angle of  $1'$  at its center.

and therefore the dihedral angle having edge  $OC$  is a right dihedral angle. From  $A$ , a plane is passed perpendicular to edge  $OB$  cutting the spherical pyramid in the triangle  $AED$ . Since  $OE$  is perpendicular to plane  $AED$ , it is perpendicular to lines  $EA$  and  $ED$ . Hence angle  $AED$  is the plane angle of the dihedral angle having  $OB$  as edge. Therefore angle  $AED$  is equal to angle  $B$ . Also, plane  $AED$  is perpendicular to plane  $COB$ , since it is perpendicular to a line in the plane. Therefore line  $AD$  is

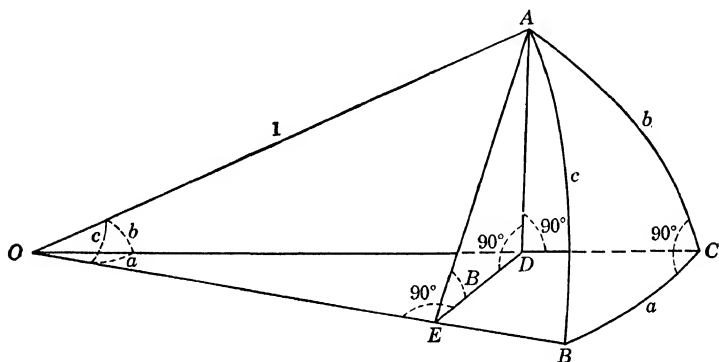


FIG. 2.

perpendicular to plane  $OBC$  because it is the intersection of the two planes  $OAD$  and  $ADE$ , both of which are perpendicular to  $OBC$ . Hence the angles  $ADO$  and  $ADE$  are right angles. Each face angle of the trihedral angle  $O-ABC$  is measured by the side of the spherical triangle intercepted by it and is therefore designated by the same letter as that side.

From Fig. 2 we read

$$\frac{DA}{1} = \sin b, \quad \frac{EA}{1} = \sin c, \quad \frac{OE}{1} = \cos c, \quad \frac{OD}{1} = \cos b. \quad (I)$$

Also from triangle  $OED$ ,  $ED/OE = \tan a$ . Replacing  $OE$  in this by  $\cos c$  from (I) and simplifying slightly, we have

$$ED = OE \tan a = \cos c \tan a. \quad (II)$$

Similarly, from triangle  $OED$ ,

$$ED = OD \sin a = \cos b \sin a. \quad (III)$$

Figure 3 is obtained from Fig. 2 by enlarging it and writing on it the values of the line segments just derived.

Both values for  $ED$ , one from (II) and the other from (III) are written on  $ED$ . From the triangle  $AED$  in Fig. 3, we read

$$\sin B = \frac{\sin b}{\sin c}, \quad (IV)$$

$$\cos B = \frac{\tan a \cos c}{\sin c},$$

$$\tan B = \frac{\sin b}{\sin a \cos b},$$

$$\tan a \cos c = \sin a \cos b.$$

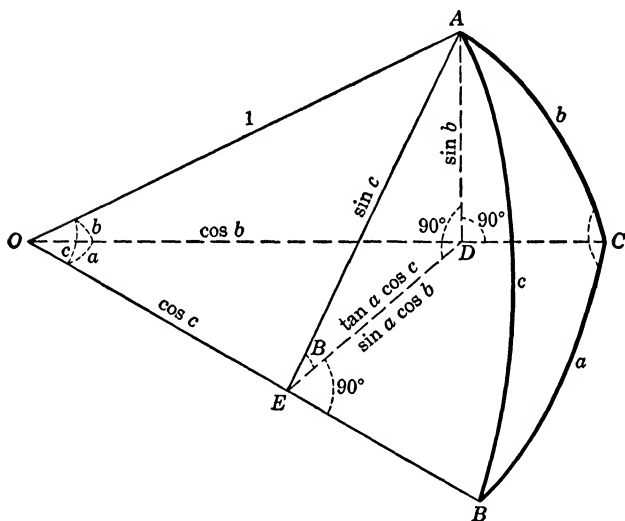


FIG. 3.

These last four equations may be written in the following form:

$$\sin b = \sin c \sin B, \quad (1)$$

$$\cos B = \tan a \cot c, \quad (2)$$

$$\sin a = \tan b \cot B, \quad (3)$$

$$\cos c = \cos a \cos b. \quad (4)$$

Similarly, by passing a plane through  $B$  of Fig. 2 perpendicular to  $OA$  and proceeding as above, we could prove the formulas

$$\sin a = \sin c \sin A, \quad (5)$$

$$\cos A = \tan b \cot c, \quad (6)$$

$$\sin b = \tan a \cot A. \quad (7)$$

Formulas (5), (6), and (7) are the result of interchanging  $a$  and  $b$

and  $A$  and  $B$  in (1), (2), and (3), respectively. From (7)  $\cot A = \sin b / \tan a$  and from (3)  $\cot B = \sin a / \tan b$ ; multiplying these two equations member by member, we obtain

$$\cot A \cot B = \frac{\sin b}{\tan a} \times \frac{\sin a}{\tan b} = \cos b \cos a,$$

or, interchanging members and replacing  $\cos b \cos a$  by  $\cos c$  from (4),

$$\cos c = \cot A \cot B. \quad (8)$$

Similarly from (2), (5), and (4), we obtain

$$\cos B = \cos b \sin A \quad (9)$$

and from (6), (1), and (4),

$$\cos A = \cos a \sin B. \quad (10)$$

**24. Napier's rules.** The ten formulas derived in §23 need not be memorized, for it is easy to write them by using two rules devised by John Napier, the inventor of logarithms.

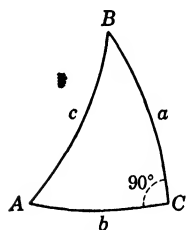


FIG. 4.

Figure 4 represents a right spherical triangle. Figure 5 contains the same letters as Fig. 4 except  $C (= 90^\circ)$ , arranged in the same order. The bars on the letters  $c$ ,  $B$ , and  $A$  mean *the complement of*; thus  $\bar{B}$  means  $90^\circ - B$ . Note that the barred parts are the hypotenuse and the two angles each of which has a side along

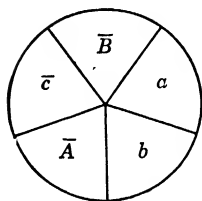


FIG. 5.

the hypotenuse. The angular quantities  $a$ ,  $b$ ,  $\bar{c}$ ,  $\bar{A}$ ,  $\bar{B}$  are called the *circular parts*. There are two circular parts contiguous with any given part and two parts that are not contiguous to it. Speaking of this given part as the *middle part*, we call the two contiguous parts the *adjacent parts*, and the two non-contiguous parts the *opposite parts*. Napier's rules may now be stated as follows:

**Napier's Rule I.** The sine of any middle part is equal to the product of the cosines of the opposite parts.

**Napier's Rule II.** The sine of any middle part is equal to the product of the tangents of the adjacent parts.



We may use the expression *sin middle = cos opposite = tan adjacent* as an aid in recalling these rules.

Thinking of any part as the middle part, we can write two formulas, one from each of the two rules. Considering each of the five parts in turn as middle part, we may write ten formulas, and these are found to be the ten formulas numbered (1) to (10) in §23.\*

**Example.** Use Napier's rules to write two formulas by using (a)  $b$  as middle part; (b)  $A$  as middle part.

*Solution.* Noting that  $\sin \bar{A} = \sin (90^\circ - A) = \cos A$ ,  $\cos \bar{A} = \cos (90^\circ - A) = \sin A$ , etc., and applying the first rule to the parts  $b, \bar{c}, \bar{B}$  (see Fig. 6), we obtain

$$\sin b = \cos \bar{c} \cos \bar{B},$$

or

$$\sin b = \sin c \sin B. \quad (a)$$

Applying the second rule, using parts  $\bar{A}, b, a$ , we obtain

$$\sin b = \tan \bar{A} \tan a = \cot A \tan a. \quad (b)$$

Similarly, using the parts  $\bar{A}, \bar{B}, a$  and the first rule, and afterwards the parts  $\bar{c}, \bar{A}, b$  and the second rule, we obtain

$$\sin \bar{A} = \cos \bar{B} \cos a, \quad \text{or} \quad \cos A = \sin B \cos a, \quad (c)$$

$$\sin \bar{A} = \tan \bar{c} \tan b, \quad \text{or} \quad \cos A = \cot c \tan b. \quad (d)$$

The formulas (a), (b), (c), and (d) are, respectively, the formulas (1), (7), (10), and (6) of §23.

### EXERCISES

1. Solve each of the following right spherical triangles for the unknown part indicated.

$$(a) \quad a = 30^\circ, \\ b = 60^\circ, \quad c = ?$$

$$(b) \quad c = 60^\circ, \\ a = 45^\circ, \quad B = ?$$

$$(c) \quad a = 45^\circ, \\ B = 60^\circ, \quad c = ?$$

$$(d) \quad a = 60^\circ, \\ B = 30^\circ, \quad A = ?$$

$$(e) \quad c = 60^\circ, \\ A = 45^\circ, \quad b = ?$$

$$(f) \quad A = 30^\circ, \\ B = 60^\circ, \quad a = ?$$

\* After the student has become familiar with the use of Napier's rules, he may save time by writing the desired formulas directly from the triangle on which the letters have been properly barred.

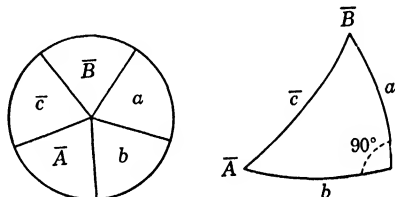


FIG. 6.

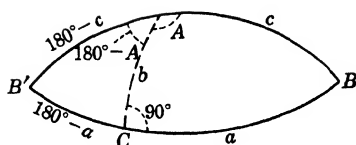


FIG. 7.

2. Using Fig. 7, show that formulas (1) to (10) hold true for the case  $a$  is greater than  $90^\circ$ ,  $c$  is greater than  $90^\circ$ ,  $b$  is less than  $90^\circ$ .

3. Solve each of the following right spherical triangles for the unknown part indicated:

- |   |   |
|---|---|
| (a) $a = 60^\circ$ ,<br>$b = 120^\circ$ ,<br>$A = ?$  | (d) $A = 135^\circ$ ,<br>$B = 60^\circ$ ,<br>$c = ?$  |
| (b) $c = 135^\circ$ ,<br>$b = 120^\circ$ ,<br>$a = ?$ | (e) $a = 30^\circ$ ,<br>$B = 120^\circ$ ,<br>$A = ?$  |
| (c) $B = 150^\circ$ ,<br>$c = 120^\circ$ ,<br>$a = ?$ | (f) $c = 120^\circ$ ,<br>$a = 135^\circ$ ,<br>$B = ?$ |

4. Corresponding to each of the following formulas pertaining to a plane right triangle, write, using Napier's rules, an analogous formula pertaining to a right spherical triangle.

- |                           |                      |                      |
|---------------------------|----------------------|----------------------|
| (a) $\sin A = a/c$ .      | (d) $\cos A = b/c$ . | (f) $\tan A = a/b$ . |
| (b) $\sin B = b/c$ .      | (e) $\cos B = a/c$ . | (g) $\tan B = b/a$ . |
| (c) $1 = \cot A \cot B$ . |                      |                      |

5. On Fig. 8 interchange  $A$  and  $B$ , also  $a$  and  $b$ . Then express the values of the line segments  $OD$ ,  $OE$ ,  $BE$ ,  $BD$ ,  $DE$  in terms of  $a$ ,  $b$ ,  $c$ ,

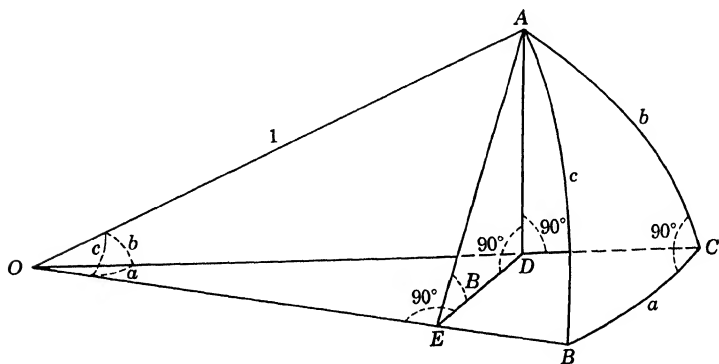


FIG. 8.

and write each of these line values on the figure. Equate two values of  $DE$  to obtain formula (4), and apply the definitions of the trigonometric functions to triangle  $BDE$  to obtain formulas (5), (6), and (7).

6. Using formula (4), show that the hypotenuse of a right spherical triangle is less than or greater than  $90^\circ$ , according as the two legs lie in the same quadrant or in different quadrants.

7. Using formula (10), show that in a right spherical triangle each leg and the opposite angle are of the same quadrant.

8. Use Napier's rules to write a formula involving the following, taking  $c$  as unknown part,

$$(a) \ c, B, A.$$

$$(b) \ c, B, a.$$

$$(c) \ c, B, b.$$

9. Use Napier's rules to write three formulas, each involving  $a$  and  $b$ .

$$10. \text{ Prove that } \tan A = \frac{\sin a}{\tan b \cos c}.$$

$$11. \text{ Prove that } \cos A = \frac{\sin b \cos a}{\sin c}.$$

**25. Two important rules.** In what follows it will be convenient to speak of an angle of the first quadrant or of the second quadrant. An angle is said to be of the first, second, third, or fourth quadrant according as its terminal side falls in the first, second, third, or fourth quadrant when laid off in the usual manner relative to rectangular coordinate axes.

From formula (10) of §23, namely,

$$\cos A = \cos a \sin B,$$

it follows that  $\cos A$  and  $\cos a$  must have the same sign since  $\sin B$  is positive in all cases. Hence both  $A$  and  $a$  must be less than  $90^\circ$ , or both must be greater than  $90^\circ$ . Formula (9) may be used to show that  $B$  and  $b$  must be of the same quadrant. The following rule expresses the relation.

**Rule (A).** In a right spherical triangle an oblique angle and the side opposite are of the same quadrant.

From formula (4), namely,

$$\cos c = \cos a \cos b,$$

it appears that the product  $\cos a \cos b$  must be positive when  $c$  is less than  $90^\circ$ ; therefore  $\cos a$  and  $\cos b$  must have the same sign, and for that reason  $a$  and  $b$  are both of the first quadrant or both of the second quadrant. From the same formula it appears that  $\cos a \cos b$  must be negative when  $c$  is greater than

$90^\circ$ ; therefore  $\cos a$  and  $\cos b$  must have opposite signs, and  $a$  and  $b$  are of different quadrants. The following rule expresses the relation.

**Rule (B).** When the hypotenuse of a right spherical triangle is less than  $90^\circ$ , the two legs are of the same quadrant; when the hypotenuse is greater than  $90^\circ$ , one leg is of the first quadrant and the other of the second.

Rules (A) and (B) enable the computer to tell the quadrant of an angle found from its sine or its cosecant.

### EXERCISES

State the quadrant of each of the unknown parts in each of the right spherical triangles indicated in the following diagram:

	$a$	$b$	$c$	$A$	$B$
1	$30^\circ$	$40^\circ$			
2	$30^\circ$		$120^\circ$		
3	$120^\circ$				$50^\circ$
4		$140^\circ$	$75^\circ$		
5				$120^\circ$	$130^\circ$
6		$35^\circ$		$100^\circ$	
7			$100^\circ$	$100^\circ$	
8			$60^\circ$		$60^\circ$

**26. Solution of right spherical triangles.** When two parts of a right spherical triangle in addition to the right angle are given, the remaining parts can be computed from formulas obtained by using Napier's rules. In solving the triangle it will be found advantageous to proceed as follows:

*a.* Draw a right spherical triangle lettered in the conventional way and encircle the given parts.

*b.* Write a formula for each unknown part by applying Napier's rules. *Each formula should contain the unknown part and both*

of the given parts. Then write a check formula connecting the three required parts.

c. Make a form.

d. Fill in the blank spaces of the form.

**Example.** Solve the right spherical triangle in which  $a = 66^{\circ}59'31''$ ,  $b = 156^{\circ}34'19''$ .

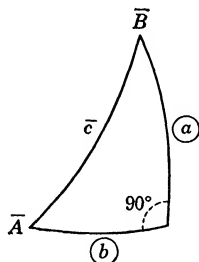


FIG. 9.

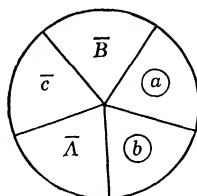


FIG. 10.

*Solution.* Figures 9 and 10 display the circular parts of a right spherical triangle, the known parts  $a$ ,  $b$  being encircled. Using Napier's rules, in connection with Fig. 10, we write

$$\sin \textcircled{b} = \tan \textcircled{a} \cot A, \quad \text{or} \quad \cot A = \sin \textcircled{b} \cot \textcircled{a}, \quad (a)$$

$$\sin \textcircled{a} = \tan \textcircled{b} \cot B, \quad \text{or} \quad \cot B = \sin \textcircled{a} \cot \textcircled{b}, \quad (b)$$

$$\cos c = \cos \textcircled{a} \cos \textcircled{b}, \quad (c)$$

$$\cos c = \cot A \cot B. \quad (d)$$

The symbols  $l \sin$ ,  $l \cot$ , etc., written in any line of a form mean log sine of the angle at the left of the line, log cotangent of that angle, etc. For convenience the negative part  $-10$  of the characteristic will be omitted in the forms.

The symbol  $(-)$  written before a logarithm in any form calls attention to the fact that the antilogarithm of that logarithm is negative. Hence an odd number of symbols  $(-)$  appearing in a column used to evaluate a product by logarithms will indicate that the product is negative. An even number of symbols  $(-)$  will indicate a positive product.

*In the forms of spherical trigonometry we shall omit the expressions  $a =$ ,  $b =$ , etc., to save space. The student will understand that each symbol refers to the number at the extreme left of its line.*

The computation of the unknown parts from the formulas (a), (b), (c), and the check by (d) is displayed on page 46.

	(a)	(b)	(c)
$a = 66^{\circ}59'31''$	$l \cot 9.62802$	$l \sin 9.96400$	$l \cos 9.59202$
$b = 156^{\circ}34'19''$	$l \sin 9.59944$	$l \cot (-)0.36319$	$l \cos (-)9.96264$
$A = 80^{\circ}25'01''$	$l \cot 9.22746$		
$B = 154^{\circ}47'25''$	$l \cot (-)0.32719$	$l \cot (-)0.32719$	
$c = 111^{\circ}1'0''$	$l \cos (-)9.55465$		$l \cos (-)9.55466$

Observe that the check obtained by adding  $\log \cot A$  to  $\log \cot B$  to get  $\log \cos c$  checks only the logarithms of the computed parts. Errors made in finding  $A$ ,  $B$ , and  $c$  from associated logarithms would not affect the check.

## EXERCISES

Solve the following right spherical triangles:

- $a = 10^{\circ}32'$ ,  
 $B = 12^{\circ}3'$ .
- $c = 46^{\circ}40'$ ,  
 $B = 20^{\circ}50'$ .
- $a = 118^{\circ}54'$ ,  
 $B = 12^{\circ}19'$ .
- $a = 43^{\circ}27'$ ,  
 $c = 60^{\circ}24'$ .
- $b = 48^{\circ}36'$ ,  
 $c = 69^{\circ}42'$ .
- $a = 168^{\circ}13'45''$ ,  
 $c = 150^{\circ}9'20''$ .
- $c = 112^{\circ}48'$ ,  
 $B = 56^{\circ}11'56''$ .
- $c = 32^{\circ}34'$ ,  
 $A = 44^{\circ}44'$ .
- $A = 116^{\circ}31'25''$ ,  
 $B = 116^{\circ}43'12''$ .
- $A = 54^{\circ}54'42''$ ,  
 $c = 69^{\circ}25'11''$ .
- $c = 55^{\circ}9'32''$ ,  
 $a = 22^{\circ}15'7''$ .
- $a = 36^{\circ}27'$ ,  
 $b = 43^{\circ}32'31''$ .
- $a = 29^{\circ}46'8''$ ,  
 $B = 137^{\circ}24'21''$ .
- $a = 144^{\circ}27'3''$ ,  
 $b = 32^{\circ}8'56''$ .
- $b = 36^{\circ}27'$ ,  
 $a = 43^{\circ}32'31''$ .
- $A = 63^{\circ}15'12''$ ,  
 $B = 135^{\circ}33'39''$ .
- $A = 67^{\circ}54'47''$ ,  
 $B = 99^{\circ}57'35''$ .
- $b = 22^{\circ}15'7''$ ,  
 $c = 55^{\circ}9'32''$ .
- $a = 118^{\circ}30'10''$ ,  
 $B = 95^{\circ}36'$ .
- $b = 92^{\circ}47'32''$ ,  
 $A = 50^{\circ}2'1''$ .

21. If angle  $A$  of a right spherical triangle is  $28^{\circ}$ , what is the maximum size of angle  $B$ ?

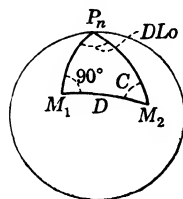


FIG. 11.

22. A plane leaves point  $M_1$  in Fig. 11 flying due east and follows a great-circle track to a point  $M_2$ . If  $M_1$  is in latitude  $40^{\circ}30'$  N., longitude  $75^{\circ}$  W. and if  $M_2$  is in longitude  $60^{\circ}$  W., find the distance  $D$  traveled, the latitude of  $M_2$ , and the course angle  $C$  at  $M_2$ .

*Hint.* The angle  $DLo$  at the north pole  $P_n$  is the difference in the longitudes of the two points  $M_1$

and  $M_2$ . The distances from the points  $M_1$  and  $M_2$  to  $P_n$  are respectively the complements of the latitudes of these points.

**23.** In the spherical triangle  $ABC$  (Fig. 12),  $p$  is the arc of a great circle perpendicular to side  $c$ . Write an expression for  $B$  in terms of  $A$ ,  $a$ , and  $b$ .

*Hint.* Find two values of  $p$  and equate them.

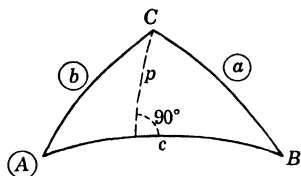


FIG. 12.

**24.** If in the triangle  $ABC$  of Exercise 23,  $A = 40^\circ 10'$ ,  $a = 46^\circ 20'$ , and  $b = 64^\circ 50'$ , find  $B$ .

**25.** All lines in Fig. 13 represent arcs of great circles. Find all unknown parts, thus solving a spherical triangle for which two angles and the included side are given.

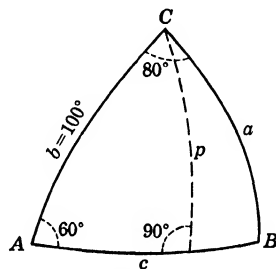


FIG. 13.

**27. The ambiguous case.** When the given parts are a side and the angle opposite, two solutions are obtained. In such cases each unknown part is found from the sine and hence may be chosen either in the first quadrant or in the second quadrant; that is, in the case of each unknown an angle and its supplement must be written. If  $A$  and  $a$  represent the given parts and  $C$  the right angle, the two triangles will form a lune as indicated in Fig. 14; for in this figure  $B'$  appears as  $180^\circ - B$ ,  $c'$  as  $180^\circ - c$ , and  $b'$  as  $180^\circ - b$ .

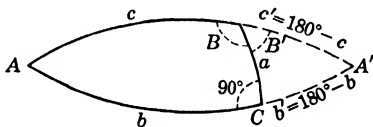


FIG. 14.

The solution of the following example will illustrate the method of finding a double solution when it exists.

**Example.** Solve the right spherical triangle in which

$$a = 46^\circ 45', \quad A = 59^\circ 12'.$$

*Solution.* Using Napier's rules in connection with Fig. 15 we obtain

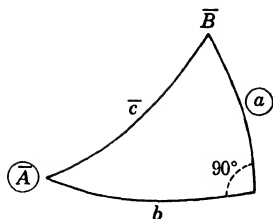


FIG. 15.

$$\sin c = \sin \textcircled{a} \csc \textcircled{A}, \quad (a)$$

$$\sin B = \sec \textcircled{a} \cos \textcircled{A}, \quad (b)$$

$$\sin b = \tan \textcircled{a} \cot \textcircled{A}, \quad (c)$$

$$\sin b = \sin c \sin B. \quad \text{Check}$$

The solution is displayed below.

	(a) and (check)	(b)	(c)
$a = 46^\circ 45'$	$l \sin 9.86235$	$l \sec 0.16419$	$l \tan 0.02655$
$A = 59^\circ 12'$	$l \csc 0.06603$	$l \cos 9.70931$	$l \cot 9.77533$
$c_1 = 57^\circ 59' 30''$	$l \sin 9.92838$		
$c_2 = 122^\circ 0' 30''$			
$B_1 = 48^\circ 21' 27''$	$l \sin 9.87350$	$l \sin 9.87350$	
$B_2 = 131^\circ 38' 33''$			
$b_1 = 39^\circ 19' 24''$	$l \sin 9.80188$		$l \sin 9.80188$
$b_2 = 140^\circ 40' 36''$			

The six answers were grouped to obtain the solutions  $b_1$ ,  $c_1$ ,  $B_1$ , and  $b_2$ ,  $c_2$ ,  $B_2$  by using the rules (A) and (B) of §25.

### EXERCISES

Solve the following right spherical triangles:

- $b = 35^\circ 44'$ ,  
 $B = 37^\circ 28'$ .
- $b = 129^\circ 33'$ ,  
 $B = 104^\circ 59'$ .
- $b = 21^\circ 39'$ ,  
 $B = 42^\circ 10' 10''$ .
- $a = 77^\circ 21' 50''$ ,  
 $A = 83^\circ 56' 40''$ .
- $a = 160^\circ$ ,  
 $A = 150^\circ$ .
- $b = 42^\circ 18' 45''$ ,  
 $B = 46^\circ 15' 25''$ .

7. Apply Napier's rules to Fig. 15 to obtain a formula involving the known parts  $a$ ,  $A$ , and the unknown part  $b$ . From this formula show that there may be no solution. Discuss the case that arises when  $a$  and  $A$  are supplementary.

Solve the following right spherical triangles:

- $b = 42^\circ 18'$ ,  
 $B = 42^\circ 18'$ .
- $a = 20^\circ 10'$ ,  
 $A = 115^\circ 20'$ .

**28. Polar triangles.** The poles of a great circle on a sphere are the points where a perpendicular to the plane of the great



circle through its center pierces the surface of the sphere. To obtain the polar triangle of a spherical triangle  $ABC$ , construct three great circles on the sphere having their poles at  $A$ ,  $B$ , and  $C$ . Two arcs, one having  $B$  as pole and the other  $C$  as pole, intersect in two points on opposite sides of arc  $BC$ . Denote by

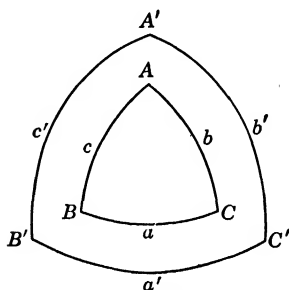


FIG. 16a.

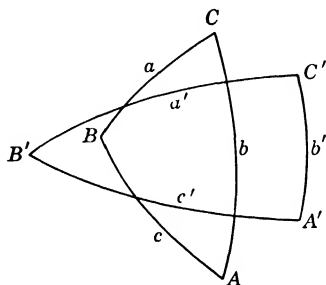


FIG. 16b.

$A'$  the point that lies on the same side of the great circle through  $BC$  as  $A$ . Locate  $B'$  and  $C'$  by an analogous procedure. Then triangle  $A'B'C'$  is the polar of triangle  $ABC$ . Figures 16 (a) and 16 (b) indicate the relations.

The following theorems from solid geometry are important:

1. If  $A'B'C'$  represents the polar triangle of spherical triangle  $ABC$ , then  $ABC$  is the polar triangle of  $A'B'C'$ .

2. An angle of any spherical triangle is the supplement of the opposite side in the polar triangle.

In accordance with Theorem 2, we have the following relations between the sides and angles represented in Figs. 16 (a) and (b):

$$\left. \begin{aligned} A' &= 180^\circ - a, & A &= 180^\circ - a', \\ B' &= 180^\circ - b, & B &= 180^\circ - b', \\ C' &= 180^\circ - c, & C &= 180^\circ - c'. \end{aligned} \right\} \quad (11)$$

If in an equation containing the quantities  $a, b, c, A, B, C$ , these quantities be replaced by their values in terms of  $a', b', c', A', B', C'$ , from (11), a new equation having reference to the polar triangle is obtained. The relations (11) will be used in the next article to solve a spherical triangle having a side equal to  $90^\circ$ .

### EXERCISES

1. Use relations (11) to find the parts of the polar triangle of each of the following spherical triangles.

- (a)  $A = 135^\circ 59.1'$ ,  $B = 100^\circ 10.1'$ ,  $C = 98^\circ 43.3'$ ,  $c = 90^\circ$ ,  $a = 135^\circ 20'$ ,  $b = 98^\circ 31.5'$ .  
 (b)  $a = 54^\circ 16.0'$ ,  $b = 114^\circ 47.0'$ ,  $C = 70^\circ 35.9'$ ,  $c = 90^\circ$ ,  $A = 49^\circ 57.9'$ ,  $B = 121^\circ 5.5'$ .  
 (c)  $a = 116^\circ 35.6'$ ,  $b = 105^\circ 14.8'$ ,  $c = 43^\circ 17.2'$ ,  $A = 112^\circ 47.4'$ ,  $B = 84^\circ 6.7'$ ,  $C = 44^\circ 59.1'$ .  
 (d)  $a = 136^\circ 19.6'$ ,  $b = 43^\circ 18.5'$ ,  $c = 114^\circ 43.3'$ ,  $A = 132^\circ 15.3'$ ,  $B = 47^\circ 19.5'$ ,  $C = 76^\circ 48.4'$ .

2. For each of the following formulas, write a new formula having reference to the polar triangle:

- (a)  $\sin a = \sin c \sin A$ .  
 (b)  $\tan b = \tan c \cos A$ .  
 (c)  $\tan a = \sin b \tan A$ .  
 (d)  $\cos c = \cos b \cos a$ .  
 (e)  $\sin b = \sin c \sin B$ .  
 (f)  $\cos a = \cos b \cos c + \sin b \sin c \cos A$ .  
 (g)  $\cos A = -\cos B \cos C + \sin B \sin C \cos a$ .  
 (h)  $\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b)}$ .  
 (i)  $\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a-b)}$ .

3. For each of the following triangles find the known parts of the polar triangle; solve these polar triangles:

- (a)  $c = 90^\circ$ ,  $a = 122^\circ 48.2'$ ,  $B = 21^\circ 35.4'$ .  
 (b)  $c = 90^\circ$ ,  $a = 49^\circ 30.0'$ ,  $B = 65^\circ 36.2'$ .

**29. Quadrantal triangles.** A spherical triangle having a side equal to  $90^\circ$  is called a *quadrantal triangle*. Evidently the polar triangle of a quadrantal triangle is a right spherical triangle. Hence this polar triangle can be solved in the usual way, and the unknown parts of the quadrantal triangle can then be obtained by using relations (11).

**Example.** Solve the spherical triangle in which  $c = 90^\circ$ ,  $A = 115^\circ 38'$ ,  $b = 139^\circ 58'$ .

*Solution.* Using (11) of §28 we obtain for the polar triangle  $C' = 180^\circ - c = 90^\circ$ ,  $a' = 180^\circ - A = 64^\circ 22'$ ,  $B' = 180^\circ - b = 40^\circ 2'$ . The solution of the polar triangle follows:

$a' = 64^{\circ}22'$	$\left  \begin{array}{l} l \cot 9.68109 \\ l \cos 9.88404 \end{array} \right $	$\left  \begin{array}{l} l \sin 9.95500 \\ l \tan 9.92433 \end{array} \right $	$\left  \begin{array}{l} l \cos 9.63610 \\ l \sin 9.80837 \end{array} \right $
$B' = 40^{\circ}2'$			
$c' = 69^{\circ}49'37''$	$\left  \begin{array}{l} l \cot 9.56513 \\ l \tan 9.87933 \end{array} \right $	$\left  \begin{array}{l} l \tan 9.87933 \end{array} \right $	
$b' = 37^{\circ}8'25''$			
$A' = 73^{\circ}50'34''$	$\left  \begin{array}{l} l \cos 9.44446 \end{array} \right $		$\left  \begin{array}{l} l \cos 9.44447 \end{array} \right $

Using equations (11) again, we obtain  $C = 180^{\circ} - c' = 110^{\circ}10'23''$ ,  $B = 180^{\circ} - b' = 142^{\circ}51'35''$ ,  $a = 180^{\circ} - A' = 106^{\circ}9'26''$ .

### EXERCISES

Solve the following right spherical triangles and then use (11) to obtain the solution of the polar triangle of each:

1.  $a = 115^{\circ}6'$ ,  
 $b = 123^{\circ}14'$ .
2.  $a = 112^{\circ}43'30''$ ,  
 $c = 85^{\circ}10'10''$ .

Solve the following quadrantal triangles:

3.  $B = 117^{\circ}54'30''$ ,  
 $a = 95^{\circ}42'20''$ ,  
 $c = 90^{\circ}$ .
4.  $B = 69^{\circ}45'$ ,  
 $A = 94^{\circ}40'$ ,  
 $c = 90^{\circ}$ .
5.  $A = 153^{\circ}16'$ ,  
 $b = 19^{\circ}3'$ ,  
 $c = 90^{\circ}$ .
6.  $b = 159^{\circ}33'40''$ ,  
 $a = 95^{\circ}18'20''$ ,  
 $c = 90^{\circ}$ .

7. In Fig. 17,  $a = 18^{\circ}12'$ ,  $B = 74^{\circ}45'$ ,  $c = 90^{\circ}$ . Solve the right triangle  $ACD$ , and from it deduce the solution of the quadrantal triangle  $ABC$ .

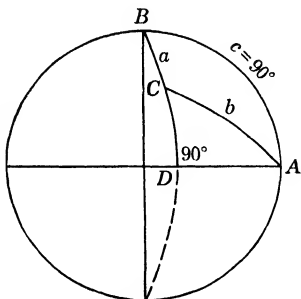


FIG. 17.

**30. The solution of the oblique triangle.** We have seen that any right spherical triangle can be solved by the use of Napier's rules. An oblique spherical triangle can be solved by dividing it into two right triangles and then using Napier's rules to solve each of them. When the given parts are two sides and the included angle, drop the perpendicular from the vertex of an

unknown angle to the opposite side. An example will serve to indicate the method.

**Example.** Solve the spherical triangle in which  $a = 88^\circ 24'$ ,  $b = 56^\circ 48'$ ,  $C = 128^\circ 16'$ .

*Solution.* Figure 18 represents a triangle with the given parts encircled and with the arc  $AD$  drawn perpendicular to the side  $BC$ . Applying Napier's rules to the right triangle  $ACD$ , we obtain the formulas

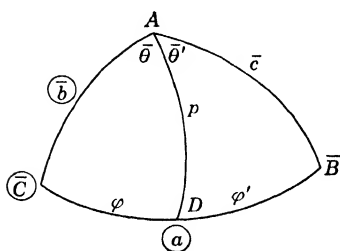


FIG. 18.

$$\tan \varphi = \tan b \cos C \quad (12)$$

$$\cot \theta = \cos b \tan C \quad (13)$$

$$\sin p = \sin b \sin C \quad (14)$$

$$\sin p = \cot \theta \tan \varphi \text{ (check)} \quad (15)$$

The solution of the right triangle  $ADC$  by using (12), (13), (14), and (15) follows.

$b = 56^\circ 48'$	$\begin{array}{l} l \tan \quad 0.18417 \\ l \cos (-) 9.79192 \end{array}$	$\begin{array}{l} l \cos \quad 9.73843 \\ l \tan (-) 0.10303 \end{array}$	$\begin{array}{l} l \sin 9.92260 \\ l \sin 9.89495 \end{array}$
$C = 128^\circ 16'$	$l \tan (-) 9.97609$		
$\varphi = 136^\circ 34' 35''$	$l \tan (-) 9.97609$		
$\theta = 124^\circ 46' 0''$	$l \cot (-) 9.84147$	$l \cot (-) 9.84146$	
$p = 138^\circ 55' 48''$	$l \sin \quad 9.81756$		$l \sin 9.81755$

After the first right triangle has been solved, the figure should

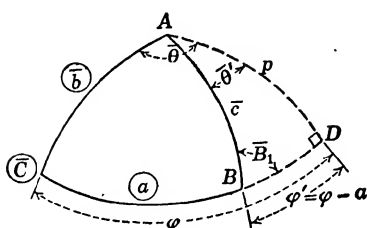


FIG. 19.

be drawn showing the perpendicular falling inside or outside the triangle according as  $\varphi$  is less than or greater than the side along which it lies.

Since  $\varphi$  is greater than  $a$ , the point  $D$  falls outside the arc  $\overline{CB}$  extended as indicated in Fig. 19.

In the triangle  $BDA$  the arcs  $p$  and  $\varphi' = \varphi - a$  are known. Applying Napier's rules to triangle  $BDA$ , we obtain the following formulas:

$$\cot B_1 = \cot p \sin \varphi' \quad (16)$$

$$\cot \theta' = \sin p \cot \varphi' \quad (17)$$

$$\cos c = \cos p \cos \varphi' \quad (18)$$

$$\text{(check)} \cos c = \cot \theta \cot B_1 \quad (19)$$

The solution of the triangle  $BDA$  follows.

$p = 138^{\circ}55'48''$	$l \cot (-) 0.05977$	$l \sin 9.81755$	$l \cos (-) 9.87732$
$\varphi' = (\varphi - a) = 48^{\circ}10'35''$	$l \sin 9.87227$	$l \cot 9.95175$	$l \cos 9.82402$
$B_1 = 130^{\circ}32'7''$	$l \cot (-) 9.93204$		
$\theta' = 59^{\circ}32'56''$	$l \cot 9.76930$	$l \cot 9.76930$	
$c = 120^{\circ}10'52''$	$l \cos 9.70134$		$l \cos (-) 9.70134$

Using Fig. 19 and the quantities obtained by the solutions, we have

$$B = 180^{\circ} - B_1 = 49^{\circ}27'48'', \quad A = \theta - \theta' = 65^{\circ}13'4'',$$

$$C = 120^{\circ}10'52''.$$

### EXERCISES

Solve the following spherical triangles by the method of this article:

- |   |   |
|---|---|
| <p>1. <math>a = 88^{\circ}24'0''</math>,<br/> <math>b = 56^{\circ}48'0''</math>,<br/> <math>C = 128^{\circ}16'0''</math>.</p> | <p>4. <math>a = 88^{\circ}37'40''</math>,<br/> <math>c = 125^{\circ}18'20''</math>,<br/> <math>B = 102^{\circ}16'36''</math>.</p> |
| <p>2. <math>b = 120^{\circ}30'0''</math>,<br/> <math>c = 70^{\circ}20'0''</math>,<br/> <math>A = 50^{\circ}10'0''</math>.</p> | <p>5. <math>a = 86^{\circ}18'40''</math>,<br/> <math>b = 45^{\circ}36'20''</math>,<br/> <math>C = 120^{\circ}46'30''</math>.</p>  |
| <p>3. <math>a = 76^{\circ}24'0''</math>,<br/> <math>b = 58^{\circ}19'0''</math>,<br/> <math>C = 116^{\circ}30'0''</math>.</p> | <p>6. <math>b = 132^{\circ}17'30''</math>,<br/> <math>c = 78^{\circ}15'15''</math>,<br/> <math>A = 40^{\circ}20'10''</math>.</p>  |

Solve the following triangles by solving the polar triangle.

- |   |  |
|---|--|
| <p>7. <math>A = 120^{\circ}10'0''</math>,<br/> <math>B = 100^{\circ}20'0''</math>,<br/> <math>c = 30^{\circ}5'0''</math>.</p> | <p>8. <math>A = 27^{\circ}22'34''</math>,<br/> <math>C = 91^{\circ}26'44''</math>,<br/> <math>b = 120^{\circ}18'33''</math>.</p> |
|---|--|

Solve the following spherical triangles by the method of this article

- |  |  |
|--|--|
| <p>9. <math>a = 40^{\circ}6'0''</math>,<br/> <math>b = 118^{\circ}22'0''</math>,<br/> <math>A = 29^{\circ}43'0''</math>.</p>     | <p>11. <math>a = 150^{\circ}57'5''</math>,<br/> <math>b = 134^{\circ}15'54''</math>,<br/> <math>A = 144^{\circ}22'42''</math>.</p> |
| <p>10. <math>a = 128^{\circ}15'0''</math>,<br/> <math>b = 129^{\circ}20'0''</math>,<br/> <math>A = 130^{\circ}25'0''</math>.</p> | <p>12. <math>a = 52^{\circ}45'20''</math>,<br/> <math>c = 71^{\circ}12'40''</math>,<br/> <math>A = 46^{\circ}22'10''</math>.</p>   |

13. Solve each of the following triangles by solving its polar triangle

- |   |  |
|---|--|
| <p>(a) <math>c = 80^{\circ}13'0''</math>,<br/> <math>C = 78^{\circ}15'0''</math>,<br/> <math>B = 75^{\circ}17'0''</math>.</p> | <p>(b) <math>a = 115^{\circ}13'4''</math>,<br/> <math>A = 120^{\circ}43'0''</math>,<br/> <math>B = 116^{\circ}38'0''</math>.</p> |
|---|--|

## CHAPTER IV ELEMENTARY APPLICATIONS

**31. Definitions and notations.** The earth revolves about a diameter called its *axis*. One point where the axis cuts the surface of the earth is called the *north pole*,  $P_n$ ; the other is called the *south pole*,  $P_s$ .

The *equator* is the great circle on the earth whose plane is perpendicular to the axis of the earth.

A *meridian* is a great circle on the earth passing through the

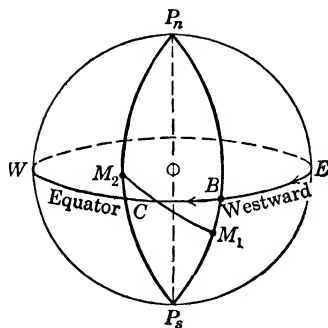


FIG. 1.

north pole and the south pole. In Fig. 1,  $P_nBP_s$  and  $P_nCP_s$  represent meridians. Since meridians cut the equator at right angles, angular distances of points on the earth from the equator are measured along meridians.

The *latitude* (Lat. or  $L$ ) of a point on the earth is the angular distance of the point from the equator. It is measured along a meridian north or south of the equator from  $0^\circ$  to  $90^\circ$ .

In Fig. 1,  $CM_2$  represents the latitude of  $M_2$ . In general, north latitude is considered positive, south latitude negative.

Because of the great importance of triangle  $M_1P_nM_2$  in connection with problems relating to distances and angles on the earth, it is called the *terrestrial triangle*. Arc  $M_1M_2$  represents the distance along the great-circle track from  $M_1$  to  $M_2$ , and the angle  $M_2M_1P_n$  gives the initial direction of the track. The angle of departure  $P_nM_1M_2$  measured from the north around through the east from  $0^\circ$  to  $360^\circ$  is called the initial course  $C_n$ . For a person situated on the northern hemisphere of the earth at a point such as  $z$  in Fig. 2, north is along the tangent to the meridian away from the equator; for a person standing at  $z$  facing north,

east is on his right, west is on his left, and south is opposite to the direction in which he is facing.

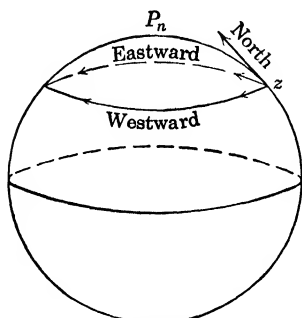


FIG. 2.

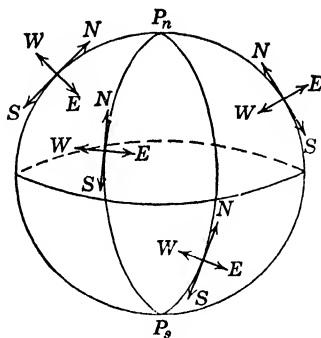


FIG. 3.

Figure 3 indicates directions at four positions on the earth.

The *longitude* (Long. or  $\lambda$ ) of a point on the earth is the angle at either pole between the meridian passing through the point and some fixed meridian known as the *prime meridian*. It is measured east or west of the prime meridian from  $0^\circ$  to  $180^\circ$ . The meridian of Greenwich, England, is the prime meridian, not only for English and American navigators but also for those of many other nations.

The latitude and longitude of a point give its position on the earth just as the two coordinates of a point give its position relative to a set of rectangular axes.

**32. Course and distance.** In general, the procedure of applying spherical trigonometry to solve problems relating to the earth consists in finding three parts of the terrestrial triangle, solving for one or more of the other three parts, and interpreting the results. Consider, for example, the problem of finding the great-circle distance between two points  $M_1$  and  $M_2$  when the latitude and the longitude of each point are known. In Fig. 4,  $P_n$  represents the north pole,  $QK_1K_2Q'$  the equator,  $P_nGQP_s$  the meridian of Greenwich, and  $M_1$  and  $M_2$  two places on the earth. The longitudes  $\lambda_1$  of  $M_1$  and  $\lambda_2$  of  $M_2$  are known; hence angle

$$M_1P_nM_2 = \lambda_2 - \lambda_1$$

is known. Also, the latitudes  $L_1 = K_1M_1$  of  $M_1$  and  $L_2 = K_2M_2$  of  $M_2$  are known; hence the arcs  $M_1P_n = 90^\circ - L_1 = co-L_1$

and  $M_2P_n = 90^\circ - L_2 = co-L_2$  are known. Thus, in triangle  $M_1P_nM_2$ , two sides  $M_1P_n = co-L_1$  and  $M_2P_n = co-L_2$  and the included angle  $M_1P_nM_2 = \lambda_2 - \lambda_1$  are known. Consequently, we can solve this triangle by the method of §30.

The *course* of a ship is the inclination of its direction of sailing to the meridian through it. Course is measured from  $0^\circ$  at north around to the right (clockwise) through  $360^\circ$ .

A *nautical mile* is 6080.27 ft. This length was chosen because it is practically the length of  $1'$  of arc on a great circle of the earth (see Example 2, §20).

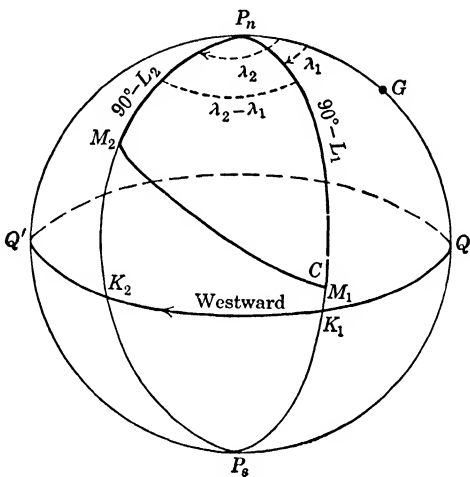


FIG. 4.

A *knot* is the unit of speed used in navigation. It is a speed of 1 nautical mile per hour.

**Example.** Compute the initial course and the distance for a great circle airplane trip from Annapolis, Lat.  $38^\circ 59'$  N., Long.  $76^\circ 29'$  W. to Seattle, Wash., Lat.  $47^\circ 36'$  N., Long.  $122^\circ 20'$  W.

*Solution.* Spherical triangle  $P_nAS$  in Fig. 6 represents the terrestrial triangle; spherical triangle  $CAB$  in Fig. 5 is the same triangle lettered in the conventional way.  $A$  represents the position of Annapolis,  $S$  the position of Seattle, and  $EE_2$  part of the equator. The given parts are the angle at  $P_n =$  the difference in longitude  $DLo = C = 122^\circ 20' - 76^\circ 29' = 45^\circ 51'$ , the



arc  $P_nA = \text{co-}L_1 = b = 90^\circ - 38^\circ 59' = 51^\circ 1'$ , and the arc  $P_nS = \text{co-}L_2 = a = 90^\circ - 47^\circ 36' = 42^\circ 24'$ .

The parts to be found are the angle at  $A$ , initial-course angle, and the length of the arc  $AS$ , the distance traveled  $c$ . The arc  $BD = p$  represents the perpendicular from  $S$  to  $CA$ . Applying Napier's rules to solve right triangle  $BDC$ , we obtain the formulas

$$\begin{aligned}\tan \varphi &= \tan a \cos C, \\ \sin p &= \sin a \sin C.\end{aligned}$$

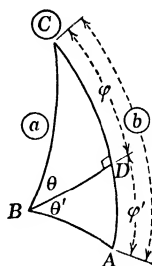


FIG. 5.

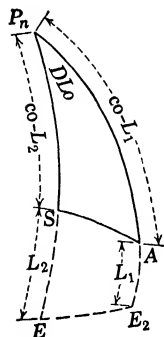


FIG. 6.

The solution is exhibited in the following form:

$a = \text{co-}L_2 = 42^\circ 24'$	$\left  \begin{array}{l} l \tan 9.96053 \\ l \cos 9.84295 \end{array} \right $	$\left  \begin{array}{l} l \sin 9.82885 \\ l \sin 9.85583 \end{array} \right $
$C = DL_0 = 45^\circ 51'$	$\left  \begin{array}{l} l \tan 9.80348 \end{array} \right $	
$\varphi = 32^\circ 27' 28''$		
$p = 28^\circ 56' 5''$		$l \sin 9.68468$

In this case  $\varphi$  is less than  $b$  and the perpendicular  $p$  falls inside the triangle as shown in Fig. 5. In triangle  $BDA$  we know  $p$  and  $\varphi' = b - \varphi = 51^\circ 1' - 32^\circ 27' 28'' = 18^\circ 33' 32''$ . Applying Napier's rules to solve the right triangle  $BDA$ , we obtain the following formulas:

$$\begin{aligned}\cot A &= \cot p \sin \varphi', \\ \cos c &= \cos p \cos \varphi' .\end{aligned}$$

The solution is exhibited in the following form:

$p = 28^\circ 56' 5''$	$\left  \begin{array}{l} l \cot 0.25742 \\ l \sin 9.50281 \end{array} \right $	$\left  \begin{array}{l} l \cos 9.94209 \\ l \cos 9.97681 \end{array} \right $
$\varphi' = 18^\circ 33' 32''$	$\left  \begin{array}{l} l \cot 9.76023 \end{array} \right $	
$A = \text{Course angle} = 60^\circ 4' 8''$		
$c = \text{Distance} = 33^\circ 56' 8''$		$l \cos 9.91890$

Here  $c$  represents the distance. Since  $1'$  of arc of a great circle on the earth has the length of 1 nautical mile (6080.27 ft.) the distance  $AS = c = 33^\circ 56' 08'' = (33 \times 60 + 56 + \frac{8}{60})$  miles = **2036.1** miles. From Fig. 6 and the quantities obtained in the

solution we get initial course =  $A = \text{N. } 60^{\circ}04'8'' \text{ W.}$  or compass reading  $299^{\circ}55'52''$ .

### EXERCISES

1. Solve the following spherical triangles:

- |  |  |
|--|--|
| (a) $a = 37^{\circ}48'12''$ ,<br>$b = 59^{\circ}44'16''$ ,<br>$C = 90^{\circ}$ .   | (d) $b = 132^{\circ}25'$ ,<br>$B = 107^{\circ}30'$ ,<br>$C = 90^{\circ}$ .       |
| (b) $A = 110^{\circ}47'50''$ ,<br>$B = 135^{\circ}35'34''$ ,<br>$c = 90^{\circ}$ . | (e) $B = 74^{\circ}45'$ ,<br>$a = 18^{\circ}12'$ ,<br>$c = 90^{\circ}$ .         |
| (c) $A = 55^{\circ}32'45''$ ,<br>$B = 101^{\circ}47'56''$ ,<br>$C = 90^{\circ}$ .  | (f) $a = 25^{\circ}18'45''$ ,<br>$A = 15^{\circ}58'15''$ ,<br>$C = 90^{\circ}$ . |

2. Solve the following isosceles spherical triangles:

- |   |   |
|---|---|
| (a) $c = 51^{\circ}8'$ ,<br>$A = B = 41^{\circ}57'$ . | (b) $C = 50^{\circ}19'40''$ ,<br>$A = B = 100^{\circ}12'30''$ . |
|---|---|

*Hint.* Draw the arc of a great circle through the vertex perpendicular to the opposite side. This perpendicular bisects the base and the angle at the vertex.

3. Two great circles on a sphere intersect at  $35^{\circ}$ . A point  $A$  on one circle is  $65^{\circ}$  from their intersection. Find the distance from the intersection to the point nearest to  $A$  on the other circle.

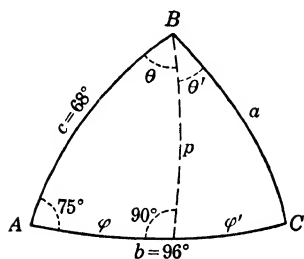


FIG. 7.

4. All lines in Fig. 7 represent arcs of great circles. Find all unknown parts, thus solving a spherical triangle for which two sides and the included angle are given.

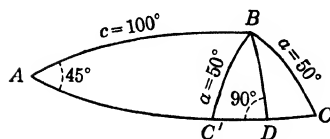


FIG. 8.

5. All lines in Fig. 8 represent arcs of great circles. Find all unknown parts, thus solving a spherical triangle for which two sides and an angle opposite one of them are given.

6. Figure 9 represents a spherical triangle, with the north pole at  $P$ , Panama in latitude  $8^{\circ}57'$  N. at  $M_1$ , and Honolulu in latitude  $21^{\circ}18'$  N. at  $M_2$ .  $M_2D$  is the arc of a great circle perpendicular to  $PM_1$  and  $DLo$  is  $78^{\circ}20'$ . Solve the right triangle I completely and afterward triangle II. From the results find the distance  $M_1M_2$  and the course angle at  $M_1$ .

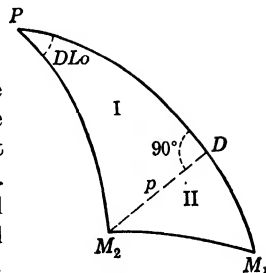


FIG. 9.

7. The northern vertex  $V$  (see Fig. 10), or point of highest latitude reached on the great-circle track from  $M_1$  to  $M_2$ , is in latitude  $L_v = 68^{\circ}27'$  N., and longitude  $\lambda_v = 20^{\circ}23'$  W. A ship sails on the great-circle track  $M_1M_2$ , starting from  $M_1$  in longitude  $\lambda_1 = 37^{\circ}18'$  W. to  $M_2$  in longitude  $\lambda_2 = 26^{\circ}28'$  W. Find the distance  $M_1M_2$ .

*Hint.*  $DLo_1 = \lambda_1 - \lambda_v$ ,  $DLo_2 = \lambda_2 - \lambda_v$ , and  $V$  is a right angle.

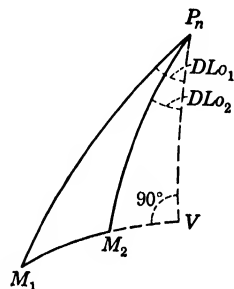


FIG. 10.

8. The initial course of a certain ship sailing from New York (latitude  $L = 40^{\circ}40'$  N., long.  $\lambda = 73^{\circ}58'30''$  W.) is due east. After she has sailed 600 nautical miles on a great circle, find her latitude, longitude, and course.

9. Find the latitude and distance from New York of the ship in Exercise 8 when her longitude is  $15^{\circ}25'$  W.

10. Find the latitude and longitude of the northernmost point on a great-circle track sailed by a ship leaving San Francisco (latitude  $L = 37^{\circ}48'$  N., long.  $\lambda = 122^{\circ}23'$  W.) on a course of  $310^{\circ}$ .

11. What is the shortest distance from New York to the great circle that passes through San Francisco and the nearest point to San Francisco on the  $180^{\circ}$  meridian?

12. Find the point on the  $180^{\circ}$  meridian that is nearest San Francisco (latitude  $L = 37^{\circ}48'$  N., long.  $\lambda = 122^{\circ}23'$  W.).

13. A ship sails from a place in longitude  $33^{\circ}14'25''$  W. 2000 nautical miles on a great circle. If the initial course is due east and if the change in longitude is  $53^{\circ}14'25''$ , find the latitude of departure and the course of arrival.

**33. Parallels of latitude.** In Fig. 11,  $C$  represents the center of the earth,  $P_n$  the north pole,  $AB$  an arc on the equator, and  $DE$

an arc of a small circle in latitude  $L$  cut out by a plane  $DEF$  parallel to the plane of the equator. From the figure it appears that angle  $ACB = \text{angle } DFE = \text{angle } DP_nE$  is the difference in longitude  $DLo$  between points  $A$  and  $B$  or between  $D$  and  $E$ . From sector  $ACB$ ,

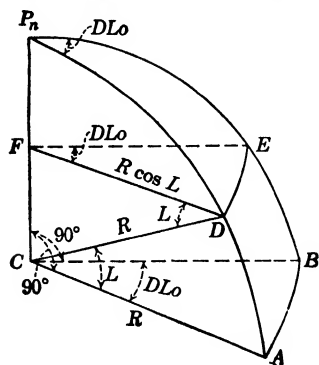


FIG. 11.

where the symbol ' indicates the quantity is measured in minutes.

Hence numerically

$$(DLo)' = R(DLo)_r. \quad (2)$$

Also from sector  $DFE$

$$(DE)_n = R(\cos L)(DLo)_r,$$

where  $(DE)_n$  denotes arc  $DE$  in nautical miles. Substituting the value of  $R(DLo)_r$  from (2) in this equation, we get

$$(DE)_n = (\cos L)(DLo)'. \quad (3)$$

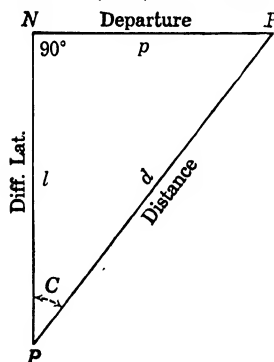


FIG. 12.

In Fig. 12  $PP'$  represents a comparatively short distance along a rhumb line which cuts meridian  $PN$  at angle  $C$ .  $NP'$  represents part of a parallel of latitude. The lengths of  $PP' = d$ ,  $PN = l$ , and  $NP' = p$  are called respectively the *distance*, the *difference in latitude*, and the *departure*. For comparatively short distances

**34. Plane sailing.** The path of a ship intersecting at the same angle all the meridians which it crosses is called a *rhumb line*. All rhumb lines except parallels of latitude are called *loxodromic curves*. Such a curve when sufficiently prolonged spirals about a pole but does not reach it.

the triangle  $PNP'$  is considered as a plane triangle and the following formulas are read from it:

$$l = d \cos C, \quad p = d \sin C. \quad (4)$$

**34a. Middle latitude sailing.** Since difference in latitude  $l$  is along a meridian, the number of nautical miles in  $l$  is the number of minutes in the difference in latitude between  $P$  and  $P'$ . Formula (3) shows that departure  $p$  must be multiplied by  $\sec L$  to get  $DLo$ . Since  $L$  is a variable between  $P$  and  $P'$ , an approximation to  $DLo$  in minutes is obtained by multiplying departure  $p$  by the secant of the mid-latitude  $(\frac{1}{2})(Lat. P + Lat. P')$ . These relations are expressed by the following formulas:

$$\begin{aligned} (\text{Diff. lat.})' &= d \cos C, \\ (DLo)' &= d \sin C \sec \frac{1}{2}(Lat. P + Lat. P'), \end{aligned} \quad (5)$$

where  $d$  is in miles. Observe that the first formula in (5) is exact, whereas the second is approximate. This method of converting departure to difference in longitude is called *middle latitude sailing*.

**Example.** An airplane flies 200 miles northeast from Annapolis Lat.  $38^{\circ}59' N.$ , Long.  $76^{\circ}29' W.$  Find the difference in latitude and the departure. Also find the latitude and longitude of the place reached.

*Solution.* Using formulas (4) we obtain

$$l = 200 \cos 45^{\circ} = \mathbf{141.4 \text{ miles}},$$

$$p = 200 \sin 45^{\circ} = \mathbf{141.4 \text{ miles}} \quad (a)$$

Hence the change in latitude is  $141.4' = 2^{\circ}21.4'$  and the required latitude is  $(38^{\circ}59' + 2^{\circ}21.4') N. = \mathbf{41^{\circ}20.4' N.}$  Using the second formula of (5), we have

$$DLo = 200' \sin 45^{\circ} \sec [38^{\circ}59' + \frac{1}{2}(2^{\circ}21.4')] = 188.5' = 3^{\circ}8.5'.$$

Hence the required longitude is

$$(76^{\circ}29' - 3^{\circ}8.5') W. = \mathbf{73^{\circ}20.5' W.}$$

### EXERCISES

1. If a ship sails on a course of  $42^{\circ}$  for 190 miles, what are the departure and difference in latitude?

2. If a ship sails a course of  $19^{\circ}$  for 201.85 miles, what is the departure?

3. A ship asks bearings from two radio stations  $A$  and  $B$ .  $A$  reports the ship's bearing  $82^{\circ}$  (Navy Compass) and  $B$  reports  $127^{\circ}$ .

Station  $B$  is known to be 127 nautical miles from  $A$  on bearing  $58^\circ$  from  $A$ . Find the difference in latitude and departure of the ship from  $A$ .

*In solving the following problems use formula (5).*

4. A ship steams due west 120.5 miles in latitude  $39^\circ$ . Find the change in its longitude.

5. A ship in latitude  $47^\circ 30'$  N. steams directly east until it has made good a difference in longitude of  $2^\circ 30'$ . Find the departure.

6. A ship at point  $M_1$ ,  $L = 41^\circ 30'$  N.,  $\lambda = 59^\circ 47'$  W., steams on course  $147^\circ$  for 290 miles. Find the latitude and longitude of the point of arrival.

7. A ship leaves a point  $M_1$ ,  $L_1 = 43^\circ 19'$  N.,  $\lambda_1 = 17^\circ 42'$  W. and arrives at point  $M_2$ ,  $L_2 = 41^\circ 13'$  N.,  $\lambda_2 = 21^\circ 14'$  W. Find the course and distance for a rhumb line track.

8. Find the course and distance on a rhumb line track from a point in latitude  $34^\circ 48.1'$  N., longitude  $22^\circ 14.2'$  W. to a point in latitude  $37^\circ 40'$  N., longitude  $25^\circ 40'$  W.

9. (a) If the difference of longitude of two places  $A$  and  $B$  on the earth is  $50^\circ$  and their latitudes are  $30^\circ$ , find the distance  $AB$  measured on the equal latitude circle.

(b) What is the distance  $AB$  measured on a great circle? The radius of the earth is approximately 3960 land miles.

10. Two points  $A$  and  $B$  are the ends of a 500-land-mile arc of a small circle in latitude  $36^\circ$  N. Find the difference in their longitudes. If  $A_1$  and  $B_1$  are both in latitude  $36^\circ$  N. and the arc of a great circle connecting them is 500 land miles long, what is the difference in their longitudes? Assume the radius of the earth is 3960 land miles.

**35. The Mercator chart.** In steaming a short distance a ship generally follows a rhumb line for the convenience of maintaining a constant course. For added convenience navigators use freely a chart on which any rhumb line will appear as a straight line. Such a chart is called a *Mercator chart*.

On a Mercator chart the meridians appear as a set of parallel lines spaced at equal distances for equal differences in longitude; the parallels of latitude appear as a set of parallel lines perpendicular to the first set. Since the meridians are represented by parallel lines and a rhumb line must cut them at the same angle, the rhumb line must appear as a straight line on the chart.

In Fig. 13 the length  $X'Y'$  represents the length  $XY$  on the equator, and  $A'B'$  represents the arc  $AB$  of a parallel of latitude.

In accordance with formula (3) arc  $AB = \text{arc } XY \cos L$ ; and, since  $A'B' = X'Y'$ , it is apparent that arc  $AB$  appears on the chart expanded to  $1/\cos L = \sec L$  times its natural size. Since the parallels of latitude are expanded in the ratio  $\sec L$ , the meridians near each parallel must be expanded in the same ratio

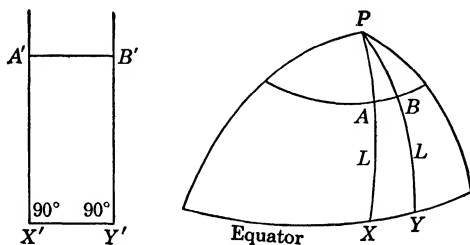


FIG. 13.

to avoid local distortion. The greater the latitude the greater the distortion; for as  $L$  increases so does  $\sec L$ . However, since the ratio of expansion is always  $\sec L$ , the length  $d$  of any short part of a rhumb line will be approximately equal to the line segment of length  $d_m$  representing this part on the map multiplied by the cosine of the mid-latitude for the segment. In symbols

$$d = d_m \cos (\text{mid. lat.}). \quad (6)$$

If  $B$  in Fig. 13 is in latitude  $L$  and the earth be assumed spherical in shape the distance  $Y'B'$  on the map would be, to some scale,  $R \log (\sec L + \tan L) = (21,600/2\pi) \log (\sec L + \tan L)$  miles.\* Because of the fact that the meridians are slightly elliptical, this formula cannot be used for large distances.

\* For those who have studied calculus it may be interesting to read the following derivation. Let  $C$  in the adjoining figure to some scale, represent the length  $Y'B'$  of Fig. 13 in map units which would represent miles along  $X'Y'$ . Then if  $AC$  represents a slight change in  $C$  and  $\Delta L$  the corresponding change in latitude we have

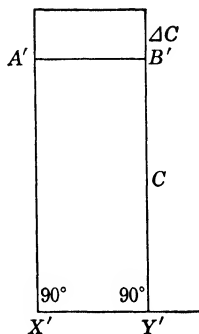
$$\Delta C = (R \Delta L) \sec L,$$

or from calculus

$$dC = R \sec L dL.$$

Hence

$$\begin{aligned} C &= \int_0^L R \sec L dL = R \log (\sec L + \tan L) \\ &= \frac{(360)(60)}{2\pi} \log (\sec L + \tan L). \end{aligned}$$



The scale for the maps shown (see Fig. 14) is such that  $\frac{1}{2}$  in. is assigned to each degree of longitude (or of latitude at the equator). Hence any length on the map can be changed to minutes, and therefore to miles by multiplying its length in inches by 120, or by *laying it off along the horizontal longitude scale and reading the corresponding number of degrees and minutes directly.*

The essential facts may be summarized as follows:

When the length  $d_m$  of any line is found in minutes of the longitude scale *the corresponding true length  $d$  may be obtained by using*

$$d = d_m \cos (\text{mid. Lat.}), (\text{approx.}). \quad (6)$$

Also the latitudes of the ends of the line may be read from the chart and used in the first of formulas (5) slightly transformed to read

$$d = (L_2 - L_1)' \sec C. \quad (7)$$

Observe that  $L_2 - L_1$  must be expressed in minutes and that  $C$ , the course angle, may be found by using a protractor.

**Example.** Figure 14 represents a Mercator chart. Approximately how many miles are represented by lines  $BC$ ,  $BA$ , and  $AC$ ?

**Solution.** Measurement of  $BC$  gives its length as  $2\frac{1}{8}$  in. The corresponding number of minutes is  $(2\frac{1}{8}) \times 120' = 247.5'$ . The mid-latitude is  $31.5^\circ$ . Hence, in accordance with (6),  $BC$  represents the length  $d$  given by

$$d = 247.5 \cos 31.5^\circ = \mathbf{211.3 \text{ miles.}}$$

The student should also find this result by applying formula (7).

Similarly  $BA$  is 1.75 in. long, and it represents the length  $l$  given by

$$l = 1.75 \times 120 \cos 31.5^\circ = \mathbf{180 \text{ miles.}}$$

Observe that it is the difference in latitude for the track  $BC$ . This could have been found by observing that  $BA$  represents the three degrees of latitude from  $30^\circ$  to  $33^\circ$  on the left of the chart. Hence it represents  $3 \times 60 = 180$  miles.

The length  $AC$  is  $1\frac{3}{8}$  in., and  $AC$  lies in latitude  $33^\circ$ . Hence in accordance with (6) it represents the length  $p$  given by

$$p = (\frac{3}{8}) \times 120 \cos 33^\circ = \mathbf{110 \text{ miles.}}$$

Observe that this is the departure for track  $BC$



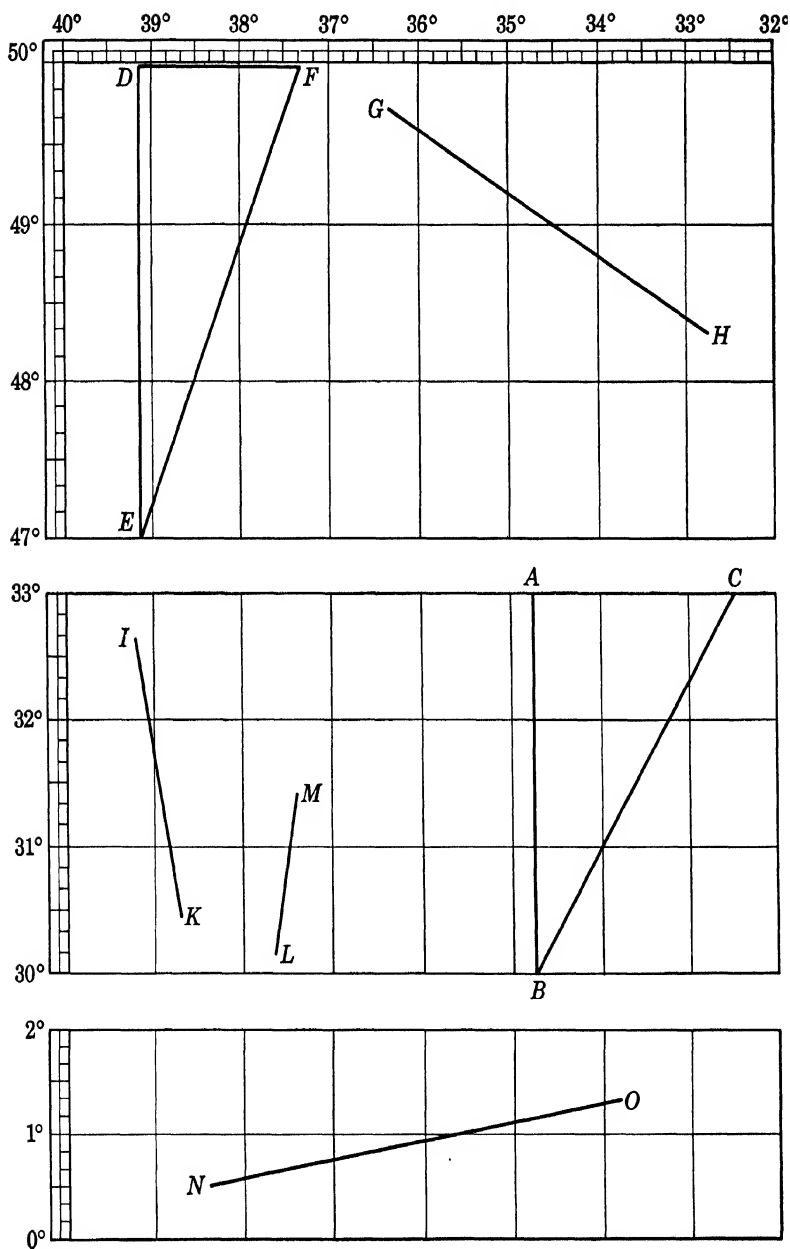


FIG. 14.

## EXERCISES

1. In Fig. 14 find approximately how many miles are represented by  $DE$ ,  $EF$ , and  $FD$ .

2. Read from the chart of Fig. 14 the latitude and longitude of each point lettered.

3. Using formula (6) find the rhumb line distance represented by each of the following lines in Fig. 14: (a)  $GH$ , (b)  $IK$ , (c)  $LM$ , (d)  $NO$ .

4. If a ship sails from  $G$  to  $H$  (see Fig. 14), find the difference in latitude and the difference in longitude (a) by reading these quantities directly from the figure, (b) by using formulas (5).

5. In Exercise 4 replace  $G$  by  $K$  and  $H$  by  $I$  and then solve the problem.

6. Plot on Fig. 14 point  $M_1$ ,  $L = 49^\circ 20'$ ,  $\lambda = 38^\circ$ , and point  $M_2$ ,  $L = 47^\circ 30'$ ,  $\lambda = 32^\circ 30'$ . Draw a line connecting these points and measure the angle (course angle) this line makes with a meridian. Measure the length of the line and use formula (6) to find the number of miles it represents.

7. In Exercise 6 change  $49^\circ 20'$  to  $47^\circ 20'$  and  $47^\circ 30'$  to  $48^\circ 10'$ . Solve the resulting problem.

8. From a point  $M_1$  in latitude  $30^\circ 30'$ , longitude  $39^\circ 40'$ , draw a line at an angle of  $50^\circ$  with the meridians and running upward and toward the right a distance of 2 in. At the upper end of this line segment make a dot and mark it  $M_2$ . Find the latitude and longitude of  $M_2$  (a) by reading these quantities from the chart, (b) by using formulas (5).

9. A ship steams from a point in latitude  $47^\circ 30'$ , longitude  $36^\circ 10'$  to a second point in latitude  $49^\circ 10'$ , longitude  $33^\circ 50'$ . Using Fig. 14, find the rhumb line distance between the two points and the rhumb line course angle. (Measure the course angle with a protractor.)

10. A ship steams on a rhumb line course of  $70^\circ$  for a distance of 45 miles from a point in latitude  $30^\circ 20'$ , longitude  $39^\circ 20'$  to a second point. Find the latitude and longitude of the second point.

11. In Exercise 9 change  $47^\circ 30'$  to  $47^\circ 10'$ ,  $33^\circ 50'$  to  $32^\circ 5'$  and solve the problem.

12. In Exercise 10 change  $30^\circ 20'$  to  $47^\circ 20'$ ,  $70^\circ$  to  $55^\circ$  and solve the problem.

13. With each of the following trips the rhumb line distance is tabulated.  $W$  represents westward sailing,  $E$  represents eastward sailing. Using (7) find, in each case, the course  $C_n$ .

Distance

(a) San Francisco  $L = 37^\circ 48' N.$  to Honolulu  $L = 21^\circ 18' N.$   $W$  2100 mi.

	Distance
(b) Honolulu $L = 21^{\circ}18' \text{ N.}$ to Manila $L = 14^{\circ}36' \text{ N.}$	$W$ 2160 mi.
(c) Manila $L = 14^{\circ}36' \text{ N.}$ to Tokyo $L = 35^{\circ}39' \text{ N.}$	$E$ 1620 mi
(d) Tokyo $L = 35^{\circ}39' \text{ N.}$ to Singapore $L = 1^{\circ}18' \text{ N.}$	$W$ 2880 mi.

14. With each of the following trips the course  $C_n$  is tabulated. Using (7) find, in each case, the rhumb line distance.

	Course
(a) Singapore $L = 1^{\circ}18' \text{ N.}$ to Darwin $L = 12^{\circ}23' \text{ S.}$	$117^{\circ}5'$
(b) New York $L = 40^{\circ}42' \text{ N.}$ to Liverpool $L = 53^{\circ}27' \text{ N.}$	$75^{\circ}10'$
(c) Dakar $L = 14^{\circ}41' \text{ N.}$ to Natal, Brazil $L = 5^{\circ}47' \text{ S.}$	$221^{\circ}$

## CHAPTER V

### THE OBLIQUE SPHERICAL TRIANGLE

**36. Law of sines.** To prepare for solving spherical triangles, we shall develop general formulas analogous to those developed for plane triangles.

The law of sines for spherical triangles, analogous to the law of sines for plane triangles, may be stated as follows:

*The sines of the sides of a spherical triangle are proportional to the sines of the angles opposite, or in symbols*

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \quad (1)$$

In Fig. 1 let  $a, b, c$  represent the sides of a spherical triangle and let  $A, B, C$  represent the opposite angles. Draw an arc

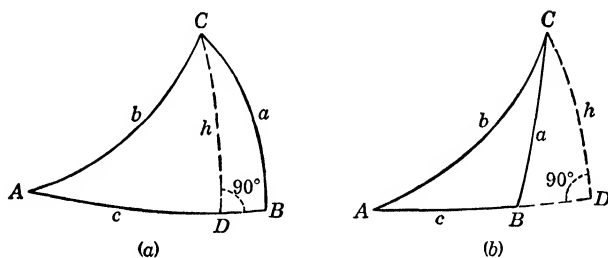


FIG. 1.

$CD(=h)$  of a great circle through the vertex  $C$  perpendicular to the side  $c$ , or the side  $c$  produced, to form the right spherical triangles  $ACD$  and  $BCD$ . Apply Napier's rules to these right triangles to obtain

$$\sin h = \sin b \sin A, \quad \sin h = \sin a \sin B.$$

Equating these two values of  $\sin h$ , we get

$$\sin a \sin B = \sin b \sin A,$$

or, dividing by  $\sin A \sin B$ ,

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}. \quad (2)$$

In like manner, by drawing an arc from  $A$  perpendicular to  $CB$  and arguing as above, we can show that

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \quad (3)$$

Equations (2) and (3) are together equivalent to (1). The law of sines may be used in the solution of a spherical triangle when a side and the angle opposite are included among the given parts.

When a part of a spherical triangle is found by means of the law of sines, there is often some difficulty in determining whether the part found is of the first quadrant or of the second quadrant; for  $\sin A = \sin (180^\circ - A)$ . Other formulas must be used in many cases. However, the following theorems from solid geometry will often enable the computer to determine the quadrant.

**The order of magnitude of the sides of a spherical triangle is the same as the order of magnitude of the respective opposite angles; or, in symbols, if**

$$a < b < c, \quad \text{then} \quad A < B < C.$$

**The sum of two sides of a spherical triangle is greater than the third side.**

### EXERCISES

1. Figure 2 represents the spherical triangle  $ABC$  with its associated trihedral angle  $O$ , the face angles of which are  $a, b, c$ .  $AF$  is the intersection of two planes, one perpendicular to  $OB$ , the other perpendicular to  $OC$ . Point  $F$  is in plane  $OCB$ . Taking  $OA = 1$  unit, express the values of all straight-line segments of the figure in terms of  $a, b, c, B$ , and  $C$ . Derive the law of sines from the result.

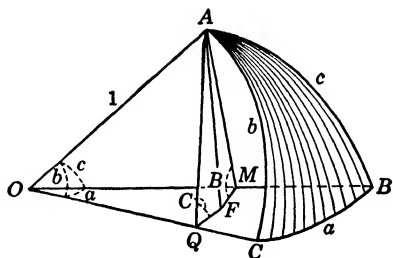


FIG. 2.

2. Check the following data by using the law of sines:

$$\begin{aligned} (a) \quad & A = 108^\circ 40', \quad B = 134^\circ 20', \quad C = 70^\circ 18', \quad a = 145^\circ 36', \\ & b = 154^\circ 45', \quad c = 34^\circ 9'. \end{aligned}$$

$$(b) A = 47^{\circ}21', B = 22^{\circ}20', C = 146^{\circ}40', a = 117^{\circ}9', b = 27^{\circ}22', c = 138^{\circ}20'.$$

$$(c) A = 110^{\circ}10', B = 133^{\circ}18', C = 70^{\circ}16', a = 147^{\circ}6', b = 155^{\circ}5', c = 32^{\circ}59'.$$

3. Use the law of sines to find the missing parts of the following right spherical triangles:

$$(a) a = 58^{\circ}8'19'', b = 32^{\circ}49'22'', B = 37^{\circ}12'53'', c = 63^{\circ}40'.$$

$$(b) a = 36^{\circ}14'6'', A = 49^{\circ}29'56'', b = 38^{\circ}45', c = 51^{\circ}1'11''.$$

4. Use the law of sines to find the missing part of each of the following spherical triangles:

$$(a) A = 130^{\circ}5'22'', B = 32^{\circ}26'6'', C = 36^{\circ}45'26'', c = 51^{\circ}6'12'', a = 84^{\circ}14'29''.$$

$$(b) A = 70^{\circ}, C = 94^{\circ}48'12'', c = 116^{\circ}, a = 57^{\circ}56'53'', b = 137^{\circ}20'33''.$$

5. Solve the polar triangles of the triangles of Exercise 3.

**37. The law of cosines for sides.** *The cosine of any side of a spherical triangle is equal to the product of the cosines of the two other sides increased by the product of the sines of the two other sides and the cosine of the angle included between them, or in symbols*

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \quad (4)$$

The following proof is analogous to the one given for the law of cosines in plane trigonometry.

In Fig. 1 let arc  $AD = \varphi$ . Then arc  $BD = c - \varphi$ . Write these values on the triangle of Fig. 1(a), and place bars over  $a, b, A$ , and  $B$  in preparation for using Napier's rules. The result is Fig. 3.

Now apply Napier's rules to triangles  $ACD$  and  $BCD$  to obtain

$$\cos a = \cos h \cos (c - \varphi), \quad (5)$$

$$\cos b = \cos h \cos \varphi. \quad (6)$$

Divide (5) by (6) member by member, and transform slightly to get

$$\frac{\cos a}{\cos b} = \frac{\cos h \cos (c - \varphi)}{\cos h \cos \varphi} = \frac{\cos c \cos \varphi + \sin c \sin \varphi}{\cos \varphi}, \quad (7)$$

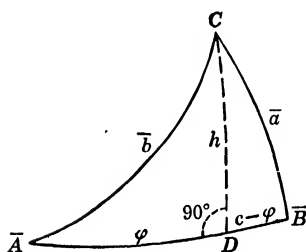


FIG. 3.

or, simplifying further,

$$\cos a = \cos b(\cos c + \sin c \tan \varphi). \quad (8)$$

Again apply Napier's rules, using parts  $b$ ,  $A$ ,  $\varphi$  of triangle  $ACD$  to obtain

$$\cos A = \cot b \tan \varphi,$$

or

$$\tan \varphi = \cos A \tan b. \quad (9)$$

Replace  $\tan \varphi$  in (8) by its value from (9) to get

$$\cos a = \cos b(\cos c + \sin c \cos A \tan b), \quad (10)$$

or, simplifying the right-hand member,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \quad (11)$$

Similarly, we may obtain

$$\cos b = \cos a \cos c + \sin a \sin c \cos B, \quad (12)$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \quad (13)$$

An argument differing slightly from the one just used shows that (11) holds for a triangle shaped like the triangle of Fig. 1(b).

The law of cosines applies to the solution of a spherical triangle when two sides and the included angle are given. Although it is not adapted to logarithmic computation, it is used in the derivation of many important formulas of spherical trigonometry.

**Example.** Find  $c$  in the spherical triangle for which  $a = 76^\circ 24' 40''$ ,  $b = 58^\circ 18' 36''$ ,  $C = 116^\circ 30' 28''$ .

*Solution.* The law of cosines may be written

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

Here it will be necessary to compute each product in the right-hand member, add the results, and then find  $c$  from a table of natural cosines; or find the logarithm of the natural cosine, and then find  $c$  from the table giving the logarithms of cosines. The computation is indicated in the following form:

	(cos a cos b)	(sin a sin b cos C)
$a = 76^{\circ}24'40''$	$l \cos 9.37098$	$l \sin 9.98767$
$b = 58^{\circ}18'36''$	$l \cos 9.72042$	$l \sin 9.92988$
$C = 116^{\circ}30'28''$		$l \cos(-)9.64965$
0.12342	log 9.09140	
-0.36915		log (-)9.56720
-0.24573		

$\therefore c = \cos^{-1}(-0.24573) = 104^{\circ}13'30''.$

**38. The law of cosines for angles.** Applying (11) to the polar triangle (see §28) of  $ABC$ , we obtain

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'. \quad (14)$$

Using equation (11) of §28 to replace  $a'$ ,  $b'$ ,  $c'$ , and  $A'$  of (14) by  $180^{\circ} - A$ ,  $180^{\circ} - B$ ,  $180^{\circ} - C$ , and  $180^{\circ} - a$ , respectively, we obtain

$$\begin{aligned} \cos (180^{\circ} - A) &= \cos (180^{\circ} - B) \cos (180^{\circ} - C) \\ &\quad + \sin (180^{\circ} - B) \sin (180^{\circ} - C) \cos (180^{\circ} - a), \end{aligned}$$

or

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a,$$

or

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \quad (15)$$

Similarly, we obtain from (12) and (13)

$$\cos B = -\cos A \cos C + \sin A \sin C \cos b, \quad (16)$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c. \quad (17)$$

Evidently this process of applying known formulas to the polar triangle of a given one is very important. It furnishes a method of deriving from every equation applying to a general spherical triangle another equation that may be called the *dual* of the first one. The role played by the sides in the given equation is played by the angles in the dual equation, and the role played by the angles in the given equation is played by the sides in the other. A similar statement applies to theorems relating to a spherical triangle. This principle of duality will come to our attention again and again in the discussion that follows.

**Example.** In a certain spherical triangle,  $A = 60^{\circ}$ ,  $B = 60^{\circ}$ , and  $c = 60^{\circ}$ . Find  $C$ .



*Solution.* Substituting  $60^\circ$  for each of the letters  $A$ ,  $B$ , and  $c$  in (17), we obtain

$$\begin{aligned}\cos C &= -\cos 60^\circ \cos 60^\circ + \sin 60^\circ \sin 60^\circ \cos 60^\circ \\ &= -\frac{1}{4} + \frac{3}{8} = \frac{1}{8}.\end{aligned}$$

Hence

$$C = \cos^{-1} \frac{1}{8} = 82^\circ 49' 9''.$$

### EXERCISES

1. Use the law of cosines to find  $a$  for each of the following spherical triangles:

(a) $b = 60^\circ$ ,	(b) $b = 45^\circ$ ,	(c) $b = 45^\circ$ ,
$c = 30^\circ$ ,	$c = 30^\circ$ ,	$c = 60^\circ$ ,
$A = 45^\circ$ .	$A = 120^\circ$ .	$A = 150^\circ$ .

2. Use the law of cosines for angles to find  $A$  for each of the following triangles:

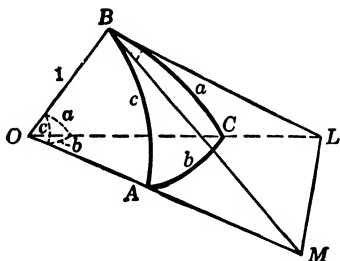
(a) $B = 120^\circ$ ,	(b) $B = 135^\circ$ ,
$C = 150^\circ$ ,	$C = 120^\circ$ ,
$a = 135^\circ$ .	$a = 30^\circ$ .

3. In a spherical triangle, given  $a = 30^\circ$ ,  $b = 45^\circ$ ,  $c = 60^\circ$ , find  $A$ .

4. Derive the law of sines algebraically from the law of cosines.

*Hint.* Solve (11) for  $\cos A$ , form  $\sin^2 A$ , and reduce the numerator to a form involving cosines only. Then show that  $\sin^2 A / \sin^2 a$  is symmetrical in  $a, b, c$ .

5. In Fig. 4,  $ABC$  represents a spherical triangle with its associated trihedral angle  $O$ .  $BLM$  is a plane through  $B$  perpendicular to  $OB$ , intersecting  $OA$  produced, in  $M$  and  $OC$  produced, in  $L$ . Taking  $OB = 1$  unit, express the values of the line segments  $OL$ ,  $OM$ ,  $BL$ ,  $BM$  in terms of  $a, b, c$ , then apply the law of cosines of plane trigonometry to the triangles  $BLM$ , and  $OLM$ , and equate two values of  $\overline{LM}^2$  to obtain after slight transformation



$$\cos b = \cos a \cos c + \sin a \sin c \cos B.$$

6. From formula (15) show that

$$\text{hav } (180^\circ - A) = \text{hav } (B + C) - \sin B \sin C \text{ hav } a,$$

remembering that  $\text{hav } A = \frac{1}{2}(1 - \cos A)$ .

7. In each of the triangles of Exercise 1 complete the solution by means of the law of sines.

8. Solve the polar triangles of the triangles of Exercises 1 and 3.

9. Using the law of cosines, prove that in a spherical triangle having three sides of the second quadrant the angles opposite are of the second quadrant.

10. What equations are dual to those expressing the law of sines?

11. Find the equation dual to the one written in Exercise 6.

12. Replace  $C$  by  $90^\circ$  in (1), (13), (15), and (17), and then obtain the resulting formulas by applying Napier's rules to the parts of a right spherical triangle.

**39. The six cases.** When three parts of a spherical triangle are given, the other three parts can be computed. Accordingly a classification of spherical triangles is made on the basis of given parts. Six cases are referred to as follows:

- I. Given the three sides.
- II. Given the three angles.
- III. Given two sides and the included angle.
- IV. Given two angles and the included side.
- V. Given two sides and an angle opposite one of them.
- VI. Given two angles and a side opposite one of them.

For purposes of solution, there are, in a sense, only three cases. If a method of solution for Case I is known, this same method may be applied to solve the polar of a triangle classified under Case II. The solution of a quadrantal triangle in §29 by the method of solving a right spherical triangle illustrates the process. Similarly, the formulas used to solve a triangle classified under Case III may be used to solve the polar of a triangle classified under Case IV; also, the same formulas may be used to solve a triangle coming under Case V and the polar of a triangle classified under Case VI.

**40. The half-angle formulas.** This article is devoted to the derivation of formulas that may be used to solve triangles for

which the given parts are three sides or three angles. Solving (11) for  $\cos A$ , we have

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}. \quad (18)$$

Equating 1 minus the left-hand member to 1 minus the right-hand member and simplifying slightly, we get

$$1 - \cos A = \frac{\sin b \sin c + \cos b \cos c - \cos a}{\sin b \sin c},$$

or, replacing  $\sin b \sin c + \cos b \cos c$  by  $\cos(b - c)$ ,

$$1 - \cos A = \frac{\cos(b - c) - \cos a}{\sin b \sin c}.$$

Now, replacing  $1 - \cos A$  by  $2 \sin^2 \frac{1}{2}A$  and changing the right-hand member by using (14) of §19 and the fact that  $\sin(-\theta) = -\sin \theta$ , we get

$$2 \sin^2 \frac{1}{2}A = \frac{2 \sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c}. \quad (19)$$

Denote half the sum of the sides by  $s$  and write

$$s = \frac{1}{2}(a + b + c). \quad (20)$$

Subtracting in succession  $a$ ,  $b$ , and  $c$  from both members of (20), we obtain

$$\left. \begin{aligned} s - a &= \frac{1}{2}(-a + b + c), & s - b &= \frac{1}{2}(a - b + c), \\ s - c &= \frac{1}{2}(a + b - c). \end{aligned} \right\} \quad (21)$$

Substituting from (21) in (19) and taking the square root of both members, we obtain

$$\sin \frac{1}{2}A = \sqrt{\frac{\sin(s - b) \sin(s - c)}{\sin b \sin c}}. \quad (22)$$

Considerations of symmetry show that

$$\sin \frac{1}{2}B = \sqrt{\frac{\sin(s - a) \sin(s - c)}{\sin a \sin c}}, \quad (23)$$

$$\sin \frac{1}{2}C = \sqrt{\frac{\sin(s - a) \sin(s - b)}{\sin a \sin b}}. \quad (24)$$

Similarly, proceeding as above, we obtain

$$\begin{aligned}
 1 + \cos A &= 1 + \frac{\cos a - \cos b \cos c}{\sin b \sin c}, \\
 &= \frac{\cos a - (\cos b \cos c - \sin b \sin c)}{\sin b \sin c}, \\
 &= \frac{\cos a - \cos (b + c)}{\sin b \sin c}, \\
 1 + \cos A &= \frac{2 \sin \frac{1}{2}(a + b + c) \sin \frac{1}{2}(-a + b + c)}{\sin b \sin c}. \quad (25)
 \end{aligned}$$

Replacing in (25)  $1 + \cos A$  by  $2 \cos^2 \frac{1}{2}A$ , using (20) and (21) and extracting the square root of both members, we get

$$\cos \frac{1}{2}A = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}}. \quad (26)$$

Considerations of symmetry show that

$$\cos \frac{1}{2}B = \sqrt{\frac{\sin s \sin (s - b)}{\sin a \sin c}}, \quad (27)$$

$$\cos \frac{1}{2}C = \sqrt{\frac{\sin s \sin (s - c)}{\sin a \sin b}}. \quad (28)$$

Dividing (22) by (26), member by member, and replacing  $\sin \frac{1}{2}A \div \cos \frac{1}{2}A$  by  $\tan \frac{1}{2}A$ , we obtain

$$\tan \frac{1}{2}A = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}}. \quad (29)$$

Multiplying numerator and denominator under the radical by  $\sin (s - a)$  and removing  $1/\sin^2 (s - a)$  from the radical, we have

$$\tan \frac{1}{2}A = \frac{1}{\sin (s - a)} \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}, \quad (30)$$

or

$$\tan \frac{1}{2}A = \frac{r}{\sin (s - a)}, \quad (31)$$

where

$$r = \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}. \quad (32)$$

Similarly,

$$\tan \frac{1}{2}B = \frac{r}{\sin(s-b)}, \quad (33)$$

$$\tan \frac{1}{2}C = \frac{r}{\sin(s-c)}. \quad (34)$$

Since  $\text{hav } A = \sin^2 \frac{1}{2}A$ , formula (22) may be written

$$\text{hav } A = \sin(s-b) \sin(s-c) \csc b \csc c. \quad (35)$$

Similar formulas for  $\text{hav } B$  and  $\text{hav } C$  may be obtained from (23) and (24). Formula (35) is often used when haversine tables are available.

**41. Cases I and II. Given three sides or given three angles.** Evidently formulas (31), (33), and (34) are adapted to solve a spherical triangle when three sides are given. To solve a spherical triangle when the three angles are given, we find the sides of the polar triangle by subtracting each of the given angles from  $180^\circ$  and then applying equations (31), (33), and (34) to find the angles of the polar triangle; subtraction of each of these angles from  $180^\circ$  gives the sides of the original triangle. Also, the formulas of Exercise 1 on page 299 may be used.

**Example.** Find  $A$ ,  $B$ , and  $C$  for a spherical triangle in which  $a = 70^\circ 14' 20''$ ,  $b = 49^\circ 24' 10''$ ,  $c = 38^\circ 46' 10''$ .

**Solution.**  $s = \frac{1}{2}(a + b + c) = 79^\circ 12' 20''$ . The solution by means of formulas (32), (31), (33), and (34) and the check by the law of sines follows. The number in parenthesis above each column refers to the formula associated with the column.

	(32)	(31)	(33)	(34)	
$s - a = 8^{\circ}58'00''$	$l \sin 9 \ 19273$	$l \csc 0.80727$			
$s - b = 29^{\circ}48'10''$	$l \sin 9 \ 69637$		$l \csc 0.30363$		
$s - c = 40^{\circ}26'10''$	$l \sin 9 \ 81197$			$l \csc 0.18803$	
$s = 79^{\circ}12'20''$	$l \csc 0.00775$				
	$2) \log 8.70882$				
$r$	$\log 9.35441$	$\log 9.35441$	$\log 9.35441$	$\log 9.35441$	
$\frac{1}{2}A = 55^{\circ}25'38''$		$l \tan 0.16168$			
$A = 110^{\circ}51'16''$					
$\frac{1}{2}B = 24^{\circ}28'2''$			$l \tan 9.65804$		
$B = 48^{\circ}56'4''$					
$\frac{1}{2}C = 19^{\circ}13'23''$				$l \tan 9.54244$	
$C = 38^{\circ}26'46''$					
Check.					
$a$	$l \sin 9.97364$	$b$	$l \sin 9.88042$	$c$	$l \sin 9.79671$
$A$	$l \sin 9.97058$	$B$	$l \sin 9.87735$	$C$	$l \sin 9.79364$
	0.00306		0.00307		0.00307

## EXERCISES

1. Write  $\sigma = \frac{A + B + C}{2}$ , and use equations (11) of §28 to derive

$$s' = \frac{a' + b' + c'}{2} = 270^\circ - \frac{A + B + C}{2} = 270^\circ - \sigma,$$

$$s' - a' = 90^\circ - (\sigma - A), \quad s' - b' = 90^\circ - (\sigma - B),$$

$$s' - c' = 90^\circ - (\sigma - C).$$

Then apply equations (22), (26), and (29) to the polar triangle to obtain

$$\cos \frac{1}{2}a = \sqrt{\frac{\cos(\sigma - B) \cos(\sigma - C)}{\sin B \sin C}},$$

$$\sin \frac{1}{2}a = \sqrt{\frac{-\cos \sigma \cos(\sigma - A)}{\sin B \sin C}},$$

$$\tan \frac{1}{2}a = \sqrt{\frac{-\cos \sigma \cos(\sigma - A)}{\cos(\sigma - B) \cos(\sigma - C)}}.$$

2. Solve the following spherical triangles:

(a) $a = 30^\circ$ , $b = 45^\circ$ , $c = 60^\circ$ ,	(c) $a = 150^\circ$ , $b = 120^\circ$ , $c = 60^\circ$ .	(e) $A = 60^\circ$ , $B = 30^\circ$ , $C = 120^\circ$ .
(b) $a = 30^\circ$ , $b = 60^\circ$ , $c = 60^\circ$ .	(d) $A = 60^\circ$ , $B = 135^\circ$ , $C = 60^\circ$ .	(f) $A = 150^\circ$ , $B = 120^\circ$ , $C = 135^\circ$ .

3. Solve the following spherical triangles:

(a) $a = 110^\circ$ , $b = 32^\circ$ , $c = 96^\circ$ .	(e) $A = 80^\circ$ , $B = 110^\circ$ , $C = 130^\circ$ .
(b) $a = 108^\circ 14'$ , $b = 75^\circ 29'$ , $c = 56^\circ 37'$ .	(f) $A = 59^\circ 55' 10''$ , $B = 85^\circ 36' 50''$ , $C = 59^\circ 55' 10''$ .
(c) $a = 78^\circ 15' 12''$ , $b = 101^\circ 20' 18''$ , $c = 112^\circ 38' 42''$ .	(g) $A = 89^\circ 5' 46''$ , $B = 54^\circ 32' 24''$ , $C = 102^\circ 14' 12''$ .
(d) $a = 70^\circ 0' 37''$ , $b = 125^\circ 30' 52''$ , $c = 63^\circ 47' 55''$ .	(h) $A = 172^\circ 17' 56''$ , $B = 8^\circ 28' 20''$ , $C = 4^\circ 23' 35''$ .

4. Solve the polar triangles of the triangles of Exercise 2.

5. Derive the following equations from (22) to (34):

$$\begin{aligned}\frac{\cos \frac{1}{2}A \cos \frac{1}{2}B}{\sin \frac{1}{2}C} &= \frac{\sin s}{\sin c}, \\ \frac{\cos \frac{1}{2}A \sin \frac{1}{2}B}{\cos \frac{1}{2}C} &= \frac{\sin (s-a)}{\sin c}, \\ \frac{\sin \frac{1}{2}A \cos \frac{1}{2}B}{\cos \frac{1}{2}C} &= \frac{\sin (s-b)}{\sin c}, \\ \frac{\sin \frac{1}{2}A \sin \frac{1}{2}B}{\sin \frac{1}{2}C} &= \frac{\sin (s-c)}{\sin c}.\end{aligned}$$

6. Prove that the following relation holds true for a right spherical triangle:

$$\tan^2 \frac{1}{2}A = \sin (c-b) \csc (c+b).$$

**42. Napier's analogies.** This article is devoted to deriving formulas that may be used to solve triangles for which the given parts are two sides and the included angle or two angles and the included side. Substituting  $A = \frac{1}{2}A$  and  $B = \frac{1}{2}B$  in (7) and (9) of §19, we get

$$\sin \frac{1}{2}(A+B) = \sin \frac{1}{2}A \cos \frac{1}{2}B + \cos \frac{1}{2}A \sin \frac{1}{2}B, \quad (36)$$

$$\sin \frac{1}{2}(A-B) = \sin \frac{1}{2}A \cos \frac{1}{2}B - \cos \frac{1}{2}A \sin \frac{1}{2}B. \quad (37)$$

Dividing (37) by (36) member by member, we get

$$\frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} = \frac{\sin \frac{1}{2}A \cos \frac{1}{2}B - \cos \frac{1}{2}A \sin \frac{1}{2}B}{\sin \frac{1}{2}A \cos \frac{1}{2}B + \cos \frac{1}{2}A \sin \frac{1}{2}B}. \quad (38)$$

Or, dividing both numerator and denominator of the right-hand member of (38) by  $\sin \frac{1}{2}A \sin \frac{1}{2}B$ ,

$$\frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} = -\frac{\cot \frac{1}{2}A - \cot \frac{1}{2}B}{\cot \frac{1}{2}A + \cot \frac{1}{2}B}. \quad (39)$$

From (31) and (33) we find  $\cot \frac{1}{2}A = \frac{\sin (s-a)}{r}$  and  $\cot \frac{1}{2}B = \frac{\sin (s-b)}{r}$ . Substituting these values in (39) and canceling  $r$ , we obtain

$$\frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} = -\frac{\sin (s-a) - \sin (s-b)}{\sin (s-a) + \sin (s-b)}. \quad (40)$$

Using (14) of §19 to transform the right-hand member of (40), we get

$$\frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} = -\frac{2 \cos \frac{1}{2}(2s - a - b) \sin \frac{1}{2}(b - a)}{2 \sin \frac{1}{2}(2s - a - b) \cos \frac{1}{2}(b - a)}. \quad (41)$$

Replacing  $(2s - a - b)$  by  $c$  in (41) and simplifying slightly, we get

$$\frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}c}. \quad (42)$$

Again, using (9) and (7) of §19 with  $A = \frac{1}{2}A$  and  $B = \frac{1}{2}B$ , we get

$$\cos \frac{1}{2}(A - B) = \cos \frac{1}{2}A \cos \frac{1}{2}B + \sin \frac{1}{2}A \sin \frac{1}{2}B, \quad (43)$$

$$\cos \frac{1}{2}(A + B) = \cos \frac{1}{2}A \cos \frac{1}{2}B - \sin \frac{1}{2}A \sin \frac{1}{2}B. \quad (44)$$

Dividing (43) by (44) member by member, then dividing numerator and denominator of the right-hand member of the resulting equation by  $\sin \frac{1}{2}A \sin \frac{1}{2}B$  and finally replacing  $\cot \frac{1}{2}A$  by  $\frac{\sin(s - a)}{r}$  and  $\cot \frac{1}{2}B$  by  $\frac{\sin(s - b)}{r}$ , we have

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\frac{\sin(s - a) \sin(s - b)}{r^2} + 1}{\frac{\sin(s - a) \sin(s - b)}{r^2} - 1}. \quad (45)$$

Replacing  $r^2$  by its value from (32) and simplifying slightly, we obtain

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\sin s + \sin(s - c)}{\sin s - \sin(s - c)}. \quad (46)$$

Treating the right-hand member of this equation in a manner similar to that employed in transforming (40), we get

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a + b')}{\tan \frac{1}{2}c}. \quad (47)$$

Applying (42) and (47) to the polar triangle, we obtain



$$\frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} = \frac{\tan \frac{1}{2}(A - B)}{\cot \frac{1}{2}C}, \quad (48)$$

$$\frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} = \frac{\tan \frac{1}{2}(A + B)}{\cot \frac{1}{2}C}. \quad (49)$$

The formulas (42), (47), (48), and (49) are known as Napier's analogies. These formulas are analogous to the law of tangents in plane trigonometry.

### EXERCISES

1. Apply (42) and (47) to the polar triangle, then proceed in a manner analogous to that pursued in this article and obtain formulas (48) and (49).

2. Use formulas (42), (47), (48), and (49) to prove the following formulas known as Gauss's equations or Delambre's analogies:

$$\sin \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}c} \cos \frac{1}{2}C,$$

$$\sin \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}c} \cos \frac{1}{2}C,$$

$$\cos \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}c} \sin \frac{1}{2}C,$$

$$\cos \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}c} \sin \frac{1}{2}C.$$

3. Show that the second of Gauss's equations can be written

$$\text{hav}(A - B) = \frac{\text{hav}(a - b)}{\text{hav } c} \text{hav}(180^\circ - C).$$

4. From formula (47), show that in any spherical triangle one-half the sum of two angles is in the same quadrant as one-half the sum of the opposite sides; that is,  $\frac{1}{2}(a + b)$  and  $\frac{1}{2}(A + B)$  are in the same quadrant.

5. (a) Divide  $\sin \frac{1}{2}(A - B) = \sin \frac{1}{2}A \cos \frac{1}{2}B - \cos \frac{1}{2}A \sin \frac{1}{2}B$  by  $\cos \frac{1}{2}(A - B) = \cos \frac{1}{2}A \cos \frac{1}{2}B + \sin \frac{1}{2}A \sin \frac{1}{2}B$ , member by member, then proceed in a manner similar to that employed in this article in deriving (42) and thus deduce formula (48).

(b) Derive formula (49) by dividing  $\sin \frac{1}{2}(A + B)$  by  $\cos \frac{1}{2}(A + B)$ .

6. (a) Divide  $\sin \frac{1}{2}(A - B)$  by  $\cos \frac{1}{2}(A + B)$  and proceed in a manner similar to that outlined in 5 (a) and derive the formula

$$\frac{\sin \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\sin \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \frac{1}{2}c \cot \frac{1}{2}C.$$

**43. Cases III and IV.** Given two sides and the included angle or given two angles and the included side. The four formulas (42), (47), (48), and (49) are used to solve a triangle when the given parts are two sides and the included angle, or two angles and the side common to them. If the law of sines is used to find the last unknown after two unknowns have been found, often the ambiguity arising may be removed by using the theorem that states that the order of magnitude of the sides of a spherical triangle is the same as that of their respective opposite angles.

Other sets of formulas may be obtained from (42) and (47) to (49) by the interchange of letters. For example, another set would result from replacing  $a$  by  $c$ ,  $c$  by  $a$ ,  $A$  by  $C$ , and  $C$  by  $A$  in (42) and (47) to (49).

**Example.** Find  $A$ ,  $B$ , and  $c$  for a spherical triangle in which  $a = 57^\circ 56' 53''$ ,  $b = 137^\circ 20' 33''$ ,  $C = 94^\circ 48' 6''$ .

*Solution.* In this example  $\frac{1}{2}(b - a) = 39^\circ 41' 50''$ ,  $\frac{1}{2}(b + a) = 97^\circ 38' 43''$ ,  $\frac{1}{2}C = 47^\circ 24' 3''$ . Formulas (48), (49), (42), and (47) may be written in the respective forms

$$\tan \frac{1}{2}(B - A) = \sin \frac{1}{2}(b - a) \csc \frac{1}{2}(b + a) \cot \frac{1}{2}C, \quad (48')$$

$$\tan \frac{1}{2}(A + B) = \cos \frac{1}{2}(b - a) \sec \frac{1}{2}(b + a) \cot \frac{1}{2}C, \quad (49')$$

$$\tan \frac{1}{2}c = \tan \frac{1}{2}(b - a) \sin \frac{1}{2}(B + A) \csc \frac{1}{2}(B - A), \quad (42')$$

$$\tan \frac{1}{2}c = \tan \frac{1}{2}(b + a) \sec \frac{1}{2}(B - A) \cos \frac{1}{2}(B + A). \quad (47')$$

The following form indicates the computation. The number in parenthesis above each column refers to the formula associated with the column.

	(48')	(49')	(42')	check (47')
$\frac{1}{2}(b - a) = 39^\circ 41' 50''$	$l \sin 9.80531$	$l \cos 9.88617$	$l \tan 9.91915$	
$\frac{1}{2}(b + a) = 97^\circ 38' 43''$	$l \csc 0.00388$	$l \sec (-)0.87602$		$l \tan (-)0.87214$
$\frac{1}{2}C = 47^\circ 24' 3''$	$l \cot 9.96356$	$l \cot 9.96356$		
$\frac{1}{2}B - A = 30^\circ 39' 2''$	$l \tan 9.77275$		$l \csc 0.29260$	$l \sec 0.06535$
$\frac{1}{2}(B + A) = 100^\circ 38' 58''$		$l \tan (-)0.72575$	$l \sin 9.99245$	$l \cos (-)9.26670$
$\frac{1}{2}c = 57^\circ 59' 56''$			$l \tan 0.20420$	$l \tan 0.20419$
$A = 69^\circ 59' 56''$				
$B = 131^\circ 18' 0''$				
$c = 115^\circ 59' 52''$				

These results could have been checked by the law of sines.

## EXERCISES

1. Solve the following spherical triangles:

- |   |  |  |
|---|--|--|
| (a) $a = 30^\circ$ ,<br>$B = 45^\circ$ ,<br>$c = 60^\circ$ .  | (c) $a = 30^\circ$ ,<br>$C = 150^\circ$ ,<br>$b = 135^\circ$ . | (e) $B = 30^\circ$ ,<br>$a = 45^\circ$ ,<br>$C = 60^\circ$ .   |
| (b) $b = 135^\circ$ ,<br>$A = 45^\circ$ ,<br>$c = 60^\circ$ . | (d) $A = 150^\circ$ ,<br>$c = 30^\circ$ ,<br>$B = 120^\circ$ . | (f) $A = 60^\circ$ ,<br>$b = 120^\circ$ ,<br>$C = 150^\circ$ . |

2. In the following triangles where two values for a part are given, select the proper value.

- (a)  $A = 65^\circ 13'$ ,  $B = 49^\circ 28'$ ,  $130^\circ 33'$ ,  $C = 128^\circ 16'$ ,  $a = 88^\circ 24'$ ,  
 $b = 56^\circ 48'$ ,  $c = 120^\circ 11'$ .
- (b)  $A = 50^\circ 10'$ ,  $B = 135^\circ 5'$ ,  $C = 50^\circ 30'$ ,  $a = 69^\circ 35'$ ,  $110^\circ 25'$ ,  
 $b = 120^\circ 30'$ ,  $c = 70^\circ 20'$ .
- (c)  $A = 127^\circ 40'$ ,  $B = 45^\circ 15'$ ,  $C = 124^\circ 42'$ ,  $15^\circ 20'$ ,  $a = 68^\circ 53'$ ,  
 $b = 56^\circ 50'$ ,  $c = 18^\circ 10'$ .
- (d)  $A = 52^\circ 20'$ ,  $B = 45^\circ 15'$ ,  $C = 124^\circ 42'$ ,  $a = 68^\circ 53'$ ,  $b = 56^\circ 50'$ ,  
 $c = 104^\circ 19'$ ,  $18^\circ 10'$ .

3. Using Napier's analogies, solve the following spherical triangles:

- |   |  |
|---|--|
| (a) $c = 116^\circ 0' 0''$ ,<br>$A = 70^\circ 0' 0''$ ,<br>$B = 131^\circ 18' 0''$ .      | (d) $a = 86^\circ 18' 40''$ ,<br>$b = 45^\circ 36' 20''$ ,<br>$C = 120^\circ 46' 30''$ . |
| (b) $a = 88^\circ 37' 40''$ ,<br>$c = 125^\circ 18' 20''$ ,<br>$B = 102^\circ 16' 36''$ . | (e) $a = 41^\circ 6' 0''$ ,<br>$b = 119^\circ 24' 0''$ ,<br>$C = 162^\circ 22' 30''$ .   |
| (c) $a = 76^\circ 24' 0''$ ,<br>$b = 58^\circ 19' 0''$ ,<br>$C = 116^\circ 30' 0''$ .     | (f) $c = 120^\circ 18' 33''$ ,<br>$A = 27^\circ 22' 34''$ ,<br>$B = 91^\circ 26' 44''$ . |

4. In the following spherical triangles, find the angles by means of Napier's analogies and the required side by using the law of sines.

- |   |  |
|---|--|
| (a) $a = 42^\circ 45' 0''$ ,<br>$b = 47^\circ 15' 0''$ ,<br>$C = 11^\circ 11' 41''$ | (b) $a = 131^\circ 15' 0''$ ,<br>$b = 129^\circ 20' 0''$ ,<br>$C = 103^\circ 37' 23''$ . |
|---|--|

**44. Cases V and VI.** *Two of the given parts are opposites. Double solutions.* For convenience of reference, a theorem from solid geometry is repeated here.

**Theorem.** The order of magnitude of the sides of a spherical triangle is the same as that of their respective opposite angles. Or if  $a$  and  $b$  are a pair of sides of a spherical triangle and  $A$  and  $B$  the respective opposite angles, we know that if

$$a < b, \quad \text{then} \quad A < B. \quad (50)$$

When the given parts of a spherical triangle are two sides and an angle opposite one of them, say,  $a$ ,  $b$ , and  $A$ , the angle  $B$  may be found by using the law of sines,

$$\sin B = \frac{\sin b}{\sin a} \sin A. \quad (51)$$

Since  $\sin B$  does not exceed 1 in magnitude,  $\log \sin B$  does not exceed zero. Hence no solution will exist when  $\log \sin B > 0$ .

When  $\log \sin B < 0$ , a positive acute angle and its supplement must be considered for  $B$ . Each value of  $B$  must be consistent with (50). Hence, there will be no solution, one solution, or two solutions according as (50) is satisfied by neither, by one and only one, or by both of the values of  $B$  obtained from (51). If  $b = a$ , then  $B = A$ , and there is one solution.

Accordingly, begin the solution of a spherical triangle in which  $a$ ,  $b$ , and  $A$  are the given parts by using (51) to find  $\log \sin B$ . If  $\log \sin B > 0$ , there is no solution. If  $\log \sin B < 0$ , find two values of  $B$ , one a positive acute angle and the other its supplement. Then, to find  $c$  and  $C$ , use the given parts with each value of  $B$  that satisfies (50) in

$$\left. \begin{aligned} \tan \frac{1}{2}c &= \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} \tan \frac{1}{2}(a - b), \\ \cot \frac{1}{2}C &= \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} \tan \frac{1}{2}(A - B). \end{aligned} \right\} \quad (52)$$

These formulas were obtained by solving Napier's analogies (42) and (48) for  $\tan \frac{1}{2}c$  and  $\cot \frac{1}{2}C$ , respectively.

A similar discussion, with the roles of sides and angles interchanged, applies when the given parts are two angles and a side opposite one of them; (51) solved for  $\sin b$  would first be used and then (52).

**Example.** Given  $a = 52^\circ 45' 20''$ ,  $b = 71^\circ 12' 40''$ ,  $A = 46^\circ 22' 10''$ . find  $c$ ,  $B$ ,  $C$ .

*Solution.* Two solutions are to be expected. First using

$$\sin B = \sin b \sin A \csc a \quad (1')$$

to find  $B$ , and afterwards using (42') and (49) to find  $c_1$ ,  $C_1$ ,  $c_2$ , and  $C_2$ , we obtain the solution indicated below.

	(1')	
$a = 52^\circ 45' 20''$	$l \csc 0.09906$	
$b = 71^\circ 12' 40''$	$l \sin 9.97622$	
$A = 46^\circ 22' 10''$	$l \sin 9.85962$	
$\{ B_1 = 59^\circ 24' 22''$	$l \sin 9.93490$	
$\{ B_2 = 120^\circ 35' 38''$		
	(42')	(49)
$\frac{1}{2}(B_1 - A) = 6^\circ 31' 6''$	$l \csc 0.94492$	
$\frac{1}{2}(B_1 + A) = 52^\circ 53' 16''$	$l \sin 9.90171$	$l \tan 0.12112$
$\frac{1}{2}(b - a) = 9^\circ 13' 40''$	$l \tan 9.21075$	$l \sec 0.00565$
$\frac{1}{2}(b + a) = 61^\circ 59' 0''$		$l \cos 9.67185$
$\frac{1}{2}c_1 = 48^\circ 46' 26''$	$l \tan 0.05738$	
$c_1 = 97^\circ 32' 52''$		
$\frac{1}{2}C_1 = 57^\circ 49' 56''$		$l \cot 9.79862$
$C_1 = 115^\circ 39' 52''$		
	(42')	(49)
$\frac{1}{2}(b - a) = 9^\circ 13' 40''$	$l \tan 9.21075$	$l \sec 0.00565$
$\frac{1}{2}(b + a) = 61^\circ 59' 0''$		$l \cos 9.67185$
$\frac{1}{2}(B_2 - A) = 37^\circ 6' 44''$	$l \csc 0.21941$	
$\frac{1}{2}(B_2 + A) = 83^\circ 28' 54''$	$l \sin 9.99718$	$l \tan 0.94211$
$\frac{1}{2}c_2 = 14^\circ 58' 35''$	$l \tan 9.42734$	
$c_2 = 29^\circ 57' 10''$		
$\frac{1}{2}C_2 = 13^\circ 30' 4''$		$l \cot 0.61961$
$C_2 = 27^\circ 0' 8''$		

This solution may be checked by the law of sines.

### EXERCISES

Solve the following spherical triangles:

1.  $a = 68^\circ 52' 48''$ ,  
 $b = 56^\circ 49' 46''$ ,  
 $B = 45^\circ 15' 12''$ .

2.  $a = 34^\circ 0' 30''$ ,  
 $A = 61^\circ 29' 30''$ ,  
 $B = 24^\circ 30' 30''$ .

$$\begin{aligned} 3. \quad a &= 42^\circ 15' 20'', \\ A &= 36^\circ 20' 20'', \\ B &= 46^\circ 30' 40''. \end{aligned}$$

$$\begin{aligned} 5. \quad b &= 80^\circ, \\ A &= 70^\circ, \\ B &= 120^\circ. \end{aligned}$$

$$\begin{aligned} 4. \quad a &= 59^\circ 28' 27'', \\ A &= 52^\circ 50' 20'', \\ B &= 66^\circ 7' 20''. \end{aligned}$$

$$\begin{aligned} 6. \quad a &= 63^\circ 29' 56'', \\ b &= 132^\circ 14' 23'', \\ C &= 61^\circ 18' 27''. \end{aligned}$$

#### 45. MISCELLANEOUS EXERCISES

Solve the following spherical triangles:

$$\begin{aligned} 1. \quad a &= 120^\circ 22' 40'', \\ b &= 111^\circ 34' 27'', \\ c &= 96^\circ 28' 35''. \end{aligned}$$

$$\begin{aligned} 2. \quad a &= 41^\circ 6' 0'', \\ b &= 119^\circ 24' 0'', \\ C &= 48^\circ 54' 38''. \end{aligned}$$

$$\begin{aligned} 3. \quad A &= 121^\circ 32' 41'', \\ B &= 82^\circ 52' 53'', \\ C &= 98^\circ 51' 55''. \end{aligned}$$

$$\begin{aligned} 4. \quad c &= 86^\circ 15' 15'', \\ A &= 153^\circ 17' 6'', \\ B &= 78^\circ 43' 32''. \end{aligned}$$

$$\begin{aligned} 5. \quad b &= 84^\circ 21' 56'', \\ A &= 115^\circ 36' 45'', \\ B &= 80^\circ 19' 12''. \end{aligned}$$

$$\begin{aligned} 6. \quad a &= 40^\circ 5' 26'', \\ b &= 118^\circ 22' 7'', \\ C &= 160^\circ 1' 23''. \end{aligned}$$

$$\begin{aligned} 7. \quad b &= 150^\circ 17' 26'', \\ A &= 61^\circ 37' 53'', \\ B &= 139^\circ 54' 34''. \end{aligned}$$

$$\begin{aligned} 8. \quad a &= 31^\circ 11' 7'', \\ b &= 32^\circ 19' 18'', \\ c &= 33^\circ 15' 21''. \end{aligned}$$

$$\begin{aligned} 9. \quad A &= 63^\circ 57' 39'', \\ B &= 35^\circ 4' 3'', \\ c &= 132^\circ 44' 8''. \end{aligned}$$

$$\begin{aligned} 10. \quad A &= 59^\circ 55' 10'', \\ B &= 85^\circ 36' 50'', \\ C &= 59^\circ 55' 10''. \end{aligned}$$

11. In a spherical triangle given  $c$ ,  $A$ ,  $a + b$ , derive

$$\tan \frac{1}{2}A \tan \frac{1}{2}B = \frac{\sin(s - c)}{\sin s}.$$

12. Given two sides and the sum of the opposite angles of a spherical triangle derive a formula from Gauss's equations (Exercise 2, §148) for computing the remaining angle.

13. Prove the relation

$$\cot a \sin b = \cot A \sin C + \cos C \cos b.$$

*Hint.* Multiply equation (13) by  $\cos b$ , substitute in (11), and then divide by  $\sin b \sin a$ , etc.

14. If  $c_1$  and  $c_2$  be the two values of the third side when  $A$ ,  $a$ ,  $b$  are given and the triangle comes under Case V, show that

$$\tan \frac{1}{2}c_1 \tan \frac{1}{2}c_2 = \tan \frac{1}{2}(b - a) \tan \frac{1}{2}(b + a).$$

15. If  $b$  is the base of an isosceles spherical triangle and if the equal sides  $a, c$  be bisected by the arc  $h$  of a great circle, show that

$$\sin \frac{1}{2}h = \frac{1}{2} \sin \frac{1}{2}b \sec \frac{1}{2}a.$$

16. Prove that

$$\sin (s - a) + \sin (s - b) + \sin (s - c) - \sin s = 4 \sin \frac{1}{2}a \sin \frac{1}{2}b \sin \frac{1}{2}c.$$

17. In a spherical triangle  $A = B = 2C$ , show that

$$8 \sin^2 \frac{1}{2}C (\cos s + \sin \frac{1}{2}C) \cos \frac{1}{2}c = \cos a.$$

18. Show that

$$\text{hav } a = \frac{\sin \frac{1}{2}E \sin (A - \frac{1}{2}E)}{\sin B \sin C}$$

where  $E = (2\sigma - 180^\circ)$  and  $\sigma = \frac{1}{2}(A + B + C)$ .

19. In an equilateral spherical triangle, show that  $2 \cos \frac{1}{2}a \sin \frac{1}{2}A = 1$ .

20. If in a spherical triangle  $C = A + B$ , show that

$$\cos C = -\tan \frac{1}{2}a \tan \frac{1}{2}b.$$

21. If the sum of the angles of a spherical triangle is  $360^\circ$ , show that

$$\cos^2 \frac{1}{2}a + \cos^2 \frac{1}{2}b + \cos^2 \frac{1}{2}c = 1.$$

46. Case III. Alternate method.

Another set of formulas sufficient to solve the spherical triangle for which two sides and the included angle are known do not contain  $p$ .

Applying Napier's rule to triangle I of Fig. 5, we obtain

$$\tan \varphi = \tan b \cos C. \quad (53)$$

Also

$$\varphi' = a - \varphi. \quad (54)$$

Again, by using Napier's rules, we obtain from triangles II and I

$$\begin{aligned} \sin \varphi' &= \cot B \tan p, \\ \sin \varphi &= \cot C \tan p. \end{aligned} \quad (a)$$

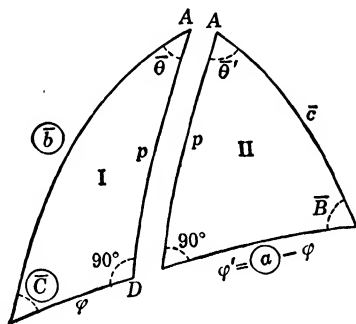


FIG. 5.

Dividing the first of these equations by the second, member by member, and solving the result for  $\cot B$ , we get

$$\cot B = \cot C \sin \varphi' \csc \varphi. \quad (55)$$

Note that the equations (a) were found by using  $\varphi'$ ,  $p$ , and  $B$  in triangle II and the homologous parts  $\varphi$ ,  $p$ , and  $C$  in triangle I. The procedure to get (55) will be followed to obtain a formula for  $\cos c$ . From triangles II and I, we get

$$\cos c = \cos \varphi' \cos p, \quad \cos b = \cos \varphi \cos p.$$

Dividing the first of these equations by the second, member by member, and solving for  $\cos c$ , we get

$$\cos c = \cos b \sec \varphi \cos \varphi'. \quad (56)$$

From triangle I

$$\cot \theta = \cos b \tan C; \quad (57)$$

from triangle II

$$\cot \theta' = \cos c \tan B, \quad (58)$$

and

$$A = \theta + \theta'. \quad (59)$$

The law of sines may be used as a check formula.

**Example.** Use formulas (53) to (59) of this article to solve the spherical triangle in which  $a = 68^\circ 20' 25''$ ,  $b = 52^\circ 18' 15''$ ,  $C = 117^\circ 12' 20''$ .

*Solution.* The solution and the check by the law of sines are displayed in the following form:

	(53)	(55)	(56)	(57)	(58)
$a = 68^\circ 20' 25''$					
$b = 52^\circ 18' 15''$	$l \tan 0.11194$		$l \cos 9.78638$	$l \cos 9.78638$	
$C = 117^\circ 12' 20''$	$l \cos (-)9.66009$	$l \cot (-)9.71100$		$l \tan (-)0.28900$	
$\varphi = 149^\circ 23' 20''$	$l \tan (-)9.77203$	$l \csc 0.29314$	$l \sec (-)0.06517$		
$\varphi' = a - \varphi = -81^\circ 3' 4''$		$l \sin (-)9.99468$	$l \cos 9.19188$		
$B = 45^\circ 4' 41''$		$l \cot 9.99882$			
$c = 96^\circ 20' 43''$			$l \cos (-)9.04343$		$l \tan 0.00118$
$\theta = 139^\circ 56' 51''$				$l \cot (-)0.07538$	
$\theta' = -83^\circ 40' 35''$					$l \cos (-)9.04343$
$A = \theta + \theta' = 56^\circ 16' 16''$					$l \cot (-)9.04461$
Check: $a$	$l \sin 9.96820$	$b$	$l \sin 9.89832$	$c$	$l \sin 9.99733$
$A$	$l \sin 9.91995$	$B$	$l \sin 9.85008$	$C$	$l \sin 9.94909$
	0.04825		0.04824		0.04824



## EXERCISES

Solve the following spherical triangles by the method of this article:

- |  |  |
|--|--|
| 1. $a = 88^{\circ}24'0''$ ,<br>$b = 56^{\circ}48'0''$ ,<br>$C = 128^{\circ}16'0''$ . | 4. $a = 88^{\circ}37'40''$ ,<br>$c = 125^{\circ}18'20''$ ,<br>$B = 102^{\circ}16'36''$ . |
| 2. $b = 120^{\circ}30'0''$ ,<br>$c = 70^{\circ}20'0''$ ,<br>$A = 50^{\circ}10'0''$ . | 5. $a = 86^{\circ}18'40''$ ,<br>$b = 45^{\circ}36'20''$ ,<br>$C = 120^{\circ}46'30''$ .  |
| 3. $a = 76^{\circ}24'0''$ ,<br>$b = 58^{\circ}19'0''$ ,<br>$C = 116^{\circ}30'0''$ . | 6. $b = 132^{\circ}17'30''$ ,<br>$c = 78^{\circ}15'15''$ ,<br>$A = 40^{\circ}20'10''$ .  |

Solve the following triangles by solving the polar triangle:

- |  |   |
|--|---|
| 7. $A = 120^{\circ}10'0''$ ,<br>$B = 100^{\circ}20'0''$ ,<br>$c = 30^{\circ}5'0''$ . | 8. $A = 27^{\circ}22'34''$ ,<br>$C = 91^{\circ}26'44''$ ,<br>$b = 120^{\circ}18'33''$ . |
|--|---|

9. Using Fig. 6, derive formulas (a) to (g):

$$\begin{aligned} \cot \theta &= \cos c \tan B, \\ \theta' &= A - \theta, \\ \tan b &= \tan c \cos \theta \sec \theta', \\ \cos C &= \cos B \csc \theta \sin \theta', \\ \tan \varphi &= \cos B \tan c, \\ \tan \varphi' &= \cos C \tan b, \\ a &= \varphi + \varphi'. \end{aligned}$$

(a)  
(b)  
(c)  
(d)  
(e)  
(f)  
(g)

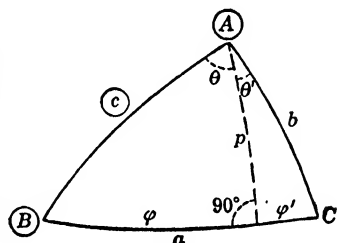


FIG. 6.

Using the formulas of Exercise 9, solve each of the following triangles:

- |   |   |
|---|---|
| 10. $a = 129^{\circ}5'28''$ ,<br>$B = 142^{\circ}12'42''$ ,<br>$C = 60^{\circ}4'54''$ . | 11. $A = 31^{\circ}34'26''$ ,<br>$B = 30^{\circ}28'12''$ ,<br>$c = 70^{\circ}2'3''$ . |
|---|---|

**47. Haversine solution of Case III.\*** Evidently the law of cosines could be used to find  $a$  when  $b$ ,  $c$ , and  $A$  are given. This would not, however, be convenient for logarithmic computation. A formula for finding  $a$  directly by using a table of haversines will be developed from the law of cosines.

\* A table of haversines is contained in "Useful Tables from the American Practical Navigator," H. O. No. 9, Part II, published by the United States Hydrographic Office, Washington, D. C.

The law of cosines may be written

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \quad (60)$$

By definition  $\text{hav } \theta = \frac{1}{2}(1 - \cos \theta)$ . Solving this for  $\cos \theta$ , we get  $\cos \theta = 1 - 2 \text{ hav } \theta$ . Hence

$$\cos a = 1 - 2 \text{ hav } a, \quad \cos A = 1 - 2 \text{ hav } A. \quad (61)$$

Substituting the expressions for  $\cos a$  and  $\cos A$  from (61) in (60), we obtain after slight simplification

$$1 - 2 \text{ hav } a = \cos b \cos c + \sin b \sin c - 2 \sin b \sin c \text{ hav } A. \quad (62)$$

Now  $\cos b \cos c + \sin b \sin c = \cos (b - c) = 1 - 2 \text{ hav } (b - c)$ . Replacing  $\cos b \cos c + \sin b \sin c$  by  $1 - 2 \text{ hav } (b - c)$  in (62) and solving for  $\text{hav } a$ , we obtain

$$\text{hav } a = \text{hav } (b - c) + \sin b \sin c \text{ hav } A. \quad (63)$$

Similarly,

$$\text{hav } b = \text{hav } (a - c) + \sin a \sin c \text{ hav } B, \quad (64)$$

$$\text{hav } c = \text{hav } (a - b) + \sin a \sin b \text{ hav } C. \quad (65)$$

After a side has been computed by the haversine formula, three sides and an angle will be known. The other two angles may then be obtained by using the law of sines. The facts that when  $a < b < c$  then  $A < B < C$  and that the sum of two sides is greater than the third side will often serve to determine the quadrant of each angle thus found. Also, since in  $\cos a - \cos b \cos c = \sin b \sin c \cos A$ ,  $\sin b$  and  $\sin c$  are always positive, it is evident that the *angle A will be in the first or second quadrant according as  $\cos a - \cos b \cos c$  is positive or negative.* When the sign of this quantity cannot be determined by inspection, the slide rule may be used. Also the result of solving (63) for  $\text{hav } A$ ,

$$\text{hav } A = \frac{\text{hav } a - \text{hav } (b - c)}{\sin b \sin c} \quad (66)$$

and the corresponding formulas for  $\text{hav } B$  and  $\text{hav } C$  are useful.

**Example.** Use (63) to find the side  $a$  of a spherical triangle in which  $b = 59^\circ 29' 30''$ ,  $c = 109^\circ 39' 40''$ ,  $A = 50^\circ 10' 10''$ ; then find  $B$  and  $C$  by the law of sines.

*Solution.* The formulas to be used are

$$\text{hav } a = \text{hav } (b - c) + \sin b \sin c \text{ hav } A, \quad (a)$$

$$\sin B = \sin b \sin A \csc a, \quad (b)$$

$$\sin C = \sin c \sin A \csc a. \quad (c)$$

The solution is displayed in the following form:

	(a)	(b)	(c)
$b = 59^{\circ}29'30''$	$l \sin 9.93529$	$l \sin 9.93529$	
$c = 109^{\circ}39'40''$	$l \sin 9.97391$		$l \sin 9.97391$
$A = 50^{\circ}10'10''$	$l \text{ hav } 9.25465$	$l \sin 9.88533$	$l \sin 9.88533$
$c - b = 50^{\circ}10'10''$			
	<hr/>	$n \text{ hav } 0.17974$	
	$\log 9.16385$	$n 0.11583$	
		$n \text{ hav } 0.32557$	
$a = 69^{\circ}34'56''$		$l \csc 0.02818$	$l \csc 0.02818$
$B = 44^{\circ}54'35''$		$l \sin 9.84880$	
$C = 129^{\circ}29'54''$			$l \sin 9.88742$

### EXERCISES

Using the haversine formula, find the unknown side in the following spherical triangles:

- $b = 125^{\circ}8'$ ,  
 $c = 64^{\circ}26'$ ,  
 $A = 100^{\circ}4'$ .
- $a = 131^{\circ}15'$ ,  
 $b = 129^{\circ}20'$ ,  
 $C = 103^{\circ}37'20''$ .
- $a = 63^{\circ}29'56''$ ,  
 $b = 132^{\circ}14'23''$ ,  
 $C = 61^{\circ}18'27''$ .
- $C = 48^{\circ}20'$ ,  
 $b = 52^{\circ}10'$ ,  
 $a = 49^{\circ}20'$ .

5. Solve Exercise 3 for  $B$  and  $A$  by using the law of sines.

6. Using the relation  $\cos \theta = 1 - 2 \text{ hav } \theta$ , derive from the cosine law  $\text{hav } c = \text{hav } (a - b) \text{ hav } (180^{\circ} - C) + \text{hav } (a + b) \text{ hav } C$ .

**48. Cases I and II.** The most expeditious method of solving a spherical triangle in which three sides are given employs formulas (31) to (34) of §40. However, one angle may be found by using

$$\cos A = (\cos a - \cos b \cos c) \csc b \csc c,$$

a formula obtained from the law of cosines, or by using (66) of §47, namely

$$\text{hav } A = [\text{hav } a - \text{hav } (b - c)] \csc b \csc c.$$

Two sides and the included angle will then be known, and the law of sines may be employed. The spherical triangle for which

three angles are given may be solved by means of its polar triangle.

### EXERCISES

Solve the following spherical triangles:

- |   |  |
|---|--|
| 1. $a = 57^\circ$ ,<br>$b = 137^\circ$ ,<br>$c = 116^\circ$ .             | 4. $A = 116^\circ 35' 36''$ ,<br>$B = 105^\circ 14' 48''$ ,<br>$C = 43^\circ 17' 12''$ . |
| 2. $A = 150^\circ$ ,<br>$B = 131^\circ$ ,<br>$C = 115^\circ$ .            | 5. $a = 77^\circ 36' 12''$ ,<br>$b = 63^\circ 16' 48''$ ,<br>$c = 107^\circ 23' 12''$ .  |
| 3. $a = 149^\circ 30'$ ,<br>$b = 131^\circ 0'$ ,<br>$c = 119^\circ 20'$ . | 6. $A = 136^\circ 19' 36''$ ,<br>$B = 43^\circ 18' 30''$ ,<br>$C = 114^\circ 43' 18''$ . |

### 49. MISCELLANEOUS EXERCISES

Solve the following spherical triangles:

- |   |  |
|---|--|
| 1. $a = 76^\circ 24' 40''$ ,<br>$b = 58^\circ 18' 36''$ ,<br>$C = 116^\circ 30' 28''$ . | 5. $a = 99^\circ 40' 48''$ ,<br>$b = 64^\circ 23' 15''$ ,<br>$A = 95^\circ 38' 4''$ .  |
| 2. $b = 99^\circ 40' 48''$ ,<br>$c = 100^\circ 49' 30''$ ,<br>$A = 65^\circ 33' 10''$ . | 6. $A = 73^\circ 11' 18''$ ,<br>$B = 61^\circ 18' 12''$ ,<br>$a = 46^\circ 45' 30''$ . |
| 3. $A = 31^\circ 34' 26''$ ,<br>$B = 30^\circ 28' 12''$ ,<br>$c = 70^\circ 2' 3''$ .    | 7. $a = 57^\circ 17'$ ,<br>$b = 20^\circ 39'$ ,<br>$c = 76^\circ 22'$ .                |
| 4. $a = 40^\circ 5' 26''$ ,<br>$b = 118^\circ 22' 7''$ ,<br>$A = 29^\circ 42' 34''$ .   | 8. $A = 86^\circ 20'$ ,<br>$B = 76^\circ 30'$ ,<br>$C = 94^\circ 40'$ .                |

9. A ship sailing on a great circle crosses the equator in longitude  $78^\circ 26'$  W. with course  $43^\circ 32'$ . Find its latitude when its longitude is  $10^\circ$  W.

10. A ship sails 5400 nautical miles from San Francisco, Lat.  $37^\circ 48'$  N., Long.  $122^\circ 23'$  W., along a great circle with initial course of  $240^\circ 25'$ . Find the position reached.

11. Find the pole ( $L, \lambda$ ) of the great circle of Exercise 10.

12. An airplane flies 7000 nautical miles along a great circle. If the initial course is  $25^\circ 32'$  and if it reaches a point in latitude  $18^\circ 15'$  N. and longitude  $12^\circ 15'$  W., find its initial position.

**13.** Using (63) and (66), find the initial course and distance for a voyage along a great circle from Los Angeles (latitude  $L = 34^{\circ}03'$  N., longitude  $\lambda = 118^{\circ}15'$  W.) to Wellington (latitude  $L = 41^{\circ}18'$  S., longitude  $\lambda = 174^{\circ}51'$  E.).

**14.** Using (66) find the three angles of the spherical triangle in which  $a = 70^{\circ}14'20''$ ,  $b = 49^{\circ}24'10''$ ,  $c = 38^{\circ}46'10''$ .

## CHAPTER VI

### APPLICATIONS

**50. Nature of applications.** Many applications of spherical trigonometry deal with time and with angular distances. These considerations of time and distance may have reference to bodies far removed from the earth (celestial) or to bodies on the earth (terrestrial).

The shape of the earth is approximately that of a sphere having a diameter of 7917 miles. In what follows we shall consider it as a sphere. Hence the problem of finding the great-circle distance between two points on the earth or of locating a point on it is a problem that may be solved by the use of spherical trigonometry. Time enters our considerations because the rotation of the earth about its axis once every day furnishes the basic unit of time.

**51. Definitions and notations.** The earth revolves about a diameter called its *axis*. One point where the axis cuts the surface of the earth is called the *north pole*,  $P_n$ ; the other is called the *south pole*,  $P_s$ .

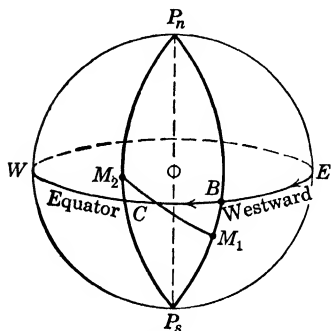


FIG. 1.

The *equator* is the great circle on the earth whose plane is perpendicular to the axis of the earth.

A *meridian* is a great circle on the earth passing through the north pole and the south pole. In Fig. 1,  $P_nBP_s$  and  $P_nCP_s$  represent meridians. Since meridians cut the equator at right angles, angular distances of points on the earth from the equator are measured along meridians.

The *latitude* (Lat. or  $L$ ) of a point on the earth is the angular distance of the point from the equator. It is measured along a

meridian north or south of the equator from  $0^\circ$  to  $90^\circ$ . In Fig. 1,  $CM_2$  represents the latitude of  $M_2$ . In general, north latitude is considered positive, south latitude negative.

Because of the great importance of triangle  $M_1P_nM_2$  in connection with problems relating to distances and angles on the earth, it is called the *terrestrial triangle*. Arc  $M_1M_2$  represents the distance along the great-circle track from  $M_1$  to  $M_2$ , and the angle  $M_2M_1P_n$  gives the initial direction of the track. The angle of departure  $P_nM_1M_2$  measured from the north around through the east from  $0^\circ$  to  $360^\circ$  is called the initial course  $C_n$ . For a person situated on the northern hemisphere of the earth at a point such as  $z$  in Fig. 2, north is along the tangent to the meridian away from the equator; for a person standing at  $z$  facing north, east is on his right, west is on his left, and south is opposite to the direction in which he is facing.

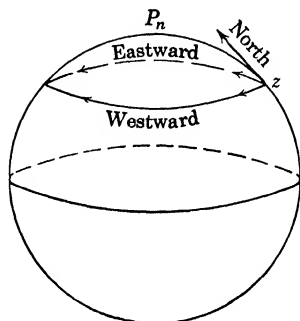


FIG. 2.

Figure 3 indicates directions at four positions on the earth.

The *longitude* (Long. or  $\lambda$ ) of a point on the earth is the angle at either pole between the meridian passing through the point and some fixed meridian known as the *prime meridian*. It is measured east or west of the prime meridian from  $0^\circ$  to  $180^\circ$ . The meridian of Greenwich, England, is the prime meridian, not only for English and American navigators but also for those of many other nations.

The latitude and longitude of a point give its position on the earth just as the two coordinates of a point give its position relative to a set of rectangular axes.

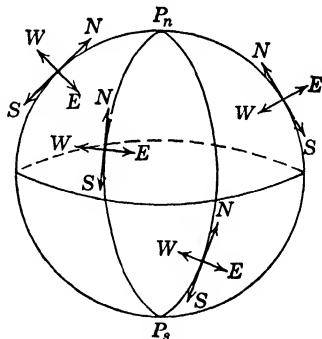


FIG. 3.

**52. Course and distance.** In general, the procedure of applying spherical trigonometry to solve problems relating to the earth consists in finding three parts of the terrestrial triangle, solving

for one or more of the other three parts, and interpreting the results. Consider, for example, the problem of finding the great-circle distance between two points  $M_1$  and  $M_2$  when the latitude and the longitude of each point are known. In Fig. 4,  $P_n$  represents the north pole,  $QK_1K_2Q'$  the equator,  $P_nGQP_n$  the

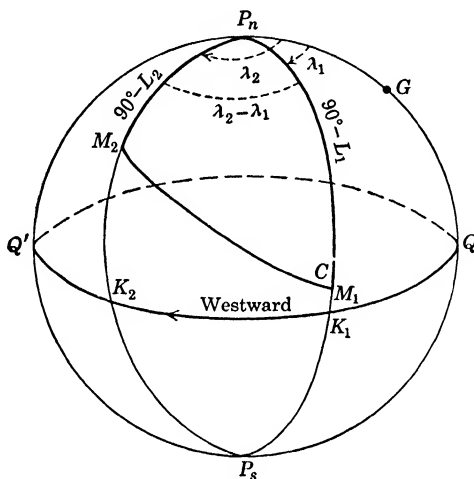


FIG. 4.

meridian of Greenwich, and  $M_1$  and  $M_2$  two places on the earth. The longitudes  $\lambda_1$  of  $M_1$  and  $\lambda_2$  of  $M_2$  are known; hence angle

$$M_1P_nM_2 = \lambda_2 - \lambda_1$$

is known. Also, the latitudes  $L_1 = K_1M_1$  of  $M_1$  and  $L_2 = K_2M_2$  of  $M_2$  are known; hence the arcs  $M_1P_n = 90^\circ - L_1 = co-L_1$  and  $M_2P_n = 90^\circ - L_2 = co-L_2$  are known. Thus, in triangle

$M_1P_nM_2$ , two sides  $M_1P_n = co-L_1$  and  $M_2P_n = co-L_2$  and the included angle  $M_1P_nM_2 = \lambda_2 - \lambda_1$  are known. Consequently, we can solve this triangle by Napier's analogies, by the method of §30 or by that of §47.

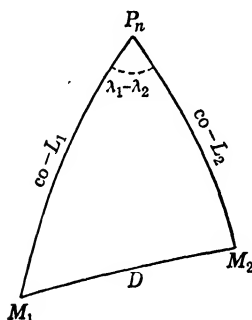


FIG. 5.

**Example.** Compute the initial great-circle course and the distance for a trip from St. Augustine lighthouse  $L_1 = 30^\circ$  N.,  $\lambda_1 = 76^\circ$  W. to the Strait of Gibraltar  $L_2 = 36^\circ$  N.,  $\lambda_2 = 5^\circ 30'$  W.



*Solution.* Substituting from Fig. 5,  $90^\circ - L_1$  for  $a$ ,  $90^\circ - L_2$  for  $b$ ,  $\lambda_1 - \lambda_2$  for  $C$ ,  $M_1$  for  $B$ , and  $D$  for  $c$  in formulas (53), (54), (55), and (56) of §46, we obtain

$$\tan \varphi = \cos (\lambda_1 - \lambda_2) \tan (co-L_2) = \cos (\lambda_1 - \lambda_2) \cot L_2, \quad (a)$$

$$\varphi' = 90^\circ - L_1 - \varphi = 90^\circ - (L_1 + \varphi), \quad (b)$$

$$\cot M_1 = \cot (\lambda_1 - \lambda_2) \sin \varphi' \csc \varphi$$

$$\text{or} \quad \cot M_1 = \cot (\lambda_1 - \lambda_2) \cos (L_1 + \varphi) \csc \varphi, \quad (c)$$

$$\cos D = \cos \varphi' \sec \varphi \cos (co-L_2) = \sin (L_1 + \varphi) \sec \varphi \sin L_2. \quad (d)$$

Substituting the given values in formulas (a), (b), (c), and (d) and evaluating  $\varphi$ ,  $M_1$ , and  $D$  from the results, we obtain the following solution:

	(a)	(c)	(d)	(Check)†
$\lambda_1 - \lambda_2 = 70^\circ 30'$	$l \cos 9.52350$	$l \cot 9.54915$	$l \sin 9.76922$	
$L_2 = 36^\circ$	$l \cot 0.13874$		$l \sec 0.04159$	
$\varphi = 24^\circ 40' 35''$	$l \tan 9.66224$	$l \csc 0.37935$	$l \sin 9.91163$	$l \tan 0.14956$
$L_1 + \varphi = 54^\circ 40' 35''$		$l \cos 9.76208$		$l \cos 9.04378$
$M_1 = N.63^\circ 52' 30'' \text{ E.}$		$l \cot 9.69058$		$l \tan 0.20666$
$D = 58^\circ 8' 43'' = 3488.7 \text{ miles}^*$			$l \cos 9.72244$	$l \tan 0.00000$
1				log 0.00000

The problem of finding course and distance is conveniently solved by using formula (65) §47 to find distance  $D$  and then using the law of sines to find the course angle. To apply (65), §47, to Fig. 5, replace  $c$  by  $D$ ,  $a$  by  $90^\circ - L_1$ ,  $b$  by  $90^\circ - L_2$ , and  $C$  by  $\lambda_1 - \lambda_2$  to obtain

$$\text{hav } D = \text{hav } (L_2 - L_1) + \cos L_1 \cos L_2 \text{hav } (\lambda_1 - \lambda_2). \quad (1)$$

The law of sines applied to Fig. 5 gives

$$\sin M_1 = \cos L_2 \sin (\lambda_1 - \lambda_2) \csc D. \quad (2)$$

So far as formula (2) is concerned the angle  $M_1$  may be of the first quadrant or of the second. A navigator usually knows the course approximately and thus knows the quadrant to be expected. Very often the quadrant of  $M_1$  can be determined by considering that the order of magnitude of the sides of a spherical

\* 1' of angle at the center of the earth subtends 1 nautical mile = 6080 ft. on a great circle of the earth. Hence, when an arc of a great circle on the earth is expressed in minutes, it is also expressed in nautical miles.

† The check formula was obtained by drawing a perpendicular from  $M_1$  to  $P_1M_2$  in Fig. 5 and applying Napier's rules.

triangle is the same as that of the opposite angles or by a rough sketch. When the suggested methods fail, the law of sines should not be employed. In such cases, the following formula may be used:

$$\text{hav } A = [\text{hav } a - \text{hav } (b - c)] \csc b \csc c.$$

### EXERCISES

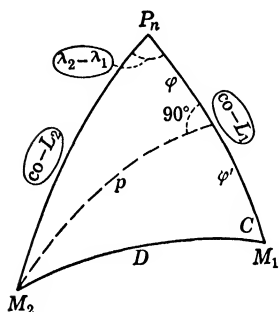


FIG. 6.

1. Figure 6 represents the terrestrial triangle with the arc of a great circle drawn through  $M_2$  perpendicular to  $P_n M_1$ . Apply Napier's rules to the figure to obtain

$$\tan \varphi = \cos (\lambda_2 - \lambda_1) \cot L_2,$$

$$\varphi' = 90^\circ - (L_1 + \varphi),$$

$$\cos D = \sin L_2 \sec \varphi \sin (L_1 + \varphi),$$

$$\cot C = \cot (\lambda_2 - \lambda_1) \csc \varphi \cos (L_1 + \varphi).$$

2. In formulas (53) to (56) of §46 substitute  $90^\circ - L_1$  for  $a$ ,  $90^\circ - L_2$  for  $b$ ,  $\lambda_2 - \lambda_1$  for  $C$ ,  $M_1$  for  $B$ , and  $D$  for  $c$  to obtain the formulas of Exercise 1.

3. Substitute for  $a$ ,  $b$ ,  $c$ , and  $C$  of formula (65) of §47 appropriate values from Fig. 6 to obtain

$$\text{hav } D = \text{hav } (L_1 - L_2) + \cos L_1 \cos L_2 \text{hav } (\lambda_2 - \lambda_1).$$

Then write a formula from the law of sines for finding the course angle  $M_1$ .

4. Substitute for  $a$ ,  $b$ ,  $c$ ,  $A$ ,  $B$ , and  $C$  appropriate values from Fig. 6 in formulas (42), (47), (48), (49) of §42 to obtain formulas for solving the triangle of Fig. 6 completely.

5. Find the initial compass course and distance in nautical miles for a great-circle voyage from San Diego ( $L_1 = 32^\circ 43' \text{ N.}$ ,  $\lambda_1 = 117^\circ 10' \text{ W.}$ ) to Hong Kong ( $L_2 = 22^\circ 9' \text{ N.}$ ,  $\lambda_2 = 114^\circ 10' \text{ E.}$ ). Use the formulas of Exercise 1.

6. The great-circle distance from Cape Flattery,  $L = 48^\circ 24' \text{ N.}$ ,  $\lambda = 124^\circ 44' \text{ W.}$ , to Tutuila,  $L = 14^\circ 18' \text{ S.}$ ,  $\lambda = 170^\circ 42' \text{ W.}$ , is 4633.7 miles. Find the course of the ship on arrival at Tutuila if it follows a great-circle track from Cape Flattery to Tutuila.

7. Find the distance by great circle from New York,  $L_1 = 40^\circ 40' \text{ N.}$ ,  $\lambda_1 = 4^{\text{h}} 55^{\text{m}} 54^{\text{s}} \text{ W.}$ , to a place near Cape of Good Hope,  $L_2 = 33^\circ 56' \text{ S.}$ ,  $\lambda_2 = 1^{\text{h}} 13^{\text{m}} 55^{\text{s}} \text{ E.}$

8. The distance from Cape Flattery,  $L = 48^{\circ}24' \text{ N.}$ ,  $\lambda = 124^{\circ}44' \text{ W.}$ , to Tutuila,  $L = 14^{\circ}18' \text{ S.}$ ,  $\lambda = 170^{\circ}42' \text{ W.}$ , is 4633.7 miles. Find the initial course for a trip from Cape Flattery to Tutuila, by great circle.

9. Find the initial course and the distance for a great-circle voyage from Cape of Good Hope  $34^{\circ}22' \text{ S.}$ ,  $18^{\circ}30' \text{ E.}$  to Singapore  $1^{\circ}17'30'' \text{ N.}$ ,  $103^{\circ}51' \text{ E.}$  Also find the latitude and longitude of the northern vertex\* (the most northerly point) of this great-circle track. Use the formulas of Exercise 3.

10. Find the initial course and the distance for a voyage along a great circle from Los Angeles  $L = 34^{\circ}03' \text{ N.}$ ,  $\lambda = 118^{\circ}15' \text{ W.}$  to Wellington  $L = 41^{\circ}18' \text{ S.}$ ,  $\lambda = 174^{\circ}51' \text{ E.}$

11. The northern vertex of the great-circle track from a place near San Francisco, Lat.  $38^{\circ}28' \text{ N.}$ , Long.  $123^{\circ}23' \text{ W.}$ , to Manila, Lat.  $14^{\circ}35' \text{ N.}$ , Long.  $120^{\circ}57' \text{ E.}$ , has Lat.  $46^{\circ}07' \text{ N.}$ , Long.  $163^{\circ}33'36'' \text{ W.}$  Find the latitude reached when the longitude is  $180^{\circ}$ .

12. The northern vertex of a great-circle track is in  $L = 60^{\circ}50'26'' \text{ N.}$ ,  $\lambda = 60^{\circ}29'37'' \text{ E.}$  Given the following positions:

Rio de Janeiro:  $L = 22^{\circ}55' \text{ S.}$ ,  $\lambda = 43^{\circ}09' \text{ W.}$ ,  
 Strait of Gibraltar:  $L = 35^{\circ}53' \text{ N.}$ ,  $\lambda = 5^{\circ}42' \text{ W.}$ ,  
 Cape St. Roque:  $L = 5^{\circ}29' \text{ S.}$ ,  $\lambda = 35^{\circ}15' \text{ W.}$ ,  
 Cape Manuel:  $L = 14^{\circ}39' \text{ N.}$ ,  $\lambda = 17^{\circ}27' \text{ W.}$

When following this track, what will be the

- (a) Longitude when in the latitude of Rio de Janeiro?
- (b) Latitude when in the longitude of Strait of Gibraltar?
- (c) Longitude when in the latitude of Cape St. Roque?
- (d) Latitude when in the longitude of Cape Manuel?
- (e) Course and distance when in the latitude of Rio de Janeiro?
- (f) Distance from vertex when in the longitude of Strait of Gibraltar?

13. A ship sails from San Francisco  $L = 37^{\circ}48' \text{ N.}$ ,  $\lambda = 122^{\circ}23' \text{ W.}$ , to Manila  $L = 14^{\circ}35'48'' \text{ N.}$ ,  $\lambda = 120^{\circ}57'18'' \text{ E.}$ , following a great-circle track. Find the course angle at departure, the course angle at arrival, and the distance traveled.

14. Substitute  $90^{\circ} - L_1$  for  $a$ ,  $90^{\circ} - L_2$  for  $b$ ,  $\lambda_1 - \lambda_2$  for  $C$ ,  $M_1$  for  $B$ ,  $M_2$  for  $A$ ,  $D$  for  $C$ , in (42), (47), (48), (49) to obtain:

$$\frac{\sin \frac{1}{2}(M_2 - M_1)}{\sin \frac{1}{2}(M_2 + M_1)} = \frac{\tan \frac{1}{2}(L_2 - L_1)}{\tan \frac{1}{2}D}$$

\* A meridian passing through the vertex of a great-circle track is perpendicular to the track.

$$\frac{\cos \frac{1}{2}(M_2 - M_1)}{\cos \frac{1}{2}(M_2 + M_1)} = \frac{\cot \frac{1}{2}(L_1 + L_2)}{\tan \frac{1}{2}D}$$

$$\frac{\sin \frac{1}{2}(L_2 - L_1)}{\cos \frac{1}{2}(L_2 + L_1)} = \frac{\tan \frac{1}{2}(M_2 - M_1)}{\cot \frac{1}{2}(\lambda_1 - \lambda_2)}$$

$$\frac{\cos \frac{1}{2}(L_2 - L_1)}{\sin \frac{1}{2}(L_2 + L_1)} = \frac{\tan \frac{1}{2}(M_1 + M_2)}{\cot \frac{1}{2}(\lambda_1 - \lambda_2)}$$

Using these formulas, solve Exercise 8.

**53. The celestial sphere.** Consider a fixed star so far away from our solar system that the light rays coming to us from this star appear to follow parallel lines independent of our position; for example, light rays coming from this star to us at one position of the earth's orbit appear to have the same direction as light rays coming from the star to us 6 months later when we are on the other side of the orbit of the earth or approximately 186 million miles from the first position. Since, to us, light rays from this star seem to travel in parallel lines, we naturally associate a fixed direction with it.

We shall speak of the *celestial sphere* as a sphere concentric with the earth and having a radius of unlimited length; by this we shall understand that any two parallel lines cut this sphere in the same point, and any two parallel planes cut it in the same great circle. With any point on this sphere is associated a fixed direction; the angular distance between two points on it may be considered, but not an actual distance in miles.

Figure 7 represents the celestial sphere with the earth at its center.

The point  $P_N$  on the celestial sphere where a line connecting the center of the earth to its north

pole cuts the celestial sphere is called the *north celestial pole*; the point  $P_S$  diametrically opposite is called the *south celestial pole*.

The plane of the equator of the earth cuts the celestial sphere in the *equinoctial* or *celestial equator*. The *celestial poles* are the poles of the celestial equator.

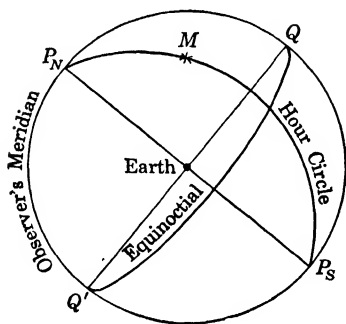


FIG. 7.

The great circles such as  $P_NMP_S$  in Fig. 7, passing through the celestial poles, are called *hour circles* or *celestial meridians*.

The point  $Z$  (see Fig. 8) directly above an observer, that is, the point where a line connecting the center of the earth to an

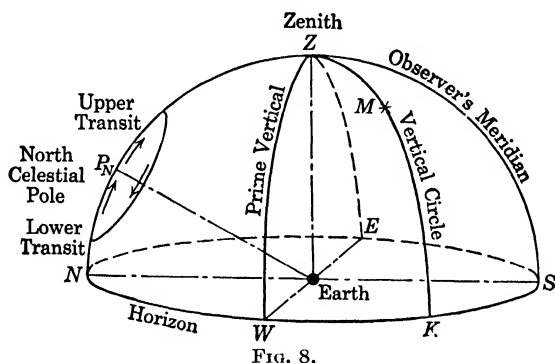


FIG. 8.

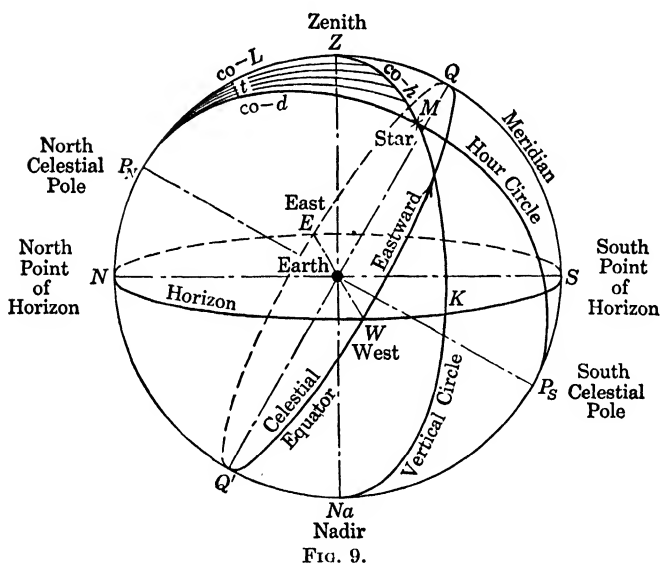


FIG. 9.

observer on it would intersect the celestial sphere, is called the *zenith*. The point on the celestial sphere diametrically opposite the zenith is called the *nadir*  $Na$  (see Fig. 9).

The *horizon*  $NWSE$  of an observer is the great circle on the celestial sphere having the zenith and nadir as poles. A plane

tangent to the earth at a point on it intersects the celestial sphere in the celestial horizon associated with the point.

The point on the horizon directly below the north celestial pole is called the *north point* of the horizon. The *south point*, the *east point*, and the *west point* of the horizon are then determined in the usual way.

The great circles, such as  $ZMK$  of the celestial sphere, which pass through the zenith, are called *vertical circles*. Evidently they are all perpendicular to the horizon. The *prime vertical* is the vertical circle  $EZW$  (see Fig. 8) passing through the zenith and the east and west points of the horizon.

Figure 9 exhibits both the equinoctial system and the horizon system.

**54. The astronomical triangle.** The spherical triangle (see Fig. 10) whose vertices are the north celestial pole, the zenith, and the projection of a heavenly body on the celestial sphere is called the *astronomical triangle*. The solution of many of the problems of astronomy and of navigation requires the solution of this triangle.

The great-circle distance of a point on the celestial sphere from the celestial equator is called the *declination*  $d$  of the point. This corresponds to the latitude of a point on the earth. Inspection of Fig. 9 shows that the arc  $P_N M$  of the astronomical triangle is  $90^\circ$  minus declination, or  $co-d$ .

The *hour angle*  $t$  of a point on the celestial sphere is the angle between the hour circle passing through the zenith of the observer and the hour circle passing through the point.\* As the earth turns on its axis, the heavenly bodies appear to move on the celestial sphere. Thus the angle through which the earth must turn to bring the celestial meridian of an observer into coincidence with the hour circle of a point on the celestial sphere appears as the hour angle of the point relative to the observer. The significance of the word hour angle appears when we consider

\* Hour angle is often expressed as so many degrees east or west, according as the body observed is in the eastern sky or in the western sky. It is often measured toward the west from  $0^h$  to  $24^h$  ( $360^\circ$ ).

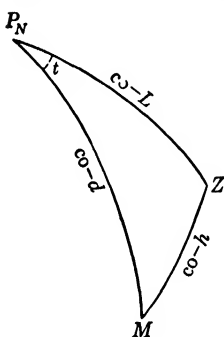


FIG. 10.

that the earth turns on its axis and moves in its orbit in such a way that the sun crosses the meridian of a place once every 24 hours.

The *altitude*  $h$  of a point on the celestial sphere is its great-circle distance from the horizon. Inspection of Fig. 9 shows that the side  $MZ$  of the astronomical triangle is  $90^\circ$  minus altitude or  $co-h$ .

The *azimuth*  $Z_n$  of a point on the celestial sphere is the angle at the zenith between the vertical circle of the point and the celestial meridian of the observer. It is usually measured from the *north point around through the east* from  $0^\circ$  to  $360^\circ$ . It is easy to write the azimuth  $Z_n$  when the angle  $Z$  of the astronomical triangle has been found.

Evidently the length  $P_NZ$  of the astronomical triangle is  $90^\circ$  minus the latitude of the observer, or  $90^\circ - L$ .

**55. Given  $t, d, L$ ; to find  $h$  and  $Z$ .\*** Figure 11 represents the astronomical triangle with the given parts encircled. Since two sides and the included angle are given, we may adapt formulas (53) to (56) of §46 to the triangle of Fig. 11, or we may con-

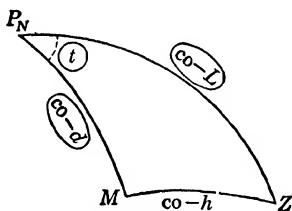


FIG. 11.

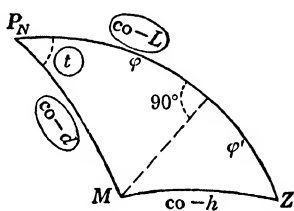


FIG. 12.

struct an arc of a great circle through  $M$  perpendicular to  $P_NZ$ , letter the triangle as shown in Fig. 12, and then apply Napier's rules to obtain

\* If a navigator wishes to observe a number of stars at a particular time, say near sunset, he knows the time and from that can find the angle  $t$ ; he knows approximately what his latitude will be, and he can find the declination of convenient stars in the Nautical Almanac. Hence he can compute the approximate positions, altitude, and azimuth of several stars in advance and thus expedite the process of locating, identifying, and observing them. Instead of computing  $h$  and  $Z$ , he can find these quantities in tables when such are available.

$$\tan \varphi = \cos t \cot d, \quad (3)$$

$$\varphi' = 90^\circ - L - \varphi = 90^\circ - (L + \varphi), \quad (4)$$

$$\cot Z = \cot t \sin \varphi' \csc \varphi = \cot t \cos (L + \varphi) \csc \varphi, \quad (5)$$

$$\sin h = \cos \varphi' \sec \varphi \sin d = \sin (L + \varphi) \sec \varphi \sin d, \quad (6)$$

$$\sin t \cos d \csc Z \sec h = 1. \quad (\text{Check}) \quad (7)$$

If  $L$  represents the latitude of a place north of the equator,  $d$  should be taken positive for a body having north declination and negative for one having south declination, or vice versa.

**Example.** Use formulas (3) to (7) to find the altitude  $h$  and the azimuth  $Z_n$  of a star having  $d = 1^\circ 9' 15''$  S.,  $t = 45^\circ 10' 30''$  east, if it is viewed by an observer in latitude  $37^\circ 30'$  N.

**Solution.** The solution found from the formulas (3), (4), (5), (6), and (7) appears below.

	(3)	(5)	(6)	(7)
$t = 45^\circ 10' 30''$ E.	$l \cos 9.84816$	$l \cot 9.99735$		$l \sin 9.85080$
$d = -1^\circ 9' 15''$	$l \cot (-) 1.69580$		$l \sin (-) 8.30411$	$l \cos 9.99991$
$L = 37^\circ 30' 0''$ N.				
$\varphi = 91^\circ 38' 13''$	$l \tan (-) 1.54396$	$l \csc 0.00018$	$l \sec (-) 1.54414$	
$L + \varphi = 129^\circ 8' 13''$		$l \cos (-) 9.80015$	$l \sin 9.88966$	
$Z = N.122^\circ 6' 43''$ E. = $Z_n$		$l \cot (-) 9.79768$		$l \csc 0.07211$
$h = 33^\circ 9' 18''$			$l \sin 9.73791$	$l \sec 0.07717$
1				$\log 9.99999$

Evidently we could have used Napier's analogies to solve the triangle of the illustrative example, or we could have adapted formula (63) of §47 to the triangle and have used the result to find  $h$ .

### EXERCISES

1. From Napier's analogies (§42) derive the formulas

$$\begin{aligned} \tan \frac{1}{2}(Z - M) &= \cot \frac{1}{2}t \sin \frac{1}{2}(L - d) \sec \frac{1}{2}(L + d), \\ \tan \frac{1}{2}(Z + M) &= \cot \frac{1}{2}t \cos \frac{1}{2}(L - d) \csc \frac{1}{2}(L + d). \end{aligned}$$

2. From formula (63) of §47, derive the formula\*

$$\text{hav co-}h = \text{hav } (L - d) + \cos L \cos d \text{ hav } t.$$

\* In the practice of navigation the method of Saint Hilaire is frequently used to determine the observer's position. In this method the value of  $Z$  is taken from azimuth tables, and  $h$  is computed by the formula of Exercise 2. The navigator then compares the computed value of  $h$  with the observed value and uses the difference between the two in determining the correction to be applied to the assumed position of his ship.



From the data of Exercises 3 to 10, compute  $h$  and  $Z_n$ .

- |   |   |
|---|---|
| 3. $d = 6^\circ 15' \text{ S.},$<br>$t = 14^\circ 6' \text{ W.},$<br>$L = 21^\circ 18' \text{ N.}$                  | 7. $d = 10^\circ \text{ N.},$<br>$t = 40^\circ \text{ W.},$<br>$L = 35^\circ \text{ S.}$      |
| 4. $d = 38^\circ 17' 24'' \text{ S.},$<br>$t = 28^\circ 30' 29'' \text{ W.},$<br>$L = 24^\circ 32' 58'' \text{ N.}$ | 8. $d = 7^\circ \text{ S.},$<br>$t = 28^\circ \text{ E.},$<br>$L = 41^\circ \text{ N.}$       |
| 5. $d = 59^\circ 56' \text{ N.},$<br>$t = 60^\circ 32' \text{ E.},$<br>$L = 44^\circ 45' \text{ N.}$                | 9. $d = 8^\circ \text{ N.},$<br>$t = 35^\circ \text{ E.},$<br>$L = 39^\circ \text{ N.}$       |
| 6. $d = 10^\circ \text{ S.},$<br>$t = 25^\circ \text{ E.},$<br>$L = 18^\circ 57' 16'' \text{ S.}$                   | 10. $d = 22^\circ 30' \text{ S.},$<br>$t = 60^\circ \text{ E.},$<br>$L = 45^\circ \text{ S.}$ |

From the data of Exercises 11 to 16, compute  $h$ .

- |   |   |
|---|---|
| 11. $t = 3^{\text{h}} \text{ P.M.},$<br>$d = 5^\circ \text{ S.},$<br>$L = 50^\circ \text{ N.}$                | 14. $t = 1^{\text{h}} 13^{\text{m}} 12^{\text{s}} \text{ P.M.},$<br>$d = 13^\circ 21' \text{ N.},$<br>$L = 15^\circ 54' \text{ S.}$ |
| 12. $t = 25^\circ \text{ E.},$<br>$d = 10^\circ \text{ S.},$<br>$L = 18^\circ 57' 16'' \text{ S.}$            | 15. $t = 4^{\text{h}} 2^{\text{m}} 8^{\text{s}} \text{ P.M.},$<br>$d = 59^\circ 56' \text{ N.},$<br>$L = 44^\circ 45' \text{ N.}$   |
| 13. $t = 2^{\text{h}} 40^{\text{m}} \text{ P.M.},$<br>$d = 10^\circ \text{ N.},$<br>$L = 35^\circ \text{ S.}$ | 16. $t = 0^{\text{h}} 56^{\text{m}} 24^{\text{s}} \text{ P.M.},$<br>$d = 6^\circ 15' \text{ S.},$<br>$L = 21^\circ 18' \text{ N.}$  |

17. Check the answers of Exercises 3 to 10 using the formulas of Exercise 1.

18. If the observer's latitude is  $29^\circ 17' 24'' \text{ N.}$ , and a star, in declination  $30^\circ 21' 14'' \text{ S.}$ , has the hour angle  $4^{\text{h}} 30^{\text{m}} 48^{\text{s}} \text{ W.}$ , find the altitude of the star. Use  $\text{hav } (90^\circ - h) = \text{hav } (L - d) + \cos L \cos d \text{ hav } t$ .

**56. To find the time and amplitude of sunrise.** Figure 13 represents a stereographic projection of the astronomical triangle  $P_N Z M$  when the body  $M$  is the sun on the horizon. The dotted line indicates the path of the sun across the sky as a small circle each of whose points is distant  $\text{co-}d$  from the pole. When the sun crosses the meridian at  $K$ , it is noon. Hence  $t$  represents the angle through which the earth must turn during the time interval from sunrise to noon. Since the earth turns through  $15^\circ$  per hour,  $t/15$  will be the number of hours from sunrise to noon if  $t$  is expressed in degrees. The declination of the sun can be found

from the Nautical Almanac,\* and the latitude of the observer is supposed known. Therefore, to find a formula for  $t$ , apply Napier's rules to right spherical triangle  $NMP_N$  (Fig. 14), and write  $\cos (180^\circ - t) = \tan d \tan L$ , or

$$\cos t = -\tan d \tan L. \quad (8)$$

The angular distance from the east point of the horizon to

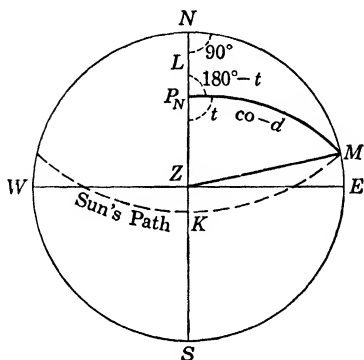


FIG. 13.

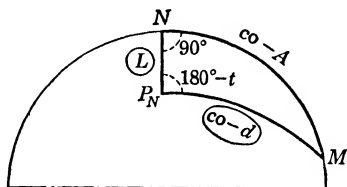


FIG. 14.

the sun at sunrise is called the *amplitude of sunrise*. From right spherical triangle  $NP_NM$  of Fig. 14 we find, by using Napier's rules,  $\sin d = \cos L \sin A$ , or

$$\sin A = \sin d \sec L. \quad (9)$$

From Fig. 14 we obtain the check formula

$$-\cot A \cot t \csc L = 1. \quad (10)$$

**Example.** Find the amplitude and the time of sunrise at *Annapolis*,  $L = 38^\circ 59' \text{ N.}$ , at a time when the declination of the sun is  $20^\circ \text{ S.}$

**Solution.** The solution found from formulas (8), (9), and (10) appears below

	(8)	(9)	(10)
$L = 38^\circ 59' 0''$	$l \tan 9.90811$	$l \sec 0.10940$	$l \csc 0.20128$
$d = -20^\circ 0' 0''$	$l \tan (-) 9.56107$	$l \sin (-) 9.53405$	
$t = 72^\circ 52' 7''$	$l \cos 9.46918$		$l \cos 9.48889$
$A = -26^\circ 6' 14''$		$l \sin (-) 9.64345$	$l \cot 0.30983$
1			$\log 0.00000$

\* Owing to refraction of the sunbeams by the earth's atmosphere, the sun will appear to be on the horizon considerably earlier than the results of this computation would indicate. In practice, corrections must be made on this account.

Since  $15^\circ$  indicates a time of  $1^h$ ,  $72^\circ 52' 7''$  will indicate  $4^h 51^m 28^s$ . As  $t$  is the time from sunrise till noon, we obtain

$$12^h - (4^h 51^m 28^s) = 7^h 8^m 32^s$$

as the local apparent time\* of sunrise. The negative sign before the amplitude indicates that the sun appeared on the horizon *south* of the east point.

### EXERCISES

1. Find the amplitude of sunrise in latitude  $38^\circ 58' 53''$  N. when the declination of the sun is  $22^\circ 29' 00''$  S.

2. At Annapolis, Lat.  $38^\circ 59'$  N., the sun in declination  $23^\circ 27'$  N. has the altitude  $0^\circ$ , bearing easterly. Find the local apparent time.

3. Find the amplitude and the local apparent time of sunrise and sunset for Annapolis, Md.,  $L = 38^\circ 58' 53''$  N., at summer and winter solstice ( $d = \pm 23^\circ 27' 7''$ ).

4. (a) Find the local apparent time of sunrise and sunset at Cape Nome,  $L = 64^\circ 23'$  N. on Mar. 21,  $d = 0^\circ 0' 0''$ , Dec. 21,  $d = 23^\circ 27'$  S., and June 21,  $d = 23^\circ 27'$  N. (b) Find the amplitude of the sun at each occurrence. (c) Find the length of the longest day and of the shortest day at Cape Nome.

5. Assuming that the declination of the sun ranges between  $23^\circ 27'$  S. to  $23^\circ 27'$  N., show that a place where the sun rises at midnight must lie within  $23^\circ 27'$  of a pole of the earth.

*Hint.* In the formula  $\cos t = -\tan L \tan d$ , let  $t = 180^\circ (= 12^h)$ .

6. For a point on the earth having latitude  $80^\circ$  N. find (a) the declination of the sun when the time of daylight is just 24 hr.; (b) the declination of the sun when the night lasts just 24 hr.; (c) the least altitude and the greatest altitude of the sun during the day when the declination of the sun is  $23^\circ 27'$  N.; (d) the declination of the sun when continuous night begins; (e) the length of the shortest possible shadow cast by a vertical pole 20 ft. long.

**57. To find the time of day.** The declination of the sun can be found from the Nautical Almanac for a given time, and the altitude of the sun can be measured with a sextant. Hence, if the latitude of the place is known, the three sides of the astro-

\* The noon of local apparent time occurs when the sun is on the meridian of the observer, and the time of day is expressed in terms of the hour angle of the sun. Owing to the fact that the sunbeams are refracted by the earth's atmosphere, the sun appears to be on the horizon slightly earlier than is indicated by the solution given.

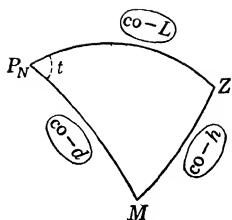


FIG. 15.

nomical triangle are known, and  $t$  can be found. Since  $t$  represents the angle through which the earth must turn before noon if the sun is in the eastern sky, and since the earth turns through  $15^\circ$  per hour,  $t/15$  will be the interval of time before noon if  $t$  is expressed in degrees. If the sun is in the western sky,  $t/15$  is the time since noon.

To obtain formulas adapted to this case, substitute from Fig. 15

$$a = 90^\circ - h, \quad b = p = (90^\circ - d), \quad c = 90^\circ - L, \\ A = t, \quad B = Z, \quad S = \frac{1}{2}(h + p + L)$$

in (22) and (23) of §40, and simplify to obtain

$$\sin^2 \frac{1}{2}t = \text{hav } t = \cos S \sin (S - h) \sec L \csc p, \quad (11)$$

$$\sin^2 \frac{1}{2}Z = \text{hav } Z = \sin (S - h) \sin (S - L) \sec h \sec L. \quad (12)$$

The law of sines may be used to obtain the check formula

$$\sin Z \csc p \csc t \cos h = 1. \quad (13)$$

Formula (11) gives the time of day, and formula (12) the angle from which the azimuth  $Z_n$  of the sun at the time of the observation may be determined.

**Example.** Find the azimuth  $Z_n$  of the sun and the local apparent time in New York,  $L = 40^\circ 43' \text{ N.}$ , at the instant when the altitude of the sun is  $30^\circ 10'$  bearing west and its declination is  $10^\circ \text{ N.}$

**Solution.** The solution obtained by using formulas (11), (12), and (13) appears below.

	(11)	(12)	(13)
$L = 40^\circ 43'$	$l \sec 0.12036$	$l \sec 0.12036$	
$h = 30^\circ 10'$		$l \sec 0.06320$	
$p = 90^\circ - d = 80^\circ$	$l \csc 0.00665$		$l \cos 9.93680$
$S = 75^\circ 26' 30''$	$l \cos 9.40031$		$l \csc 0.00665$
$S - h = 45^\circ 16' 30''$	$l \sin 9.85156$	$l \sin 9.85156$	
$S - L = 34^\circ 43' 30''$		$l \sin 9.75560$	
$t = 58^\circ 34' 9''$	$l \text{ hav } 9.37888^*$		$l \csc 0.06891$
$= 3^{\text{h}} 54^{\text{m}} 17^{\text{s}}$		$l \text{ hav } 9.79072^*$	
$Z = \text{N. } 103^\circ 36' 20'' \text{ W.}$			$l \sin 9.98764$
$Z_n = 256^\circ 23' 40''$			$\log 0.00000$
1			

\* Those who do not use haversine tables may divide  $\log \text{hav } t$  and

Since  $58^{\circ}34'9''$  is equivalent to  $3^{\text{h}} 54^{\text{m}} 17^{\text{s}}$  and the sun is in the western sky, the time is  $3^{\text{h}} 54^{\text{m}} 17^{\text{s}}$  7. P.M.

### EXERCISES

1. In formulas (22) and (23) of §40, substitute  $a = 90^{\circ} - h$ ,  $b = p = (90^{\circ} - d)$ ,  $c = 90^{\circ} - L$ ,  $A = t$ ,  $B = Z$ ,  $S = \frac{1}{2}(h + p + L)$ , and simplify to obtain formulas (11) and (12).

2. An observation of the altitude of the sun was made in each of the following cities. Find the azimuth of the sun and the local apparent time of observation in each case.

(a) Pensacola, Fla.,  $L = 30^{\circ}21' \text{ N.}$ , sun's altitude  $h = 24^{\circ}30'$  bearing east, declination  $20^{\circ}42' \text{ N.}$

(b) Philadelphia, Pa.,  $L = 40^{\circ}0' \text{ N.}$ ,  $h = 26^{\circ}0' \text{ E.}$ ,  $d = 20^{\circ}0' \text{ N.}$

(c) Annapolis, Md.,  $L = 39^{\circ}0' \text{ N.}$ ,  $h = 22^{\circ}0' \text{ E.}$ ,  $d = 20^{\circ}0' \text{ N.}$

Given the following data, find  $t$  and  $Z$ .

3.  $L = 42^{\circ}45'0'' \text{ N.}$ ,  
 $d = 18^{\circ}27'0'' \text{ N.}$ ,  
 $h = 38^{\circ}36'0'' \text{ E.}$

5.  $L = 45^{\circ}0'0'' \text{ N.}$ ,  
 $d = 22^{\circ}30'0'' \text{ N.}$ ,  
 $h = 30^{\circ}0'0'' \text{ W.}$

4.  $L = 25^{\circ}35'0'' \text{ N.}$ ,  
 $d = 10^{\circ}24'0'' \text{ S.}$ ,  
 $h = 35^{\circ}19'0'' \text{ E.}$

6.  $L = 30^{\circ}0'0'' \text{ N.}$ ,  
 $d = 15^{\circ}0'0'' \text{ N.}$ ,  
 $h = 45^{\circ}0'0'' \text{ W.}$

### 58. Ecliptic. Equinoxes.

#### Right ascension. Sidereal time.

The earth rotates about its axis once a day, and it also moves around the sun once a year. To an observer on the earth, the sun seems to move about the earth, describing a great circle on the celestial sphere called the *ecliptic*. The plane of the ecliptic is inclined at an angle of approximately  $23^{\circ}27'$  to the plane of the celestial equator (see Fig. 16).

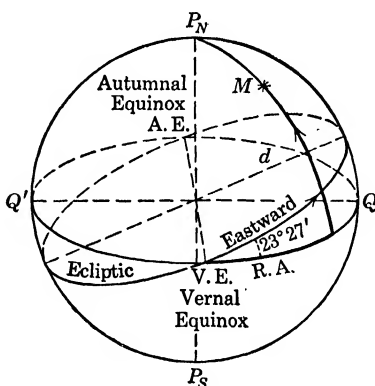


FIG. 16.

To an observer on the earth the sun appears to move eastward on the ecliptic, crossing the celestial equator while moving

log hav  $Z$  by 2 to obtain log sin  $t/2$  and log sin  $Z/2$ , respectively, and then find  $t/2$  and  $Z/2$  from the table of logarithms of trigonometric functions.

\* This angle  $23^{\circ}27'$  is called the *obliquity of the ecliptic*.

northward at the *vernal equinox* V.E. and while moving southward at the *autumnal equinox* A.E.

The *right ascension*  $RA$  of a body on the celestial sphere is the angle *measured eastward* from the hour circle of the vernal equinox to the hour circle of the body; thus the right ascension of the sun varies from  $0^\circ$  to  $360^\circ$ . Evidently a point is located on the celestial sphere by its right ascension and its declination just as a point on the earth is located by its longitude and its latitude.

Relative to the stars, the earth turns about its axis once in approximately  $23^h 56^m$  mean solar time. This period of time, called the *sidereal day*,\* is divided into 24 equal parts called sidereal hours, and the sidereal hours are divided into 60 equal sidereal minutes of 60 equal sidereal seconds each. Relative to the stars, the earth rotates through  $15^\circ$  each sidereal hour. The sidereal time of a place is measured from the time when the vernal equinox crosses the meridian of the place. *Hence the right ascension of the zenith of a place when expressed in hours, minutes, and seconds in the usual way is the sidereal time at that place.* From this it follows that the difference in the sidereal times of two points on the earth measures the hour angle between their celestial meridians; hence *the difference in the sidereal times of two points measures the difference in their longitudes.* A corollary to this may be stated: *the difference in sidereal time of Greenwich and that of a second place measures the longitude of the second place relative to Greenwich as prime meridian.*

**Example.** At a certain instant the sidereal time at one place is  $2^h$ , and at a second place it is  $4^h 30^m$ . Find the longitude of the second place if that of the first place is (a)  $0^\circ$ , (b)  $60^\circ$  E., (c)  $60^\circ$  W.

\* Besides sidereal time, we shall consider two other kinds, namely, *local apparent time* and *mean solar time*. The noon of local apparent time occurs when the sun is on the meridian of the observer, and the time of day is expressed in terms of the hour angle of the sun. Mean solar time is defined in terms of a fictitious sun that travels along the celestial equator at a uniform rate and makes a complete circuit in the same time as the actual sun. It is mean solar noon when the fictitious sun is on the meridian, and the mean solar time at any instant is the hour angle of the fictitious sun. This fictitious sun is used in order that we may have a day of uniform length throughout the year.

*Solution.* In Fig. 17 the circle represents the equator. V.E. represents the position of the vernal equinox, and  $A$ ,  $B$ , and  $G$  represent, respectively, the points on the equator where the meridian of the first place, that of second place, and that of Greenwich meet the celestial equator. Since the sidereal time of  $A$  is  $2^h$ , arc  $VEA$  is  $2 \times 15^\circ = 30^\circ$ . Similarly,  $VEB$  is  $67\frac{1}{2}^\circ$  and  $AB = 37\frac{1}{2}^\circ$ . In case (a), Greenwich and  $A$  have the same meridian; hence the longitude of  $B$  is  $37\frac{1}{2}^\circ$  E.

In Case (b), the meridian of Greenwich must be represented at  $G_2$  in Fig. 17, since  $A$  is in longitude  $60^\circ$  E. Hence the longitude of  $B$  in this case is  $60^\circ + 37\frac{1}{2}^\circ = 97\frac{1}{2}^\circ$  E.

In Case (c), Greenwich must have the position  $G_3$  in Fig. 17, since  $A$  is  $60^\circ$  west of Greenwich. Hence the longitude of  $B$  is  $60^\circ - 37\frac{1}{2}^\circ = 22\frac{1}{2}^\circ$  W.

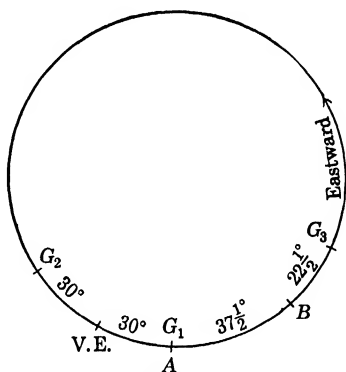


FIG. 17.

### EXERCISES

1. When it is  $0^h$  (sidereal time) in Greenwich, it is  $4^h$  at a certain place; find the longitude of this place.

2. At a place in longitude  $81^\circ 15'$  W. the sidereal time is  $10^h 17^m 30^s$ . Find the sidereal time at Greenwich.

3. The longitude of a first place differs from that of a second place by  $95^\circ 30'$ . When the sidereal time of the first place is  $10^h$ , find the sidereal time of the second place if it is (a) east of the first place; (b) west of the first place.

4. An observer in longitude  $24^\circ 30'$  W. observes a star whose  $RA$  is  $12^h 31^m 10^s$ . A radio signal gives Greenwich sidereal time at the instant of the observation as  $4^h 20^m 30^s$ . Find the hour angle of the star.

5. If  $ST_1$  is the sidereal time at a first place in longitude  $\lambda_1$  west of Greenwich and  $ST_2$  the sidereal time of a second place farther west, find the longitude of the second place.

6. On Jan. 13, 1932, the  $RA$  of the star Vega was  $18^h 34^m 36^s$ . What was the hour angle of Vega at the instant when the local sidereal time was  $12^h 54^m 16^s$ ?

7. At a certain time, the Greenwich hour angle for the Star Rigel was  $279^{\circ}42'$  W. Find the local hour angle of Rigel for an observer in Long.  $76^{\circ}38'30''$  E.

**59. The time sight.** The data and formulas considered in §57 may be used to find the longitude of an observer whose latitude is known. This method of determining longitude at sea is called *the time sight*. In Fig. 18,  $P_N G$  represents the celestial meridian of Greenwich,  $P_N O$  the celestial meridian of the observer and  $P_N M$  the celestial meridian of the sun. The angle  $t$  found

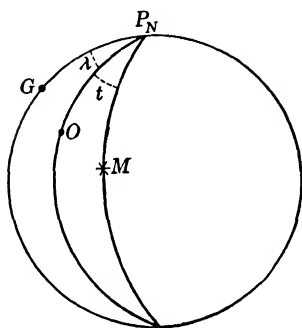


FIG. 18.

by the method of §57 determines the local apparent time at  $O$ ; the angle  $GP_N M$  determines the local apparent time of Greenwich. Hence the longitude in degrees

$$\lambda = \text{angle } GP_N O = \text{angle } GP_N M - t$$

of  $O$  is obtained by multiplying by 15 the difference in hours between the local apparent time of Greenwich and that of  $O$ . Sometimes it will be necessary to add angle  $GP_N M$  and angle  $t$

and sometimes to subtract them, depending on their relative positions. The local apparent time of Greenwich is obtained by radio, by telegraph, or by computing it from Greenwich mean time shown by a chronometer. The longitude is east or west according as the local time is later or earlier than Greenwich local time.

If the object  $M$  is a star, we still have

$$\lambda = \text{angle } GP_N M - t,$$

where  $t$  is computed as in §57, and the angle  $GP_N M$  is obtained by subtracting Greenwich sidereal time (computed from Greenwich mean time as given by a chronometer) from the right ascension of the star (obtained from a Nautical Almanac).

### EXERCISES

In each of the following sets of data,  $ST$  refers to sidereal time of Greenwich,  $RA$  to the right ascension of an observed star,  $d$  to its declination,  $h$  to its altitude, and  $L$  to the latitude of the observer. Find the longitude of the observer for each situation.



- |   |   |
|---|---|
| <p>1. <math>L = 30^{\circ}0'0''</math> N.,<br/> <math>d = 22^{\circ}30'0''</math> N.,<br/> <math>h = 45^{\circ}0'0''</math> W.,<br/> <math>ST = 4^h 10^m</math>,<br/> <math>RA = 13^h 5^m</math>.</p> | <p>4. <math>L = 30^{\circ}30'0''</math> N.,<br/> <math>d = 15^{\circ}30'0''</math> N.,<br/> <math>h = 44^{\circ}30'0''</math> W.,<br/> <math>ST = 17^h 15^m 24^s</math>,<br/> <math>RA = 10^h 5^m 6^s</math>.</p> |
| <p>2. <math>L = 12^{\circ}0'0''</math> S.,<br/> <math>d = 5^{\circ}0'0''</math> N.,<br/> <math>h = 45^{\circ}0'0''</math> W.,<br/> <math>ST = 10^h 6^m</math>,<br/> <math>RA = 8^h 7^m</math>.</p>    | <p>5. <math>L = 40^{\circ}0'0''</math> N.,<br/> <math>d = 8^{\circ}0'0''</math> N.,<br/> <math>h = 20^{\circ}0'0''</math> E.,<br/> <math>ST' = 0^h 47^m 24^s</math>,<br/> <math>RA = 1^h 5^m 7^s</math>.</p>      |
| <p>3. <math>L = 39^{\circ}0'0''</math> N.,<br/> <math>d = 20^{\circ}0'0''</math> N.,<br/> <math>h = 22^{\circ}0'0''</math> E.,<br/> <math>ST = 5^h 8^m</math>,<br/> <math>RA = 2^h 0^m</math>.</p>    | <p>6. <math>L = 43^{\circ}30'0''</math> N.,<br/> <math>d = 15^{\circ}0'0''</math> N.,<br/> <math>h = 20^{\circ}0'0''</math> W.,<br/> <math>ST = 13^h 5^m 15^s</math>,<br/> <math>RA = 0^h 15^m 20^s</math>.</p>   |

**60. Meridian altitude.** To find the latitude of a place on the earth. Figure 19 represents the cross section of the earth and of the surrounding celestial sphere by the plane of the meridian of an observer.  $qq'$  represents the equator of the earth;  $z$ , the position of the observer; and  $P_N P_S$ , the axis of the earth.  $QQ'$ ,  $Z$ ,  $P_N P_S$ ,  $N$ , and  $S$  represent, respectively, the celestial equator, the zenith, axis of celestial sphere, north point of the horizon, and south point of the horizon. Since  $qz$  represents the latitude of the observer and since arc  $qz = \text{arc } QZ = \text{arc } NP_N$ , it appears that the *latitude of an observer on the earth is equal to the declination of his zenith and to the altitude of the pole elevated above his horizon*.

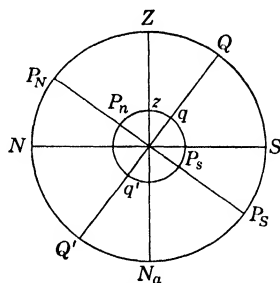


FIG. 19.

If, then, an observer knows the declination  $d$  of\* a star  $M$  (see Fig. 20) and observes its altitude  $h$ † just as it crosses his meridian above the pole, he can find his latitude by writing

$$L = NP_N = h - (90^{\circ} - d).$$

\* The declination of a star can be found from the Nautical Almanac.

† Various corrections to the observed altitude are generally necessary to obtain the true altitude.

The student should draw a figure for each case. First, a figure like Fig. 20 should be drawn showing the circle,  $Z$ ,  $N$ , and  $S$ . Then the star  $M$  should be located on the figure so that arc  $NM = h$  if the star bears north or so that  $SM = h$  if it bears south.

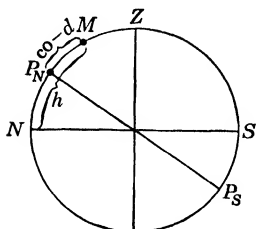


FIG. 20.

Next, the pole should be located so that arc

$$MP_N(\text{or } MP_S) = 90^\circ - d.$$

Finally, the altitude of the pole elevated above the horizon should be computed from the figure.

**Example.** Find  $L$  if the declination of a star is  $62^\circ$  S. and if its altitude as it crosses the meridian at upper culmination\* is  $50^\circ$  bearing south.

*Solution.* Since the star bears south and since it appears in the sky  $50^\circ$  above the horizon, it is represented in Fig. 21 on the right side of the circle so that arc  $SM = 50^\circ$ . Next

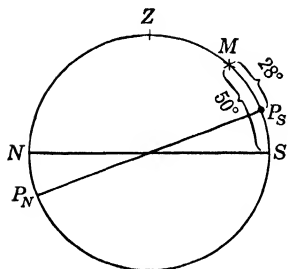


FIG. 21.

$$MP_S = 90^\circ - d = 90^\circ - 62^\circ = 28^\circ$$

is laid off to locate  $P_S$ . Hence the latitude is

$$L = 50^\circ - 28^\circ = 22^\circ \text{ S.}$$

The observer must have been in south latitude since the south pole was elevated above the horizon.

### EXERCISES

From the meridian altitude  $h$ , the declination  $d$ , and the bearing of the observed body as indicated, find the latitude of the observer in each of the following cases:

\* The stars appear to move through the sky, each describing a small circle, one of whose poles is the celestial north pole, the other, the celestial south pole. Thus each star crosses the plane of the meridian of a place twice every 24 hr., the first time on one side of the pole and the second time on the opposite side. The greater of the two altitudes of meridian transit is the altitude of upper culmination; the lesser is the altitude of lower culmination.

Assume in each of the Exercises 1 to 12 that the body is in upper culmination.

$d$	$h$	$d$	$h$
1. $50^\circ$ N.	$40^\circ$ N.	7. $41^\circ 39'$ N.	$82^\circ 11'$ N.
2. $40^\circ$ S.	$20^\circ$ S.	8. $37^\circ 15'$ N.	$40^\circ 21'$ N.
3. $20^\circ$ N.	$60^\circ$ S.	9. $11^\circ 0'$ N.	$70^\circ 19'$ N.
4. $50^\circ 25'$ S.	$35^\circ 29'$ S.	10. $17^\circ 39'$ S.	$72^\circ 21'$ S.
5. $30^\circ 15'$ S.	$47^\circ 35'$ N.	11. $47^\circ 23'$ S.	$35^\circ 26'$ S.
6. $28^\circ 10'$ N.	$71^\circ 12'$ S.	12. $23^\circ 13'$ N.	$75^\circ 40'$ S.

Assume in each of the Exercises 13 to 16 that the body is in lower culmination.

13. $59^\circ 49'$ N.	$44^\circ 11'$ N.	15. $73^\circ 16'$ N.	$28^\circ 48'$ N.
14. $77^\circ 54'$ S.	$25^\circ 18'$ S.	16. $42^\circ 29'$ N.	$25^\circ 23'$ S.

17. Two observers,  $A$  and  $B$ , are at different places on the same meridian. At the same instant each observer measured the meridian altitude of a star having declination  $26^\circ 16'$  S.  $A$  observed the star bearing south at an altitude  $30^\circ 17'$ ,  $B$  observed the star bearing north at an altitude  $60^\circ 17'$ . Find the great-circle distance between  $A$  and  $B$ .

**61. Given  $t$ ,  $d$ ,  $h$ , to find  $L$ .** This is the double-solution case, since the given parts of the astronomical triangle are two sides and the angle opposite one of them. A method of finding  $L$  when  $t$ ,  $d$ , and  $h$  are given is obtained by applying Napier's rules to the right triangles in Fig. 22. From triangle I, we have  $\cos t = \tan \varphi \tan d$  or

$$\tan \varphi = \cos t \cot d. \quad (14)$$

From triangles I and II, we get

$$\begin{aligned} \sin d &= \cos p \cos \varphi, \\ \sin h &= \cos p \cos \varphi'. \end{aligned}$$

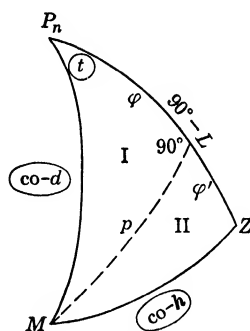


FIG. 22.

Dividing the second of these equations by the first, member by member, and solving the result for  $\cos \varphi'$ , we obtain

$$\cos \varphi' = \cos \varphi \sin h \csc d. \quad (15)$$

Then  $90^\circ - L = \varphi + \varphi'$ , or

$$L = 90^\circ - (\varphi + \varphi'). \quad (16)$$

Two solutions are obtained by choosing  $\varphi'$  from (15), first positive and then negative. Since approximate position is generally known, only the desired value need be computed. If north declination be considered as negative, the latitude found from (16) will be north if  $90^\circ - (\varphi + \varphi')$  is positive and south if  $90^\circ - (\varphi + \varphi')$  is negative.

### EXERCISES

1. From the following data, compute in each case the latitude.

$$\begin{aligned}(a) \quad t &= 35^\circ \text{ W.}, \\ d &= 0^\circ \text{ N.}, \\ h &= 42^\circ.\end{aligned}$$

$$\begin{aligned}(b) \quad t &= 29^\circ \text{ W.}, \\ d &= 7^\circ \text{ S.}, \\ h &= 34^\circ.\end{aligned}$$

2. From the following data, compute in each case the latitude and azimuth.

$$\begin{aligned}(a) \quad t &= 30^\circ \text{ W.}, \\ d &= 15^\circ \text{ N.}, \\ h &= 60^\circ.\end{aligned}$$

$$\begin{aligned}(c) \quad t &= 31^\circ 12' 13'' \text{ W.}, \\ d &= 15^\circ 12' 7'' \text{ N.}, \\ h &= 59^\circ 11' 44''.\end{aligned}$$

$$\begin{aligned}(b) \quad t &= 32^\circ \text{ W.}, \\ d &= 26^\circ \text{ N.}, \\ h &= 40^\circ.\end{aligned}$$

$$\begin{aligned}(d) \quad t &= 10^\circ \text{ E.}, \\ d &= 23^\circ \text{ S.}, \\ h &= 22^\circ.\end{aligned}$$

### 62. MISCELLANEOUS EXERCISES

1. From  $\cos x = 1 - 2 \text{ hav } x$  prove

$$\begin{aligned}\sin x \sin y &= \text{hav } (x + y) - \text{hav } (x - y), \\ \cos x \cos y &= 1 - \text{hav } (x + y) - \text{hav } (x - y),\end{aligned}$$

and thence, from the law of cosines:

$$\begin{aligned}\text{hav } a &= \text{hav } (b + c) \text{hav } A + \text{hav } (b - c) \text{hav } (180^\circ - A), \\ \text{hav } B &= \frac{\text{hav } b - \text{hav } (c - a)}{\text{hav } (c + a) - \text{hav } (c - a)},\end{aligned}$$

or

$$\text{hav } (180^\circ - B) = \frac{\text{hav } (c + a) - \text{hav } b}{\text{hav } (c + a) - \text{hav } (c - a)}.$$

2. Given  $t = 45^\circ 10' 30'' \text{ W.}$ ,  $d = 1^\circ 9' 15'' \text{ S.}$ ,  $L = 37^\circ 30' \text{ N.}$ , find the azimuth  $Z_n$ .

3. Given  $t = 55^\circ \text{ E.}$ ,  $d = 15^\circ \text{ S.}$ , and  $L = 42^\circ \text{ N.}$ , find  $h$  and  $Z$ .

4. Given  $t = 30^\circ \text{ W.}$ ,  $d = 45^\circ \text{ N.}$ ,  $h = 60^\circ$ , find  $L$  and  $Z$ .

5. Given  $t = 30^\circ \text{ E.}$ ,  $d = 15^\circ \text{ S.}$ ,  $h = 60^\circ$ , find  $L$  and  $Z$ .

6. From the following data, compute in each case the latitude and azimuth.

$$\begin{aligned}(a) \quad h &= 68^\circ, \\ t &= 10^\circ \text{ E.}, \\ d &= 23^\circ \text{ S.}\end{aligned}$$

$$\begin{aligned}(b) \quad t &= 30^\circ 11' \text{ E.}, \\ d &= 22^\circ 29' \text{ N.}, \\ h &= 44^\circ 57' .\end{aligned}$$

7. In each of the following exercises,  $L$  represents the latitude of the observer,  $d$  the declination of a star, and  $h$  its altitude. Find in each case the hour angle  $t$  and the azimuth  $Z_n$  of the star.

$$(a) \quad L = 45^\circ \text{ N.}, d = 22^\circ 30' \text{ N.}, h = 30^\circ \text{ W.}$$

$$(b) \quad L = 30^\circ \text{ S.}, d = 15^\circ \text{ N.}, h = 37^\circ 30' \text{ E.}$$

8. An airplane following a great-circle track travels from a place having  $L = 37^\circ 50' \text{ N.}$ ,  $\lambda = 122^\circ 20' \text{ W.}$  (near Oakland, Calif.) to a place having  $L = 40^\circ 40' \text{ N.}$ ,  $\lambda = 74^\circ 10' \text{ W.}$  (near Newark, N. J.). How close does it pass to a point for which  $L = 41^\circ 50' \text{ N.}$ ,  $\lambda = 87^\circ 40' \text{ W.}$  (near Chicago, Ill.)?

9. Compute the distance and the initial course for a voyage along a great circle from Yokohama,  $L = 35^\circ 26' 41'' \text{ N.}$ ,  $\lambda = 139^\circ 39' 0'' \text{ E.}$ , to Diamond Head, Hawaii,  $L = 21^\circ 15' 8'' \text{ N.}$ ,  $\lambda = 157^\circ 48' 44'' \text{ W.}$

10. Compute the distance and the initial course for a voyage along a great circle from Brisbane, Australia,  $L = 27^\circ 27' 32'' \text{ S.}$ ,  $\lambda = 153^\circ 1' 48'' \text{ E.}$ , to Acapulco,  $L = 16^\circ 49' 10'' \text{ N.}$ ,  $\lambda = 99^\circ 55' 50'' \text{ W.}$  Also find the latitude and longitude of the southern vertex of the track.

11. Compute the distance and initial course for a great-circle voyage from a point having  $L = 37^\circ 42' \text{ N.}$ ,  $\lambda = 123^\circ 4' \text{ W.}$ , near Farallon Island Lighthouse, to a point having  $L = 34^\circ 50' \text{ N.}$ ,  $\lambda = 139^\circ 53' \text{ E.}$ , near the entrance to the Bay of Tokyo.

12. Find distance and the initial course of a great-circle voyage from San Diego,  $L = 32^\circ 43' \text{ N.}$ ,  $\lambda = 117^\circ 10' \text{ W.}$ , to Cavite,  $L = 14^\circ 30' \text{ N.}$ ,  $\lambda = 120^\circ 55' \text{ E.}$

13. Find where the track of the preceding exercise crosses the meridian of  $157^\circ 49' \text{ W.}$  and at what distance from the harbor of Honolulu,  $L = 21^\circ 16' 5'' \text{ N.}$ ,  $\lambda = 157^\circ 49' \text{ W.}$ , then due south.

14. The initial course by great-circle track from San Francisco,  $L = 37^\circ 50' \text{ N.}$ ,  $\lambda = 122^\circ 30' \text{ W.}$ , to a place near Yokohama,  $L = 35^\circ 30' \text{ N.}$ ,  $\lambda = 140^\circ \text{ E.}$ , is  $302^\circ 59' 05''$ . Find the longitude of the most northerly point of this path.

15. Find the latitude and longitude of the most northerly point reached by a plane flying from San Francisco, Lat.  $37^\circ 48' \text{ N.}$ , Long.  $122^\circ 28' \text{ W.}$ , to Calcutta, Lat.  $22^\circ 33' \text{ N.}$ , Long.  $88^\circ 19' \text{ E.}$

16. An airplane follows a great-circle track from New York,  $L = 40^{\circ}40' \text{ N.}$ ,  $\lambda = 74^{\circ}10' \text{ W.}$ , to  $L = 56^{\circ}30' \text{ N.}$ ,  $\lambda = 3^{\circ}0' \text{ W.}$  (near Edinburgh, Scotland). Where will it make its nearest approach (a) to the North Pole? (b) To  $L = 46^{\circ}50' \text{ N.}$ ,  $\lambda = 71^{\circ}10' \text{ W.}$  (near Quebec, Canada)?

17. Find the distance in degrees between the sun and the moon when their right ascensions are, respectively,  $15^{\text{h}} 12^{\text{m}}$ ,  $4^{\text{h}} 45^{\text{m}}$  and their respective declinations are  $21^{\circ}30' \text{ S.}$ ,  $5^{\circ}30' \text{ N.}$

18. Find the distance in degrees between Regulus  $RA = 10^{\text{h}}$ ,  $p = 77^{\circ}19'$  and Antares  $RA = 16^{\text{h}} 20^{\text{m}}$ ,  $p = 116^{\circ}06'$ .

19. An observer in Lat.  $60^{\circ}23'20'' \text{ S.}$  finds the altitude of a star when crossing the prime vertical\* to be  $38^{\circ}23'20''$ , bearing east. Find the declination of the star.

20. A star in declination  $47^{\circ}52'15'' \text{ S.}$ , bearing east, makes its prime-vertical transit in altitude  $58^{\circ}20'00''$ . Find the hour angle of the star.

21. What is the latitude of the place at which the sun rises exactly in the northeast on the longest day of the year?

22. Find the local apparent time of sunrise and sunset at

(a) London:  $L = 51^{\circ}29' \text{ N.}$ , if  $d$  of sun =  $13^{\circ}17' \text{ N.}$

(b) Panama:  $L = 8^{\circ}57' \text{ N.}$ , if  $d$  of sun =  $18^{\circ}29' \text{ N.}$

(c) New Orleans:  $L = 29^{\circ}58' \text{ N.}$ , if  $d$  of sun =  $4^{\circ}30' \text{ N.}$

(d) Sydney:  $L = 33^{\circ}52' \text{ S.}$ , if  $d$  of sun =  $4^{\circ}30' \text{ N.}$

23. Find the length (a) of the longest day; (b) of the shortest day at Leningrad  $L = 59^{\circ}56'30'' \text{ N.}$ ,  $\lambda = 30^{\circ}19'22'' \text{ E.}$

24. Find the hour angle and amplitude of moonrise at Washington, D. C.,  $L = 38^{\circ}59' \text{ N.}$ , on a day when the moon's declination is  $25^{\circ}28' \text{ N.}$

25. If twilight continues until the sun is  $18^{\circ}$  below the horizon, find the length of dawn, dark night, bright day, and twilight in Annapolis,  $L = 38^{\circ}58'53'' \text{ N.}$  (a) at summer solstice ( $d = 23^{\circ}27'7'' \text{ N.}$ ); (b) winter solstice ( $d = 23^{\circ}27'7'' \text{ S.}$ ); (c) when the sun is at an equinox.

26. The following observations have been made of a heavenly body in upper culmination. Find the latitude in each case.

	Declination	Observed altitude	Bearing
(a)	$28^{\circ}10' \text{ N.}$	$71^{\circ}12'$	South
(b)	$73^{\circ}02' \text{ N.}$	$58^{\circ}40'$	North
(c)	$44^{\circ}17' \text{ S.}$	$65^{\circ}23'$	South
(d)	$30^{\circ}15' \text{ S.}$	$47^{\circ}35'$	North
(e)	$50^{\circ}25' \text{ S.}$	$35^{\circ}29'$	South
(f)	$40^{\circ}16' \text{ N.}$	$40^{\circ}14'$	North

\* For definition of prime vertical, see §53.

**27.** What relations must exist between  $L$  and  $d$  for a lower culmination to be visible? What relation always exists at a visible lower culmination between  $h$  and  $d$ ?

**28.** In each of the following observations of a lower culmination, find the latitude:

	Declination	Observed altitude	Bearing
(a)	88°50' N.	37°20'	North
(b)	46°22' S.	32°15'	South
(c)	59°49' N.	44°11'	North
(d)	77°54' S.	25°18'	South

**29.** The right ascension of the sun is  $45^\circ$ ; find (a) the length of the night at a point in latitude  $60^\circ$  N.; (b) the length of the shadow cast by a vertical stick 10 ft. long at 10 A.M. (local apparent time) at a point in latitude  $40^\circ$  N.; (c) the direction of a wall that casts no shadow at 10 A.M. at a place having latitude  $40^\circ$  N.

*Hint.* Compute the declination of the sun and then draw the astronomical triangle.

**30.** At a place in Lat.  $51^\circ 32'$  N., the altitude of the sun is  $35^\circ 15'$  bearing west and its declination is  $21^\circ 27'$  N. Find the local apparent time.

**31.** In London,  $L = 51^\circ 31'$  N., for an afternoon observation the altitude of the sun is  $15^\circ 40'$ . If its declination is  $12^\circ$  S., find the local apparent time.

**32.** (a) A navigator in latitude  $15^\circ 23' 36''$  S. observes a star having  $RA = 12^h 27^m 32^s$ ,  $d = 22^\circ 16' 36''$  N., at an altitude  $h = 17^\circ 26' 30''$  W. If the sidereal time  $ST$  of Greenwich at the instant of observation is  $10^h 27^m 34^s$ , find the longitude of the navigator.

(b) Also find the longitude of a second navigator in latitude  $62^\circ 21' 39''$  N. who at the same instant observes a star having  $RA = 6^h 27^m 30^s$ ,  $d = 26^\circ 55' 21''$  N. at an altitude  $h = 33^\circ 17' 44''$  W.

**33.** Find to the nearest minute the direction of the shadow of a vertical staff in Lat.  $38^\circ 59'$  N. at 6 A.M. local apparent time, when the declination of the sun is  $23^\circ 27'$  N.

**34.** Find the direction of a wall in Lat.  $52^\circ 30'$  N. that casts no shadow at 6 A.M. on the longest day of the year.

**35.** An explorer claimed to have reached the north pole. He took the picture of a flagpole 6 ft. high. From measurements made on the photograph it appeared that the 6-ft. pole cast a shadow 10.1 ft. long. Prove that he must have been at least  $7^\circ$  from the pole.

Find the shortest length of shadow that a stick 6 ft. long could possibly cast on level ground when held vertical at the north pole.

36. If the altitude of the north pole is  $45^\circ$  and if the azimuth of a star on the horizon is  $135^\circ$ , find the polar distance of the star.

37. Find the time of day when the sun bears due east and when it bears due west on the longest day of the year at Leningrad (Lat.  $59^\circ 56'$  N.).

38. Two points on the earth are in latitude  $40^\circ$  N. and their difference in longitude  $DLo = 70^\circ$ . How much does the parallel of latitude joining these points exceed in length the arc of the great circle joining them? How far apart are the mid-points of the two tracks? (Use 3437 nautical miles for the radius of the earth.)

39. Find the altitude of the sun at  $6^h$  A.M. at Munich (Lat.  $48^\circ 9'$  N.) on the longest day of the year.

### 63. Ageton's method.

The solution of a spherical triangle when two sides and the included angle are known is the most important one for navigation. A short method for solving the astronomical triangle when  $t$ ,  $d$ , and  $L$  are known was devised by Commander Arthur A. Ageton. It is widely used in the United States Navy.

Ageton's formulas involve only secants and cosecants. They may be easily derived by applying Napier's rules to Fig. 23. These formulas are

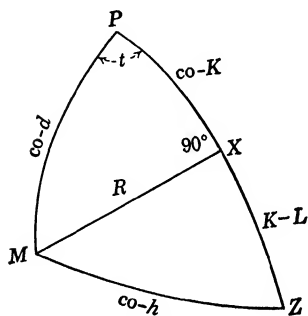


FIG. 23.

$$\left. \begin{aligned} \csc R &= \csc t \sec d, \\ \csc K &= \frac{\csc d}{\sec R}, \\ \csc h &= \sec R \sec (K - L), \\ \csc Z &= \frac{\csc R}{\sec h}. \end{aligned} \right\} \quad (17)$$

Since  $t$ ,  $d$ , and  $L$  are known, it appears that the formulas (17) can be used to solve for  $R$ ,  $K$ ,  $h$ , and  $Z$  in succession, and, from the results, azimuth and altitude can be computed.

64. Dreisonstok's method. Another method for obtaining azimuth and altitude by solving the astronomical triangle was devised by Lieutenant Commander Joseph Y. Dreisonstok (Retired). This method is also used widely in the United States



Navy and is especially useful in aerial navigation. It has reference to Fig. 24.  $t$ ,  $d$ , and  $L$  are assumed known. Then by means of special tables the legs  $a$  and  $b$  and angle  $Z'$  of the right triangle  $PXZ$  can be read. Finally by using Napier's rules and Fig. 24, we deduce the formulas

$$\left. \begin{aligned} B &= (90^\circ - d) - b, \\ \csc h &= \sec a \sec B, \\ \tan Z'' &= \csc a \tan B, \\ Z &= Z' + Z''. \end{aligned} \right\} (18)$$

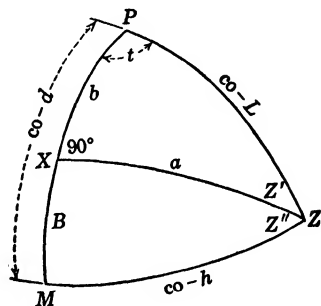


FIG. 24.

Azimuth and altitude are easily computed from the results obtained by using these formulas.

**65. Tables of computed altitude and azimuth.** By means of the United States Hydrographic Office Publication H. O. No. 214 it is possible to solve the astronomical triangle without trigonometric computation for altitude and azimuth, when declination, hour angle, and latitude are known. Although the range of the tables is limited, it is sufficient to deal with practically all useful cases. Also many other problems including course and distance problems in great circle sailing come within their range. The convenience of these tables is considerable although their bulkiness makes them unsuitable for some purposes.

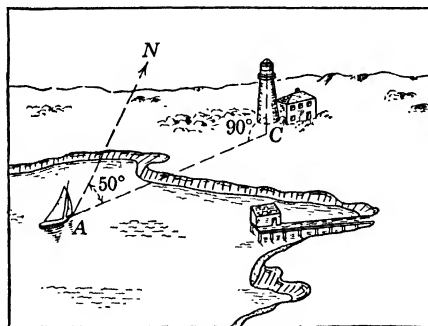


FIG. 25.

**66. Lines of position.** **Fix.** A *line of position* for an observer is a line passing through his position. For example, if an observer sees a lighthouse bearing  $50^\circ$  (see Fig. 25), then a straight

line passing through the lighthouse and bearing  $50^\circ$  or  $50^\circ + 180^\circ = 230^\circ$  is a line of position for this observer. Lines of posi-

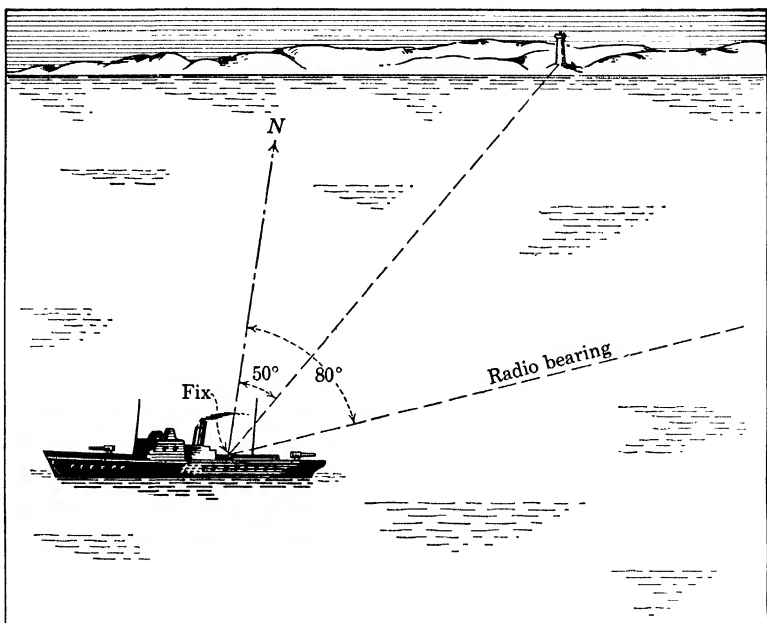


FIG. 26.

tion are frequently obtained from radio bearings. If an observer can draw two straight lines of position on a map, his position will be indicated by their intersection. Such an intersection is called a *fix* (see Fig. 26). Evidently an observer could plot his position on a map by plotting two known objects, observing their respective bearings, and drawing the corresponding lines of position.

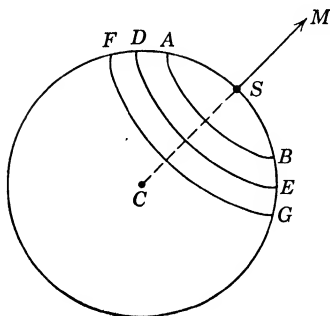


FIG. 27.

**67. Circles of equal altitudes for a star used to make a fix.** The point in which a straight line connecting

the center of the earth to a star cuts the earth's surface is called the *sub-astral point* of the star. Any circle on the earth having

the sub-astral point of the star as pole is a circle from each of whose points the star has the same altitude. In Fig. 27  $S$  represents the sub-astral point of the star  $M$  and circles  $AB$ ,  $DE$ , and  $FG$ , having  $S$  as pole, are *circles of equal altitudes* for the star.

The circles of equal altitudes through an observer's position are of great importance in navigation. If the star is in the observer's zenith, its altitude is  $90^\circ$  and the circle of equal altitudes is a point. But if the zenith distance of the star is greater than  $4^\circ$  the observer's circle is so large that, for practical purposes, the representation of a small portion of it on a map may be taken as a straight line, called a *Sumner line*. A Sumner line is a very useful line of position to be used in making a fix.

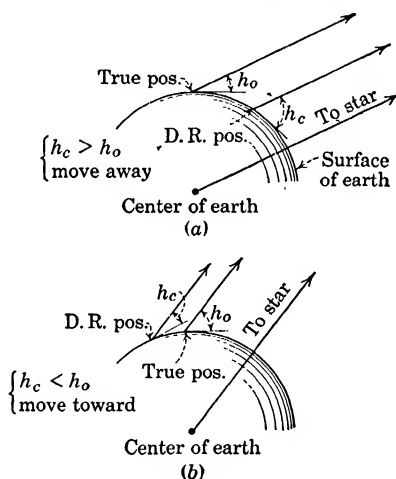


FIG. 28.

The use made of the circle of equal altitudes will now be considered. By a process known as *dead reckoning* approximate values of the ship's latitude and longitude are obtained by applying to the last well-determined position the run that has since been made using for the purpose the distance sailed and the course. The declination is obtained from the Nautical Almanac and  $t$  is obtained by using the longitude of the D.R. position in the formula of §59. Hence,  $t$ ,  $d$ , and  $L$  being known,  $h_c$  for the star may be computed by the method of §55. Also the altitude  $h_o$  and the azimuth  $Z$  can be measured directly by means of instruments. Since  $h_c$  is computed from data obtained by dead reckoning it will be slightly in error; whereas  $h_o$  and  $Z$ , being

observed from the actual position will be correct. If  $h_e$  is greater than  $h_0$  the true position is farther from the sub-astral point of the star than the D.R. (dead reckoning) position, as may be seen by considering Fig. 28(a). If  $h_e$  is less than  $h_0$  the true position is nearer to the sub-astral point than the D.R. position [see Fig. 28(b)]. The mnemonic **Coast Guard Academy** with the initial letters *C, G, A* suggests *computed greater away* or  $h_e$  greater than  $h_0$  go away from subastral point.

Hence to draw the line of position corresponding to the observer's circle of equal altitudes for a star, observe the altitude  $h_0$  and the azimuth  $Z$  of a star, using known values of  $t$ ,  $d$ , and  $L$ , compute  $h_e$  by the method of §55, plot the D.R. position, draw through it a line having a bearing equal to the azimuth  $Z$  of the star, plot a point on this line distant from the D.R. position the difference between  $h_e$  and  $h_0$  in the appropriate direction, and finally draw the desired line of position through this latter point perpendicular to the first line.

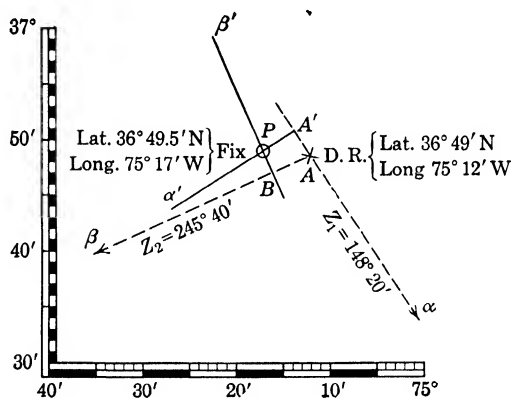


FIG. 29.

Two lines of position may be drawn by using two stars, and their intersection gives a corrected position. In practice three or even more lines of position are drawn when conditions are favorable. The following example will illustrate the procedure.

**Example.** The captain of a ship near Cape Henry found his approximate position by dead reckoning to be Lat.  $36^{\circ}49' N.$ , Long.  $75^{\circ}12' W.$  He found for the star  $\alpha$  Scorpii, declination  $26^{\circ}17'12'' S.$ , altitude  $h_0 = 19^{\circ}24'14''$ , azimuth  $148^{\circ}20'$ , and the computed altitude  $h_e = 19^{\circ}26'45''$ . For the star  $\beta$  Leonis declination  $14^{\circ}57'6'' N.$ , he found altitude  $h_0 = 51^{\circ}4'5''$ , azimuth

245°40' and by computation  $h_c = 51^\circ 00' 37''$ . Draw a fix from this data and read the latitude and longitude of his position.

*Solution.* For star  $\alpha$  we have

$$h_c - h_0 = 19^\circ 26' 45'' - 19^\circ 24' 14'' = 2' 31''.$$

Since  $h_c > h_0$  the correction is **2'31'' = 2.5 miles away on bearing  $Z_1 = 148^\circ 20'$** . For star  $\beta$  we have  $h_0 - h_c = 51^\circ 4' 5'' - 51^\circ 00' 37'' = 3' 28''$ . Since  $h_c < h_0$  the correction is **3'28'' = 3.5 miles towards on bearing  $Z_2 = 245^\circ 40'$** . On Fig. 29 the dead reckoning position is plotted at  $A$ . Line  $A\alpha$  is drawn bearing  $148^\circ 20'$  and on it  $AA'$  is laid off equal to  $2.5' = 2.5$  miles.  $A'\alpha'$  is drawn perpendicular to  $A\alpha$ . Similarly  $A\beta$  is drawn bearing  $245^\circ 40'$ ,  $AB$  is laid off equal to  $3.5' = 3.5$  miles and  $B\beta'$  is drawn perpendicular to  $A\beta$ . Lines  $A'\alpha'$  and  $B\beta'$  intersect in the position  $P$ , the fix required. From the map we read Lat. =  $36^\circ 49.5' N.$ , Long. =  $75^\circ 17' W.$  (The scale of the chart in Fig. 29 is 1 in. = 12.5 mi.)

### EXERCISES

1. Using the observed altitude  $h_0$ , the computed altitude  $h_c$ , and the bearing of the observed body as indicated, draw a figure showing an assumed dead reckoning position D.R.,  $L = 37^\circ N.$ ,  $\lambda = 75^\circ 30' W.$ , the bearing, and the line of position in each of the following:

	$h_0$	$h_c$	Bearing
(a)	30°40'	30°47'	75°30'
(b)	42°55'	43°	35°
(c)	27°55'	27°58'	82°30'
(d)	40°48'	40°44'	50°50'
(e)	39°7'	38°58'	65°40'
(f)	72°50'	72°44'	147°30'
(g)	68°40'	68°35'	285°20'
(h)	32°24'	32°30'	205°30'
(i)	57°28'	57°34'	345°10'
(j)	26°32'	26°27'	210°

2. Draw a figure showing an assumed dead reckoning position, two lines of position, and a fix obtained by using the data of the exercises indicated in each of the following:

- |                 |                 |
|-----------------|-----------------|
| (A) 1(a), 1(b); | (C) 1(d), 1(h); |
| (B) 1(c), 1(g); | (D) 1(i), 1(j). |

3. Draw a figure showing a fix determined by the following data using the D.R. position  $L = 37^\circ \text{ N.}$ ,  $\lambda = 75^\circ 30' \text{ W.}$ :

$h_0$	$h_c$	Bearing
$38^\circ 44'$	$38^\circ 40'$	$50^\circ$
$27^\circ 36'$	$27^\circ 39'$	$197^\circ$
$62^\circ 40'$	$62^\circ 38'$	$147^\circ$

*Hint:* The three lines of position will not intersect in a point. Take as your correct position the point of intersection of the medians of the small triangle formed by the lines of position.

4. Given  $d = 22^\circ 30' \text{ S.}$ ,  $t = 60^\circ \text{ E.}$ ,  $L = 45^\circ \text{ S.}$ ,  $\lambda = 32^\circ \text{ W.}$ ,  $h_0 = 36^\circ 36' 18''$ . Using the given values of  $d$ ,  $t$ , and  $L$  compute  $h_c$  and  $Z_n$ . Then show on a figure the given D.R. position, the bearing, a line of position. See §55.

5. The navigator of a cruiser at D.R. position Lat.  $37^\circ 17' \text{ N.}$ , Long.  $75^\circ 27' \text{ W.}$  observes the sun for a line of position and finds  $h_0 = 15^\circ 43'$ ,  $Z_n = 107^\circ 11'$ . At the same instant the assistant navigator finds that the true bearing of Hog Island Light (Lat.  $37^\circ 24' \text{ N.}$ , Long.  $75^\circ 42' \text{ W.}$ ) is  $285^\circ$ . Using his value of  $h_c = 15^\circ 40'$  obtain a fix and read the latitude and longitude of his position.

**68. Aerial navigation.** The parts of this treatment applying to navigation could be used for aerial navigation as well as navigation on the ocean. The theory used in location of position by an airplane pilot is essentially the same as that used by the captain of a ship. As soon as a pilot conducts aircraft on long oversea passages out of sight of land or over strange terrain, he must determine his position by using methods the same in fundamentals as those used by a surface navigator. However, the aerial navigator has many obstacles to overcome. The following paragraph will suggest some of them.

A ship travels through the water on a given course and at a speed known within close limits of accuracy. Ocean currents are known, and the effect of these currents and of winds can be estimated with relative accuracy and due allowance made for their effect. On the other hand, aircraft travel at high rates of speed; the supporting medium moves rapidly in three dimensions; winds cannot be charted as can ocean currents; aircraft encounter fog, haze, storms, and heavy cloud formations so that it is difficult to compute drift and take observations; and many other difficulties imposed by travel in three dimensions make aerial

navigation difficult and comparatively hazardous. The aerial navigator does not have a stable platform from which to take observations. The high speed of aircraft demands that solutions of the astronomical triangle be found expeditiously. Confined spaces in airplane compartments, disturbing effects of air stream, generally unstable characteristics of planes in flight, all contribute to difficulties of accurate observation and subsequent working up of data. Consequently in this field of work the aim has been to shorten navigational methods to the greatest possible extent without undue sacrifice of accuracy and to use many instruments of light weight and small bulk adaptable to convenient handling and stowage.

It therefore appears that positions in the air cannot generally be found with the same degree of ease and accuracy as positions on the surface of the earth. Nevertheless, the best possible results should be attempted to avoid the risk of missing the destination and to obtain the economy in time and fuel incident with the most direct route.





## APPENDIX A

**1. The mil.** The *mil* is an angular unit equal to  $\frac{1}{6400}$  of four right angles.

The word *mil*, meaning one-thousandth, originated from the idea of adopting as a unit the angle that subtends an arc equal to  $\frac{1}{1000}$  of the radius. Such an angle subtends 1 ft. at a distance of 1000 ft., 1 yd. at a distance of 1000 yd., etc. This manifestly furnishes a quick method of estimating the distance of an object whose size is known. There would under these circumstances

be  $\frac{2\pi}{0.001}$  or 6283.18+ such units subtended by a circle. This number is too inconvenient to be of practical use in calibrating instruments. The circle is therefore divided into 6400 equal parts, and each of these is called a mil. The arc subtended by a

central angle of 1 mil therefore equals  $\frac{2\pi R}{6400}$  or  $(0.00098+)\text{ }R$ , or so nearly  $\frac{1}{1000}$  of the radius that it may be so taken for purposes not demanding great accuracy. This property, coupled with the knowledge that in small angles the chord very nearly equals the arc, enables us to say for rapid and rough approximation:

*A mil subtends a chord equal to  $\frac{1}{1000}$  of the distance to the chord.*

*With due regard to the degree of approximation, a small number of mils (several hundred) subtends a chord equal to the small number times  $\frac{1}{1000}$  of the distance to the chord, or, in symbols*

$$s = \frac{r\theta}{1000}$$

*where  $\theta$  is in mils and  $s$  and  $r$  are expressed in the same unit.*

The methods of rapid approximate measurement of angles and distances by the use of the mil system were first developed by the Field Artillery in computing firing data. Their use was extended to mapping, sketching, and reconnaissance. During the World War the Infantry adopted the system, and it has now become general.

The mil as a unit has the advantage that it is convenient in size for certain military measurements.

**Example 1.** Two points,  $A$  and  $B$ , are 50 yd. apart and 2000 yd. away. How many mils should they subtend (see Fig. 1)?

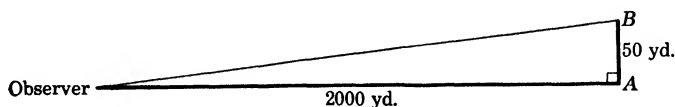


FIG. 1.

*Solution.* 50 divided by  $\frac{2000}{1000} = 25$ .

Or, at 2000 yd., 2 yd. corresponds to 1 mil; therefore 50 yd. corresponds to 25 mils.

**Example 2.** An observer measures the angular distance between two points,  $A$  and  $B$ , 5000 yd. away, to be 30 mils. How far apart are  $A$  and  $B$ ?

*Solution.*  $\frac{5000}{1000} \times 30 = 150$ .

Or, at 5000 yd., 1 mil subtends 5 yd.; therefore 30 mils subtends 150 yd.

**Example 3.** The angular distance between  $A$  and  $B$  is observed to be 40 mils. They are 100 yd. apart. How far away are they?

*Solution.*  $\frac{100}{40} \times 1000 = 2500$ .

Or 40 mils corresponds to 100 yd.; therefore 1 mil corresponds to  $2\frac{1}{2}$  yd., but  $2\frac{1}{2}$  is  $\frac{1}{1000}$  of 2500 yd.

### EXERCISES

1. A battery with a front of 60 m. is observed from a point 3000 m. away, measured on a line normal to the battery. What angle does the battery subtend? (Or what is its front in mils?)

2. A four-gun battery 4000 m. away has a front of 15 mils. How many meters between muzzles?

3. The guns in your battery have wheels  $1\frac{1}{2}$  m. in diameter. You measure a wheel as 5 mils. How far are you from the battery?

4. An observer measures the front of a target to be 40 mils at a point 6000 m. away. What should a scout (a) 3000 m. in front of the same observer measure it to be? (b) 4000 m. in front of the observer?

5. Two targets,  $T$  and  $t$ , are 20 m. apart. The range  $TG$ , perpendicular to the line of targets, is 5000 m. Two guns,  $G$  and  $g$ , are also 20 m. apart, the angle  $TGg$  being 1500 mils. Take  $t$  and  $g$  both on the same side of  $TG$ .

(a) What is angle  $tgG$  in order that the gun  $g$  may be laid on  $t$ ?

(b) What change in deflection of  $G$  must be given to lay it on  $t$ ?

6. A hostile trench measures 80 mils from your position. A scout 500 meters in front of you measures it 100 mils. What is the distance of the trench from your position?

7. You signal to a man at a distant tree to post himself 20 yd. from the tree (measured perpendicular to the line from the tree to you). The man is now 8 mils from the tree. How far away is the tree?

8. An observer finds that he is on the same level with the top of a distant tower that is 34 yd. high. The angular depression of the base of the tower is 8 mils. How far away is the tower?

9. From  $D$  a distant object  $B$  appears to the right of an object  $A$ , which is 6000 meters away. An observer at  $D$  measures the angle  $ADB$  to be 35 mils. He moves to  $C$ , 180 meters to the right on a line normal to  $AD$ , and measures the angle  $ACB$  to be 15 mils. How far away is  $B$ ?

*Hint.* Sum of angles of a triangle is constant.

10. From Trophy Point, near the U. S. Military Academy, the angular elevation of Fort Putnam is 210 mils, and its distance is 600 yd. Also, the elevation of the top of the West Academic Building is 120 mils, and its distance is 250 yd. The West Academic Building and Fort Putnam are 500 yd. apart. What is the angular elevation of Fort Putnam as measured from the top of the West Academic Building?

## APPENDIX B

**2. The range finder.** A range finder is an instrument designed to obtain the distance of an object from the instrument. Essentially it is a mechanism in a tube by means of which images caught at the ends of the tube can be brought into alignment by turning a thumbscrew.

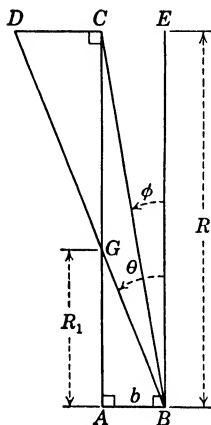


FIG. 2.

In Fig. 2 line  $AB$  represents a range finder of length  $b$ .  $AC$  and  $BE$  are lines perpendicular to  $AB$ . When the two images of point  $C$  caught at the ends  $A$  and  $B$  are brought into alignment, the distance  $AC = R$  can be read on a dial. When the image of point  $C$  caught at end  $A$  is brought into alignment with the image of point  $D$  caught at  $B$ , the distance  $AG = R_1$  is registered on the dial.

The distances  $R$  and  $R_1$  in Fig. 2 must be so great as compared with  $b$  that the errors in the equations

$$R\phi = b, \quad R_1\theta = b, \quad (1)$$

$$\phi = \frac{b}{R}, \quad \theta = \frac{b}{R_1}, \quad (2)$$

are negligible. On the other hand when the range of an object is so great that the angles represented by  $\phi$  and  $\theta$  in Fig. 2 are small, relative to the errors inherent in the mechanism of the range finder, trustworthy results cannot be obtained. A 12-ft. range finder is effective for distances from 100 to 25,000 yd.; a 26-ft. instrument, for ranges from 1200 to 50,000 yd.; a 30-ft. instrument, from 2400 to 60,000 yd.

The following examples illustrate the principles involved in the use of range finders.

**Example 1.** Let Fig. 2 represent a range finder of length  $b$  set parallel to line  $CD$ . If  $b = 10$  yd. and if the distance

$R_1 = 2500$  yd. and  $R = 10,000$  yd. have been found by using the instrument, find the length of  $CD$ . Also find  $CD$  in terms of  $R$ ,  $R_1$  and  $b$ .

*Solution.* Denote angle  $EBC$  by  $\phi$  and angle  $EBD$  by  $\theta$ . Since these angles are small, use equations (2) to obtain

$$\frac{b}{R} = \frac{10}{10000}, \quad \frac{b}{R_1} = \frac{10}{2500}.$$

By using (1), we obtain

$$CD = R\theta - R\phi = 10000\left[\frac{10}{2500} - \frac{10}{10000}\right] = 30 \text{ yd. (approx.)}.$$

To find  $CD$  in terms of  $R$ ,  $R_1$ , and  $b$ , use (2) and (1) to obtain

$$\phi = \frac{b}{R}, \quad \theta = \frac{b}{R_1}, \quad CD = R(\theta - \phi), \text{ (approx.)}.$$

Replacing  $(\theta - \phi)$  in the last equation by their values from the first two, we obtain

$$CD = R\left(\frac{b}{R_1} - \frac{b}{R}\right) = \frac{bR(R - R_1)}{RR_1} = \frac{b(R - R_1)}{R_1}. \quad (3)$$

**Example 2.** Figure 3 indicates how a range finder may be used to obtain the direction angle  $\alpha$  for an object  $CD$  of small known length  $a$  by means of the ranges  $R$  and  $R_1$  which may be read from the instrument. Find angle  $\alpha$  in terms of  $a$ ,  $b$ ,  $R$ , and  $R_1$ , assuming that  $a$  and  $b$  are small as compared with  $R$  and  $R_1$ . Find  $\alpha$  if  $a = 50$  yd.,  $R = 3000$  yd.,  $R_1 = 1000$  yd., and  $b = 10$  yd.

*Solution.* Referring to Fig. 3, observing that  $CF$  is small and using (3) in the solution of Example 1, we have

$$FD = \frac{b(R - R_1)}{R_1} \text{ (approx.)}.$$

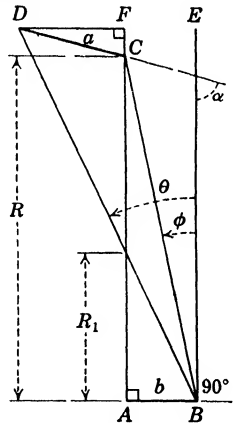


FIG. 3.

Since angle  $FCD = \alpha$ ,  $\sin \alpha = \sin (FCD) = FD/a$ , or replacing  $FD$  by the value just found,

$$\sin \alpha = \frac{b(R - R_1)}{aR_1}. \quad (4)$$

For the values mentioned in the example,

$$\sin \alpha = \frac{10(3000 - 1000)}{50(1000)} = \frac{2}{5}, \quad \text{and} \quad \alpha = 23^\circ 35'.$$

**Example 3.** A range finder is poorly adjusted. Show how the range given by such an instrument may be corrected.

*Solution.* When a range finder is not well adjusted it will register inaccurate distances. Referring to Fig. 4, we may say in such a case, that the ranges  $R$  and  $R_1$  are based on angles  $\phi \pm d$  and  $\theta \pm d$  where  $d$  is the error due to poor adjustment of the instrument. Hence

$$\phi \pm d = \frac{b}{R}, \quad \theta \pm d = \frac{b}{R_1} \quad (5)$$

If  $x$  is the corrected range, we have  $x$   
 $(\theta - \phi) = a$ , since  $\theta$  and  $\phi$  are the true angles. Then we may write

$$x = \frac{a}{\theta - \phi} = \frac{a}{(\theta \pm d) - (\phi \pm d)}, \quad (6)$$

or, replacing  $\theta \pm d$  by  $b/R_1$  and  $\phi \pm d$  by  $b/R$  from (5), we obtain the corrected range

$$x = \frac{a}{\frac{b}{R_1} - \frac{b}{R}} = \frac{aRR_1}{b(R - R_1)}. \quad (7)$$

For example, if  $a = 50$  yd.,  $R = 12,000$  yd.,  $R_1 = 2100$  yd., and  $b = 10$  yd., the corrected range would be

$$x = \frac{50(12,000)(2100)}{10(12,000 - 2100)} = 12,727 \text{ yd.},$$

and the correction increment is 727 yd.

### EXERCISES

1. In Fig. 2 find (a)  $CD$  if  $R = 10,000$  yd.,  $R_1 = 2000$  yd., and  $b = 30$  ft. (b)  $R$  if  $R_1 = 1500$  yd.,  $CD = 180$  ft.,  $b = 36$  ft. (c)  $CD$  if  $\theta = 990''$ ,  $\phi = 165''$ ,  $b = 36$  ft.

2. In Fig. 3 find (a)  $\alpha$  if  $R = 10,000$  yd.,  $R_1 = 2500$  yd.,  $a = 180$  ft.,  $b = 36$  ft. (b) find  $\alpha$  if  $\phi = 188''$ ,  $\theta = 960''$ ,  $a = 165$  ft.,  $b = 30$  ft. (c) find  $\alpha$  if  $\alpha = 9^\circ 30'$ ,  $R = 3500$  yd.,  $R_1 = 1000$  yd.,  $b = 30$  ft.

3. In Fig. 4 find the correction increment (a) if  $R = 15,000$  yd.,  $R_1 = 2800$  yd.,  $CD = 165$  ft.,  $b = 36$  ft. (b) if  $\phi = 185''$ ,  $\theta = 545''$ ,  $b = 48$  ft.,  $CD = 300$  ft.

4. In Fig. 5  $b = 36$  ft., (a) find  $DT$  if  $R = 14,000$  yd.,  $R_1 = 2000$  yd.,  $R_2 = 800$  yd. (b) find  $DT$  and  $DC$  if  $\alpha = 70^\circ$ ,  $\phi = 155''$ ,  $\theta_1 = 1710''$ ,  $\theta_2 = 4200''$ .

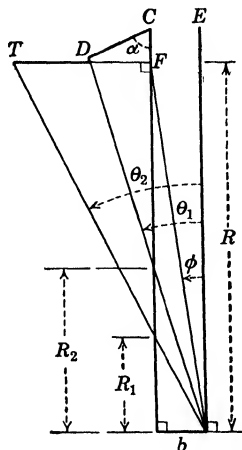


FIG. 5.

5. The captain of a vessel equipped with a coincident range finder of effective length 30 ft. desires to find the distance between two channel buoys  $C$  and  $D$ . He trains his range finder on buoy  $C$  and reads range  $R_c = 14,000$  yd. He then aligns the image of  $D$  with the image of  $C$  and reads on the dial  $R_1 = 2000$  yd. If the range finder is parallel to  $CD$  for the readings, find the distance between the buoys.

6. Two masts on a freighter are 165 ft. apart. The captain of a cruiser wishes to find the distance to the freighter with a range finder that is poorly adjusted. He trains the range finder on the right-hand mast and reads on the dial 15,000 yd. He then aligns the image of the second mast with that of the first and reads on the dial 2800 yd. If the range finder is parallel to the freighter, find the corrected range and the angular error of  $\theta$  for his instrument.

## APPENDIX C

**3. Stereographic projections.** In the applications of this chapter, the student will frequently find it convenient to draw a figure showing the main features of the problem under consideration. For this reason the following facts relating to stereographic projections are presented.

Consider a plane through the center of the sphere in Fig. 6 and the poles  $P_n$  and  $P_s$  of the great circle in which the plane intersects the sphere. A straight line connecting any point  $P$  on the sphere to  $P_s$  cuts the plane in a point called the *stereographic projection* of the point. The stereographic projection of a curve lying on the sphere is the locus of the stereographic

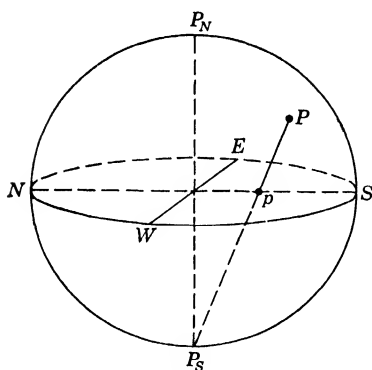


FIG. 6.

projections of its points. The point  $P_s$  is called the *center of projection*, the plane is called the *primitive plane*, and the great circle cut out by the primitive plane is called the *primitive circle*. The angular measure of an arc of a great circle that has a given arc as a projection is called the *true length* of the given arc.

Figure 6 represents the sphere with center of projection  $P_s$ , with primitive plane  $WSEN$ , and with  $p$  the stereographic projection of  $P$ . The truth of the following statements, numbered I, II, III, IV, and V, is easily perceived.

**I.** The points of the hemisphere on the same side of the primitive plane as  $P_s$  project outside the primitive circle, and the points on the other hemisphere project inside the primitive circle.

**II.** The projection of any great circle through the center of projection  $P_s$  is a straight line through the center of the primitive circle.

**III.** The primitive circle projects into itself.



IV. The projection of any great circle passes through the ends of a diameter of the primitive circle. For the plane of the great circle cuts the primitive circle in a diameter and the ends of this diameter project into themselves.

V. The part of the projection of an arc of a great circle that lies inside the primitive circle has a true length of  $180^\circ$ , and if this arc is bisected each part has a true length of  $90^\circ$ .

The following statements, numbered VI and VII, are of fundamental importance. The proofs are omitted.

VI. The stereographic projection of a circle lying on a sphere is a circle or a straight line.

VII. The angle of intersection of two arcs on a sphere is equal to the angle of intersection of their stereographic projections.

**4. Construction of some simple projections.** The projection of a great circle can be drawn when the two points where it

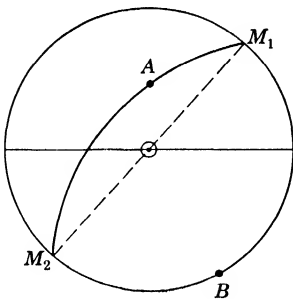


FIG. 7.

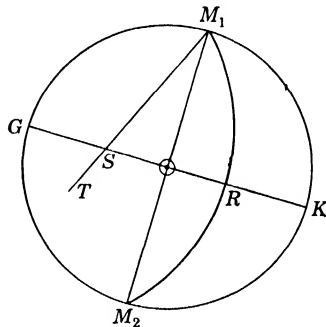


FIG. 8.

crosses the primitive circle at the ends of a diameter and the projection of another point are known. For, by VI, §3, the projection is a circle three points of which are known. For example, suppose that a great circle cuts the primitive circle shown in Fig. 7 at point  $M_1$  and that  $A$  is the projection of another of its points. If  $O$  is the center of the primitive circle,  $M_1$  lies on the projection by IV, §3. Therefore the circle through  $M_1$ ,  $A$ , and  $M_2$  is the required projection. Only the stereographic projection of one-half of a great circle is shown in Fig. 7.

Again, the projection of a great circle can be drawn when a point where the great circle cuts the primitive circle and the inclination of the plane of the circle to the primitive plane are

known. For, by IV, §3, two points at the ends of a diameter are known, by VI the projection is a circle, and by VII the angle between the primitive circle and the projection are known.

Suppose that the great circle whose stereographic projection is to be drawn cuts the primitive circle  $GM_1K$  shown in Fig. 8, at  $M_1$  and that its plane is inclined  $35^\circ$  to the primitive plane. Draw the mutually perpendicular diameters  $M_1M_2$  and  $GK$ , construct with a protractor the line  $M_1T'$ , making an angle of  $35^\circ$  with  $OM_1$  and meeting  $GK$  at  $S$ . With  $S$  as a center and  $SM_1$  as radius, draw the required circle  $M_1RM_2$ . The circle symmetrical over  $M_1M_2$  with the one drawn also satisfies the given conditions.

### EXERCISES

1. What great circles project into straight lines?
2. What is the nature of the projection of any circle passing through the center of projection?
3. What is the true length of the arc  $M_1R$  in Fig. 3? Give a reason for your answer.
4. Construct the projections of the great circles whose planes are inclined at  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $120^\circ$ , and  $150^\circ$ , respectively, with the primitive plane, assuming that each one passes through a point  $M_1$  chosen on the circumference of the primitive circle.
5. Draw a circle to be used as primitive circle. Through the ends of one of its diameters construct a circle. This second circle is the projection of a great circle. Now construct the projections of two other great circles through the ends of the same diameter, each of whose planes is inclined at  $30^\circ$  to the plane of the great circle whose projection is drawn first.

**5. To find the true length of a projected arc.** The actual magnitude of an arc of a great circle that has a given arc as its projection has been called the *true length* of the given arc. The object of this article is to give, without proof, a method of finding the true length of any arc that is the stereographic projection of a part of a great circle.

Let arc  $ACB$  in Fig. 9 represent the projection of a great circle on the primitive plane  $ABF$ . It passes through the ends  $A$  and  $B$  of a diameter and cuts the perpendicular diameter  $EF$  at  $C$ . Draw line  $AC$  and prolong it to meet the primitive circle in  $D$ ,

lay off arc  $DG$  equal to  $90^\circ$  toward the inside of the projected circle, and draw  $GA$  meeting  $EF$  at  $X$ . The true length of arc  $ST$  is then obtained by drawing  $XS$  and  $XT$  to meet the

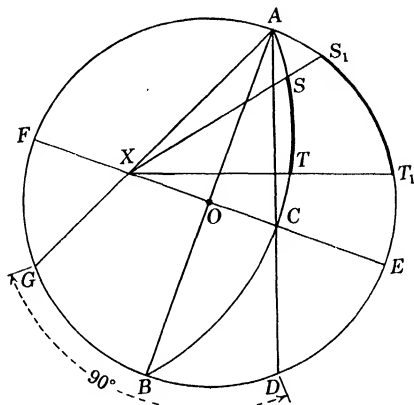


FIG. 9.

primitive circle in  $S_1$  and  $T_1$ , respectively, and then using a protractor to find the length in degrees of arc  $S_1T_1$ .

If the method just described be applied to find the true length of a part of a diameter, the point  $X$ , will be found to fall at the end of the perpendicular diameter. Hence, the true length of  $OC$  in Fig. 9 is the arc  $BD$ , and the true length of  $XC$  is the arc  $GD$  or  $90^\circ$ . It now appears that  $X$  is the projected pole of the great circle represented by  $ACB$  in Fig. 9; consequently we may refer to  $X$  as the pole of great circle  $ACB$ .

Evidently we can now lay off an arc of any desired true length from a given point on a projection of a great circle. Thus, to lay off  $50^\circ$  from  $A$  along the arc  $ACB$  in Fig. 10, lay off arc  $AT$  equal to  $50^\circ$ , locate the pole  $X$  of arc  $ACB$ , and draw  $XT$  meeting arc  $ACB$  in  $E$ . The arc  $AE$  has a true length of  $50^\circ$ .

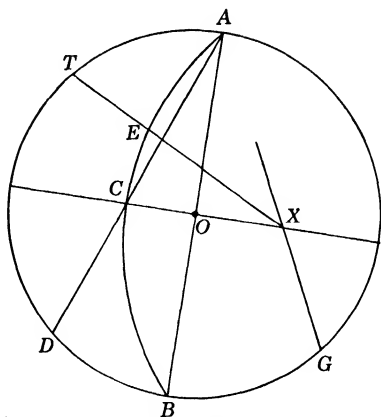


FIG. 10.

Note that arc  $AC = 90^\circ$ , and arc  $AO = 90^\circ$ . Therefore, in accordance with a theorem from solid geometry, angle  $OAC$  is measured by the true length of arc  $CO$ , or by arc  $DB$ . A little reflection on the processes just illustrated will enable the draftsman to measure with facility angles and arcs defined by projections of great circles.

*To measure the angle between two projected arcs of great circles through point A, lay off arc  $AD = 90^\circ$  on one circle and arc  $AE = 90^\circ$  on the other, draw straight lines  $AD$  and  $AE$  to meet the primitive circle in  $D$  and  $E$ , respectively, and measure arc  $DE$  with a protractor. Since  $A$  is the pole of arc  $DE$  and angle  $A$  is measured by the true length of arc  $DE$ , the reason for the construction is apparent.*

*Also, the angle between two arcs may be obtained by measuring the angle between their radii drawn to the point of intersection.*

### EXERCISES

1. Draw a primitive circle and the projections of three great circles making  $45^\circ$ ,  $90^\circ$ , and  $135^\circ$  angles, respectively, with the primitive and all passing through the ends of the same diameter. Divide each arc inside the primitive circle into six parts, each having a true length of  $30^\circ$ . Also check the angle between the primitive and the projection by finding the true lengths of parts of the diameter perpendicular to the one having its end on the projected circle.

2. Draw the projections of two great circles meeting in a point  $A$  inside the primitive circle. Lay off arc  $AD = 90^\circ$  on one projection and arc  $AE = 90^\circ$  on the other. Now find the true length of arc  $ED$ ; that is, measure the angle  $EAD$ . Perform this operation three or four times, using different great circles in each case.

3. Through the ends  $A$  and  $B$  of the diameter of a primitive circle draw a projected circle making a  $60^\circ$  angle with the primitive circle. Lay off arc  $AC$  equal to  $60^\circ$  on the primitive circle and draw through the ends  $C$  and  $D$  of a diameter the projection of a great circle making a  $45^\circ$  angle with the primitive. Now measure all arcs and angles formed inside the primitive circle.

6. **To measure the parts of a spherical triangle by stereographic projection.** A spherical triangle can be solved graphically by drawing its projection and measuring its sides and angles. An example will illustrate the method.

**Example.** Use stereographic projection to solve the triangle in which side  $b = 120^\circ$ , side  $c = 75^\circ$ , and the included angle  $A = 60^\circ$ .

*Solution.* The solution will be explained by referring to Fig. 11. Draw the primitive circle  $ACF$ . Then draw any diameter  $AE$  and the perpendicular diameter  $DF$ . Lay off arc  $ADC = b = 120^\circ$ . Draw  $AO_1$  so that angle  $OAO_1 = 60^\circ$ . With  $O_1$  as center, draw circular arc  $ABE$ . Then angle  $DAB =$

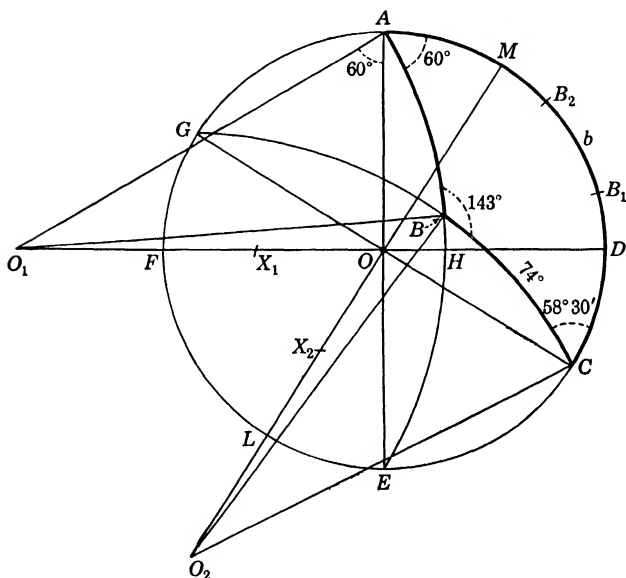


FIG. 11.

$60^\circ$ . Find the pole  $X_1$  of arc  $ABE$ , lay off arc  $AB_1 = 75^\circ$ , draw  $B_1X_1$  to meet arc  $ABE$  in  $B$ . Then arc  $AB$  has a true length of  $75^\circ$ . Now draw diameter  $CG$  and construct the circular arc  $CBG$  with center  $O_2$ . Then triangle  $ABC$  is a stereographic projection of the required triangle. To measure the unknown parts, draw diameter  $LM$  perpendicular to  $CG$ , and locate the pole  $X_2$  of arc  $CBG$ . Draw  $X_2B$  to meet the primitive circle in  $B_2$ . Then the true length of  $CB$  is equal to arc  $CB_2$ , which is found by means of a protractor to be  $74^\circ$ . Next draw  $O_2C$ . Then angle  $BCD$  is equal to angle  $GCO_2 = 58^\circ 30'$ . Also, angle  $CBA$  is  $180^\circ - \text{angle } O_1BO_2$  or  $131^\circ 30'$ .

## EXERCISES

1. Draw the stereographic projection of a spherical triangle in which  $a = 60^\circ$ ,  $b = 90^\circ$ ,  $C = 60^\circ$ , and measure  $B$  and  $c$ .

2. Draw a stereographic projection of each of the spherical triangles that have the given parts indicated, and measure the unknown parts:

(a)  $a = 60^\circ$ ,

$b = 60^\circ$ ,

$C = 90^\circ$ .

(b)  $A = 60^\circ$ ,

$B = 60^\circ$ ,

$c = 120^\circ$ .

(c)  $A = 120^\circ$ ,

$b = 75^\circ$ ,

$c = 150^\circ$ .

(d)  $b = 120^\circ$ ,

$c = 120^\circ$ ,

$A = 75^\circ$ .

## APPENDIX D

**7. Vectors.** A vector  $AB$  (see Fig. 12) is a straight line containing an arrowhead at  $B$  to indicate a direction from its initial

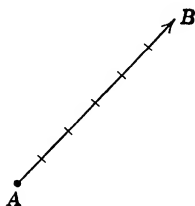


FIG. 12.

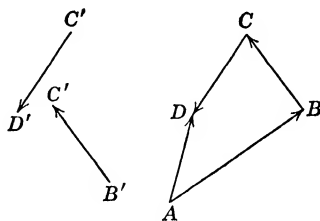


FIG. 13.

point  $A$  to its terminal point  $B$ . The length of the line segment indicates the magnitude of the vector, and the line with attached arrowhead indicates direction. If the line  $AB$  is 6 units long and is directed at  $N. 45^\circ E.$ , it could be used to represent a velocity of 6 knots in the direction  $N. 45^\circ E.$

Figure 13 indicates the method of adding vectors. To add the vectors  $AB$ ,  $B'C'$ , and  $C'D'$  through the tip  $B$  of  $AB$  draw  $BC$  parallel and equal in length to  $B'C'$ , through  $C$  draw  $CD$  parallel and equal to  $C'D'$ . The vector  $AD$  is the required sum. A similar method may be used to add a number of vectors.

If a ship starting from point  $A$  (see Fig. 14) sails 20 miles on course  $45^\circ$  to  $B$  and then 10 miles on course  $315^\circ$  to  $C$ , the vector  $AC$  represents the distance and bearing of point  $C$  from  $A$ . By direct measurement or by computation  $C$  is found to bear  $18^\circ 26'$  from  $A$  and to be 22.4 miles distant from it.

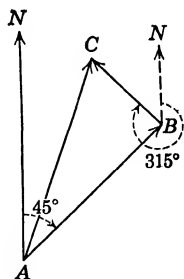


FIG. 14.

### EXERCISES

1. A ship sails due east 25 miles and then due south 25 miles. Find its distance and bearing from its starting point.
2. If a ship starting from a point  $A$  steams 40 miles on course  $135^\circ$  to point  $B$  and then steams 30 miles on course  $45^\circ$ , find its bearing and distance from  $A$ .

3. An airplane when leaving its base flies 80 miles on course  $70^{\circ}12'$  and then changes course to  $180^{\circ}$ . After traveling 27 miles on this course find the bearing and distance to its base.

4. A ship 68.2 miles due south of a lighthouse steams on course  $46^{\circ}58'$  a distance 31.6 miles. Find the bearing of the lighthouse and its distance from the ship.

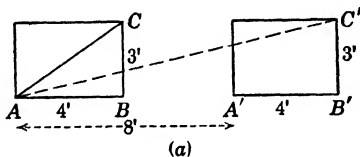
5. A man walks on course  $34^{\circ}14'$  for a distance of 6.75 miles. He then changes his direction to course  $190^{\circ}45'$  for a distance of 5.68 miles. Find the bearing and distance of his initial position reckoned from his final position.

6. A ship sailing north at 10 knots is drifting, owing to a 2-knot current toward the east. Find the distance the ship moves in 2 hr.

7. A ship is carried by the wind at 2.5 knots in direction  $300^{\circ}$ , by the current at 3 knots in direction  $180^{\circ}$ , and is steaming 12 knots on course  $120^{\circ}$ . Find the course and distance covered in 2 hr.

8. A ship is carried by the wind 4 knots on a course  $30^{\circ}$ , by the current at 1.75 knots on a course  $180^{\circ}$ , and it is steaming at the rate of 12 knots on a course  $270^{\circ}$ . Find the actual speed and course.

**8. Relative movement or maneuvering and mooring board problems.\*** The platform  $AC$  of Fig. 15(a) moves rightward 1



(a)

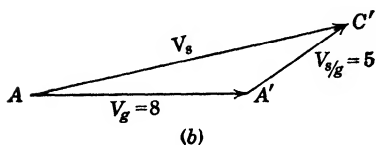


FIG. 15.

sec. at 8 ft. per sec. while a weight on the platform moves 5 ft. per sec. **relative** to it along its diagonal. During a second the weight moves from  $A$  to  $C'$ . Hence  $AC'$  represents its velocity in magnitude and direction. Figure 15(b) illustrates the velocity  $V_s$  of the weight composed of two velocities, the velocity of the platform  $V_g$  and the velocity of the weight relative

to the platform  $V_{s/g}$ . This relation is expressed by the vector equation

$$V_s = V_{s/g} + V_g. \quad (1)$$

This important equation will be applied to solve some problems arising in the movements of ships.

\* The graph paper used by the United States Navy in solving relative movement problems and many others is known as the Mooring and Maneuvering Board.



When a group of ships are sailing in formation all of them may have the same velocity as a certain one called the guide. Some particular ship may be ordered to take a new position relative to the guide while the rest of the group moves along. Let the vector  $V_g$  in Fig. 16(a) represent the velocity of the guide, and suppose that a ship  $S$  is ordered to move from  $A$  to  $B$  relative to the guide. Equation (1) applies to the motion provided  $V_g$  represents the velocity of the guide,  $V_{s/g}$  the velocity of the ship  $S$  relative to the guide, and  $V_s$  the velocity of the ship. Observing that

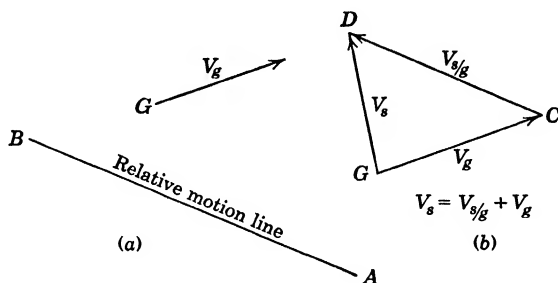


FIG. 16.

vector  $V_{s/g}$  along the line  $CD$  must be parallel to the relative motion line  $AB$ , the relation between the vectors  $V_g$ ,  $V_{s/g}$ , and  $V_s$  is readily seen in Fig. 15(b). Vector  $V_s$  may be chosen rather arbitrarily, unless some condition such as direction or speed is specified. From Fig. 16(b) the magnitude and bearing of  $V_s$  can be read. The time required for the movement is obtained by dividing relative distance  $AB$  by the magnitude of  $V_{s/g}$ . The figure should be drawn to scale and all quantities found by measurement. The scale for distance need not be the same as that for velocity. Assume that 1 knot = 2000 yd. per hr.

The beginner may find the following suggestions helpful:

(a) On a piece of polar coordinate paper plot the initial position  $A$  and the final position  $B$  of ship  $S$  and draw line  $AB$ .

(b) Beginning at the center  $G$  of the paper lay off the vector  $V_g$ .

(c) Through the tip of  $V_g$  draw a line parallel to the line found in step (a).

(d) Draw  $V_s$  from center  $G$  of the paper in accordance with any specified conditions.

(e) Measure  $V_s$ , magnitude and direction, and measure  $V_{s/g}$ .

(f) The time required for the movement is given by  $AB \div V_{s/g}$  (magnitude).

and hidden within the young seeds (ovules) are the parasitic female plants. The germ cells, that is, the eggs and sperms, are not produced directly by the flowers. Instead, flowers develop these small sexual plants which in turn bear the eggs and sperms.



FIG. 76.—An apricot flower bud just before opening. The ovary, covered with hair, is seen in the center, and above are the anthers. (Photograph furnished by Division of Pomology, California College of Agriculture.)

Within each anther there are developed a number of **spores**, a peculiar type of cell which, unlike eggs and sperms, is capable of growing into a plant without entering into the mysterious process of fertilization. Each of the spores in the anther grows into a minute male plant, a pollen grain. When the anther dries up and splits open, powdery masses of yellow male plants are carried by insects or wind to the pistils, inside of which the female plants are waiting.

**Exercise 86. The pollen grain.** With the compound microscope examine the pollen grains of some flowering plant. In specially stained pollen grains will be seen the protective coat enclosing two cells. The nuclei of these cells are visible. Thus, it is seen

that the pollen grain is not a single cell, but in reality a small sexual plant consisting of but two cells.

**Exercise 87. Germination of pollen grains.** The pollen grains of many plants will germinate in a 10 per cent solution of cane sugar. Prepare hanging

## EXERCISES

1. The fleet guide is steaming at 9 knots on course  $110^\circ$ . A destroyer bearing  $180^\circ$  from the guide, distant 3600 yd. is ordered to proceed at 15 knots on course  $20^\circ$  until she bears  $315^\circ$  from the guide. Find the time required.

2. The guide of a fleet is steaming on course  $240^\circ$  at 12 knots. A destroyer distant 4000 yd. from the fleet guide bears  $150^\circ$  from it.

(a) What course should the destroyer steer to take position bearing  $190^\circ$ , distant 2000 yd. from the guide if she is to steam to the new position at 24-knot speed?

(b) How long does she take to reach the position at 24 knots?

(c) If the destroyer is required to reach position in exactly 10 min. what should be her course and speed?

3. The fleet guide  $G$  moves on course  $0^\circ$  at 20 knots. A destroyer, distant 1000 yd. and bearing  $180^\circ$  from  $G$ , is ordered to take a position 1000 yd. bearing  $90^\circ$  from  $G$  and to complete the maneuver in 3 min. Find the course and speed of the destroyer while changing position.

4. A ship on course  $315^\circ$ , speed 30 knots, sends up a plane with orders to scout to a distance of 200 nautical miles from the ship on course  $300^\circ$ . Find the plane's speed if it maintains a constant bearing of  $296^\circ$  from the ship.

5. A fleet guide is steaming on course  $20^\circ$  at 12 knots. A destroyer due west of the guide and distant 4 miles is ordered to take a position 3 miles astern of the guide by steaming at 18 knots. Find the course the destroyer should steer.

6. A flagship is steaming at 12 knots on course  $295^\circ$ . A cruiser distant 6000 yd. from the flagship and bearing  $160^\circ$  from it is ordered to take a position distant 8 miles and bearing  $7^\circ$ . If the cruiser proceeds at 20 knots find its course and the time required.

7. A cruiser sights an enemy ship 7 miles distant, bearing  $85^\circ$ , and steaming at 12 knots on course  $10^\circ$ . If the cruiser steams at 20 knots find the course she should steer to overhaul the ship.

8. A destroyer fires a torpedo at ship  $A$ , distant 6000 yd. and bearing  $70^\circ$  from the destroyer. Ship  $A$  is steaming at 15 knots on course  $150^\circ$ . If the torpedo has a speed of 24 knots on what course should the torpedo be set?



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## ANSWERS

### §5. Page 8

1. 0	5. 2	9. 4	13. 3
2. 5	6. 1	10. 2	14. 4
3. 1	7. 8 - 10	11. 5 - 10	15. 9 - 10
4. 0	8. 9 - 10	12. 7 - 10	16. 6 - 10

### §9. Page 11

1. 1.60733	5. 9.33333 - 10	9. 8.43198 - 10
2. 0.48391	6. 7.58371 - 10	10. 9.26133 - 10
3. 4.00864	7. 8.93677 - 10	
4. 2.03411	8. 5.88152 - 10	

### §10. Page 12

1. 0.04592	5. 0.0093962	9. 12.594
2. 7903	6. 997.15	10. 0.00035304
3. 207,320	7. 7.4962	
4. 0.50119	8. 2.6448	
11. (a) 0.45347	(c) 0.00074363	
(b) 0.0038615	(d) 0.68973	

### §11. Page 14

1. 433.90	3. 3.1414	5. 0.51514	7. 0.24406
2. 224.09	4. 1.3205	6. 5.2686	8. 0.062086

### §12. Page 15

2. (a) 5.0187	(c) 0.00041391
(b) 147.54	(d) 5058.6

### §14. Pages 17 to 19

1. 8.5398	12. 3.1414	23. 1.6478
2. 0.010894	13. 18.636	24. 3463.4
3. 33,451	14. 0.72132	25. 27.278
4. 1019.4	15. 0.26868	26. -22.582
5. 200,530	16. 0.39770	27. 15.353
6. 0.19835	17. 0.39510	28. 0.00021360
7. 24.682	18. 1.2390	29. 18.666
8. 17.843	19. 1.1605	30. -22.302
9. 0.65684	20. 0.53670	31. -1.2552
10. 0.0067010	21. 107.42	32. -5.2060
11. 437.88	22. 3630.8	

**33.** 0.0074500**34.** 1.56026;  $(-)$ 1.46098; 9.05621 - 10; 2.08309**35.** 46.693**38.** 266.46 lb.**41.** 151,370 gal.**36.** 8.6458**39.** 2283.2 lb.**42.** 1.01 sec.**37.** 0.028375**40.** 6.2691 ft.**43.** 142.5 tons**44.** Volume = 13,330, surface = 2719**45.**  $1051 \times 10^7$ **47.** 834,200**49.** 0.608**46.** 11,660**48.** 1,476,000**§16. Pages 21, 22****1.** 2.3666**10.** 1.7895**2.** -90.006**11.** 339.86**3.** -1.7354**12.** 2.7183**4.** -1.9034**13.** 0.42767**5.** 1.5372**14.** 0.41639**6.** 4.9168**15.** 0.11699**7.** -0.15421**16.** -0.37979**8.** -0.76206**17.**  $x = 3.0484$ ,  $y = 2.0484$ **9.** 6.0110**18.** 17.677**19.** 0,  $\pm 1.3169$ **22.** 18,360**25.**  $x = \frac{e^2 - 1}{3}$ **20.** 3.96**23.**  $k = 0.126$ **26.**  $x = 25$  and  $-4$ **21.** 0.00003772**24.** 5.5 minutes**§18. Pages 23 to 27****1.** 222.91**8.** 4.4787**15.** 34.801**22.** 0.031072**2.** 0.037367**9.** 3.0675**16.** 67.535**23.** 4.6249**3.** 72.888**10.** 0.00079018**17.** 42.620**24.** 3.5064**4.** 0.0093936**11.** 0.37665**18.** 2362.9**25.** 1.5509**5.** 24.491**12.** 0.28926**19.** -4.2098**26.** 0.036016**6.** 1.2142**13.** 0.96048**20.** -0.86048**7.** 12.377**14.** 1.7867**21.** -0.21423**27.** (a) 0.093180; (b) 168.20; (c) 0.44668**28.** 35.239**30.** 2.92 %**29.** 31.594**31.** 1963.6 ft. per sec.**33.** 16,874 ft.**34.**  $x = 523$  ft.,  $y = 5902.6$  ft.**35.** 10.08 lb. per sq. in., 8.3516 lb. per sq. in.**36.** 1205.3 lb.**37.** 4.79 sec.**38.** (a) 823.69 ft.**39.** 15.82 min.(b)  $49^\circ 38'$ **40.** 67.188 min.

(c) 251.1 ft.

## §20. Pages 32 to 34

1. (a) 226.20 ft. (c) 217.92 ft. (e) 0.13264 ft.  
(b) 358.14 ft. (d) 4.2935 ft. (f) 4a ft.
2. (a)  $36^\circ$ ; (b)  $1^\circ 12'$ ; (c)  $7' 12''$ ; (d)  $1^\circ 26' 24''$ ; (e)  $336^\circ 50' 24''$
4. 7.5 ft. 5.  $94^\circ 4'$  6. 75 yd. 7.  $\frac{1}{83}$
8. 247.16 r.p.m., 25.882 radians per second
9. 0.00098175, 1018.1 18. 17.045 miles per hour
11. 72 yd. 19. 7.3304 ft. per sec.
12. 0.015708 20. 846.40 ft.
13. 69.088 miles, 932.71 miles 21. 222.67 ft., 4583.8 ft.
14. 2160 miles 22. 589.33 ft.
15. 2.2270 ft. 23. 20.944 ft., 200 ft.
16. 62.857 radians per second 24. 294.51 ft.
17. 1760 radians per minute 25. 2.9630 mils
26. (a) 10 miles; (b) 9 miles; (c) 6.25 miles

## §22. Pages 36, 37

3. Each side =  $5\pi$  in.
5. 3000 miles, 3638 miles,  $2750\frac{1}{3}$  miles
8. (a)  $c = 30^\circ$ ,  $a = 90^\circ$ ,  $b = 90^\circ$

## §24. Pages 41 to 43

1. (a)  $c = \cos^{-1} \frac{\sqrt{3}}{4}$  3. (a)  $A = \tan^{-1} 2$   
(b)  $B = \sec^{-1} \sqrt{3}$  (b) Impossible  
(c)  $c = \tan^{-1} 2$  (c)  $a = \tan^{-1} \frac{3}{2}$   
(d)  $A = \sec^{-1} 4$  (d)  $c = \pi - \sec^{-1} \sqrt{3}$   
(e)  $b = \tan^{-1} \sqrt{\frac{3}{2}}$  (e)  $A = \cos^{-1} \frac{3}{4}$   
(f) Impossible (f)  $B = \sec^{-1} \sqrt{3}$
8. (a)  $\cos c = \cot A \cot B$

## §26. Pages 46, 47

1.  $b = 2^\circ 14' 5''$ ,  $c = 10^\circ 45' 55''$ ,  $A = 78^\circ 9' 22''$
2.  $a = 44^\circ 43' 49''$ ,  $b = 14^\circ 59' 33''$ ,  $A = 75^\circ 21' 53''$
3.  $b = 10^\circ 49' 17''$ ,  $c = 118^\circ 20' 20''$ ,  $A = 95^\circ 55' 2''$
4.  $A = 52^\circ 16' 26''$ ,  $B = 57^\circ 26' 33''$ ,  $b = 47^\circ 7' 32''$
5.  $a = 58^\circ 21' 28''$ ,  $A = 65^\circ 11' 30''$ ,  $B = 53^\circ 6' 40''$
6.  $b = 27^\circ 37' 26''$ ,  $B = 68^\circ 42' 11''$ ,  $A = 155^\circ 48' 0''$
7.  $a = 127^\circ 4' 30''$ ,  $b = 50^\circ 0' 0''$ ,  $A = 120^\circ 3' 50''$

8.  $a = 22^\circ 15' 43''$ ,  $b = 24^\circ 24' 19''$ ,  $B = 50^\circ 8' 21''$
9.  $a = 119^\circ 59' 46''$ ,  $b = 120^\circ 10' 3''$ ,  $c = 75^\circ 26' 58''$
10.  $a = 50^\circ 0' 0''$ ,  $b = 56^\circ 50' 49''$ ,  $B = 63^\circ 25' 4''$
11.  $b = 51^\circ 53'$ ,  $A = 27^\circ 28' 38''$ ,  $B = 73^\circ 27' 11''$
12.  $c = 54^\circ 20'$ ,  $A = 46^\circ 59' 43''$ ,  $B = 57^\circ 59' 19''$
13.  $b = 155^\circ 27' 54''$ ,  $c = 142^\circ 9' 13''$ ,  $A = 54^\circ 1' 16''$
14.  $c = 133^\circ 32' 26''$ ,  $A = 126^\circ 40' 24''$ ,  $B = 47^\circ 13' 43''$
15.  $c = 54^\circ 20'$ ,  $B = 46^\circ 49' 43''$ ,  $A = 57^\circ 59' 19''$
16.  $a = 50^\circ 0' 4''$ ,  $b = 143^\circ 5' 12''$ ,  $c = 120^\circ 55' 34''$
17.  $a = 67^\circ 33' 27''$ ,  $b = 100^\circ 45'$ ,  $c = 94^\circ 5'$
18.  $a = 51^\circ 53'$ ,  $B = 27^\circ 28' 38''$ ,  $A = 73^\circ 27' 11''$
19.  $b = 96^\circ 21' 59''$ ,  $c = 86^\circ 58' 0''$ ,  $A = 118^\circ 21' 15''$
20.  $a = 49^\circ 59' 58''$ ,  $c = 91^\circ 47' 40''$ ,  $B = 92^\circ 8' 23''$
22.  $D = 690.98$  miles,  $L_2 = 39^\circ 31' 18''$ ,  $C = 80^\circ 19' 23''$
24.  $B = 53^\circ 48' 27''$

## §27. Page 48

1.  $a_1 = 69^\circ 50' 24''$ ,  $c_1 = 73^\circ 45' 15''$ ,  $A_1 = 77^\circ 54'$   
 $a_2 = 110^\circ 9' 36''$ ,  $c_2 = 106^\circ 14' 45''$ ,  $A_2 = 102^\circ 6'$
2.  $a_1 = 18^\circ 54' 38''$ ,  $c_1 = 127^\circ 2' 27''$ ,  $A_1 = 23^\circ 57' 19''$   
 $a_2 = 161^\circ 5' 22''$ ,  $c_2 = 52^\circ 57' 33''$ ,  $A_2 = 156^\circ 2' 41''$
3.  $a_1 = 25^\circ 59' 28''$ ,  $c_1 = 33^\circ 20' 13''$ ,  $A_1 = 52^\circ 53' 0''$   
 $a_2 = 154^\circ 0' 32''$ ,  $c_2 = 146^\circ 39' 47''$ ,  $A_2 = 127^\circ 7' 0''$
4.  $b_1 = 28^\circ 14' 31''$ ,  $c_1 = 78^\circ 53' 20''$ ,  $B_1 = 28^\circ 49' 57''$   
 $b_2 = 151^\circ 45' 29''$ ,  $c_2 = 101^\circ 6' 40''$ ,  $B_2 = 151^\circ 10' 3''$
5.  $b_1 = 39^\circ 4' 51''$ ,  $c_1 = 136^\circ 50' 23''$ ,  $B_1 = 67^\circ 9' 43''$   
 $b_2 = 140^\circ 55' 9''$ ,  $c_2 = 43^\circ 9' 37''$ ,  $B_2 = 112^\circ 50' 17''$
6.  $a_1 = 60^\circ 36' 10''$ ,  $c_1 = 68^\circ 42' 59''$ ,  $A_1 = 69^\circ 13' 47''$   
 $a_2 = 119^\circ 23' 50''$ ,  $c_2 = 111^\circ 17' 1''$ ,  $A_2 = 110^\circ 46' 13''$

## §28. Pages 49, 50

1. (a)  $a' = 44^\circ 0.9'$ ,  $b' = 79^\circ 49.9'$ ,  $c' = 81^\circ 16.7'$ ,  $C' = 90^\circ$ ,  $A' = 44^\circ 40'$ ;  
 $B' = 81^\circ 28.5'$
2. (a)  $\sin A' = \sin C' \sin a'$
3. (b)  $a' = 133^\circ 9.7'$ ,  $B' = 108^\circ 18.3'$ ,  $c' = 73^\circ 35.3'$

## §29. Page 51

1.  $a = 68^\circ 36' 13''$ ,  $b = 59^\circ 19' 4''$ ,  $C = 103^\circ 26' 36''$
2.  $a = 67^\circ 46' 12''$ ,  $b = 78^\circ 21' 32''$ ,  $B = 77^\circ 24' 34''$
3.  $b = 117^\circ 45' 28''$ ,  $A = 96^\circ 27' 1''$ ,  $C = 93^\circ 0' 51''$
4.  $a = 94^\circ 22' 46''$ ,  $b = 69^\circ 48' 42''$ ,  $C = 88^\circ 23' 11''$
5.  $a = 106^\circ 56' 53''$ ,  $B = 8^\circ 49' 46''$ ,  $C = 28^\circ 3' 4''$
6.  $A = 105^\circ 21' 16''$ ,  $B = 160^\circ 13' 48''$ ,  $C = 104^\circ 25' 45''$

## §30. Page 53

1.  $c = 120^\circ 10' 52''$ ,  $A = 65^\circ 13' 4''$ ,  $B = 49^\circ 27' 53''$
2.  $a = 69^\circ 34' 44''$ ,  $B = 135^\circ 5' 14''$ ,  $C = 50^\circ 29' 54''$

3.  $c = 104^\circ 12' 52''$ ,  $B = 51^\circ 46' 38''$ ,  $A = 63^\circ 48' 24''$
4.  $b = 100^\circ 47' 46''$ ,  $A = 96^\circ 2' 12''$ ,  $C = 125^\circ 43' 46''$
5.  $c = 108^\circ 39' 11''$ ,  $B = 40^\circ 23' 17''$ ,  $A = 64^\circ 48' 55''$
6.  $a = 65^\circ 28' 34''$ ,  $B = 148^\circ 14' 43''$ ,  $C = 44^\circ 9' 3''$
7.  $a = 145^\circ 24' 53''$ ,  $b = 139^\circ 45' 58''$ ,  $C = 49^\circ 46' 16''$
8.  $a = 23^\circ 57' 9''$ ,  $c = 118^\circ 2' 15''$ ,  $B = 102^\circ 5' 52''$
9.  $B_1 = 42^\circ 37' 30''$ ,  $C_1 = 160^\circ 1' 43''$ ,  $c_1 = 153^\circ 39' 4''$   
 $B_2 = 137^\circ 22' 30''$ ,  $C_2 = 50^\circ 19' 3''$ ,  $c_2 = 90^\circ 5' 18''$
10.  $B = 131^\circ 25' 11''$ ,  $C = 108^\circ 18' 55''$ ,  $c = 78^\circ 21' 6''$
11.  $B_1 = 120^\circ 47' 28''$ ,  $C_1 = 97^\circ 42' 38''$ ,  $c_1 = 55^\circ 41' 57''$   
 $B_2 = 59^\circ 12' 18''$ ,  $C_2 = 29^\circ 9' 0''$ ,  $c_2 = 23^\circ 57' 27''$
12.  $C_1 = 59^\circ 24' 20''$ ,  $B_1 = 115^\circ 40' 1''$ ,  $b_1 = 97^\circ 33' 11''$   
 $C_2 = 120^\circ 35' 40''$ ,  $B_2 = 26^\circ 59' 51''$ ,  $b_2 = 29^\circ 57' 19''$
13. (a)  $b = 76^\circ 47' 13''$ ,  $a = 96^\circ 46' 12''$ ,  $A = 99^\circ 24' 13''$   
 (b)  $b_1 = 109^\circ 49' 57''$ ,  $c_1 = 98^\circ 21' 33''$ ,  $C_1 = 109^\circ 55' 11''$   
 $b_2 = 70^\circ 10' 3''$ ,  $c_2 = 168^\circ 48' 53''$ ,  $C_2 = 169^\circ 22' 45''$

## §32. Pages 58, 59

1. (a)  $c = 66^\circ 32' 6''$ ,  $A = 41^\circ 55' 45''$ ,  $B = 70^\circ 19' 15''$   
 (b)  $a = 104^\circ 53' 1''$ ,  $b = 133^\circ 39' 48''$ ,  $C = 104^\circ 41' 37''$   
 (c)  $a = 54^\circ 41' 35''$ ,  $b = 104^\circ 21' 28''$ ,  $c = 98^\circ 14' 24''$   
 (d)  $a_1 = 20^\circ 11' 16''$ ,  $c_1 = 129^\circ 16' 38''$ ,  $A_1 = 26^\circ 28' 31''$   
 $a_2 = 159^\circ 48' 44''$ ,  $c_2 = 50^\circ 43' 22''$ ,  $A_2 = 153^\circ 31' 29''$   
 (e)  $b = 85^\circ 17' 16''$ ,  $A = 17^\circ 35' 57''$ ,  $C = 104^\circ 31' 13''$   
 (f) Impossible
2. (a)  $a = b = 32^\circ 45' 6''$ ,  $C = 105^\circ 49' 32''$   
 (b)  $c = 46^\circ 15' 12''$ ,  $a = b = 112^\circ 32' 20''$
3.  $60^\circ 20' 56''$
5.  $C_1 = 65^\circ 22' 31''$ ,  $C_2 = 114^\circ 37' 29''$ ,  $b_1 = 130^\circ 24' 35''$ ,  $b_2 = 77^\circ 35' 39''$ ,  
 $B_1 = 135^\circ 20' 37''$ ,  $B_2 = 64^\circ 21' 40''$
7. 247.95 miles
8.  $L = 39^\circ 55' 24''$  N.,  $\lambda = 60^\circ 53' 17''$  W.,  $C = 98^\circ 29' 7''$
9.  $L = 24^\circ 8' 22''$  N.,  $D = 3067.7$  miles
10.  $L = 52^\circ 45' 4''$  N.,  $\lambda = 176^\circ 14' 16''$  W.
11. 1971.3 nautical miles
12.  $L = 55^\circ 22' 33''$  N.,  $\lambda = 180^\circ$

## §34a. Pages 61, 62

1. 127.2, 141.2
2. 65.714 miles
3. 23.34, 166.1
4.  $2^\circ 35'$
5. 101.3 miles
6.  $L = 37^\circ 26.8'$  N.  
 $\lambda = 56^\circ 22.4'$  W.
7.  $C = 231.2^\circ$   
 $D = 201.1$  miles
8.  $C = 316^\circ$   
 $D = 239.0$  miles
10.  $8^\circ 56' 31''$ ,  $8^\circ 57' 18''$

## §35. Pages 66, 67

1. 179.5 miles, 221 miles, 72.2 miles
2.  $E: L = 47^\circ, \lambda = 39^\circ 10'$   
 $F: L = 50^\circ, \lambda = 37^\circ 19'$   
 $H: L = 48^\circ 20', \lambda = 32^\circ 45'$   
 $O: L = 1^\circ 20', \lambda = 34^\circ 10'$
3. (a) 167.5 miles  
 (b) 134 miles  
 (c) 80.5 miles  
 (d) 277 miles
4.  $1^\circ 32', 3^\circ 37'$
5.  $2^\circ 10', 39'$
6.  $26^\circ 48', 243$  miles
7.  $12^\circ 20', 224$  miles
8.  $L = 32^\circ 15', \lambda = 36^\circ 40'$
9. 139.5 miles
10.  $L = 30^\circ 35', \lambda = 38^\circ 31'$
11. 203 miles
12.  $L = 47^\circ 46', \lambda = 38^\circ 35'$
13. (a)  $241^\circ 50'$   
 (b)  $259^\circ 16'$   
 (c)  $38^\circ 50'$   
 (d)  $224^\circ 20'$
14. (a) 1800 mi.  
 (b) 2990 mi.  
 (c) 1620 mi.

## §36. Pages 69, 70

3. (a)  $A = 71^\circ 23' 00''$   
 (b)  $B = 53^\circ 37' 47''$
4. (a)  $b = 44^\circ 13' 45''$   
 (b)  $B = 131^\circ 18'$

## §38. Pages 73, 74

1. (a)  $a = 42^\circ 20' 12''$   
 (b)  $a = 64^\circ 10' 34''$   
 (c)  $a = 100^\circ 10' 58''$
2. (a)  $137^\circ 40'$   
 (b)  $79^\circ 49'$
3.  $A = 33^\circ 11' 19''$
7. (a)  $B = 114^\circ 35' 50'', C = 31^\circ 39' 55''$   
 (b)  $B = 42^\circ 52' 8'', C = 28^\circ 45' 18''$   
 (c)  $B = 21^\circ 3' 6'', C = 26^\circ 6' 0''$
8. (a)  $A' = 137^\circ 39' 48'', b' = 65^\circ 24' 10'', c' = 148^\circ 20' 5''$   
 (b)  $A' = 115^\circ 49' 26'', b' = 137^\circ 7' 52'', c' = 151^\circ 14' 42''$   
 (c)  $A' = 79^\circ 49' 2'', b' = 158^\circ 56' 54'', c' = 153^\circ 54'$

## §41. Pages 78, 79

2. (a)  $A = 33^\circ 11' 20'', B = 50^\circ 43' 44'', C = 108^\circ 31' 52''$   
 (b)  $A = 34^\circ 46' 44'', B = 81^\circ 6' 4'', C = 81^\circ 6' 4''$   
 (c)  $A = 145^\circ 13' 20'', B = 98^\circ 54' 0'', C = 81^\circ 6' 4''$   
 (d)  $a = 76^\circ 9' 49'', b = 127^\circ 33' 10'', c = 76^\circ 9' 49''$   
 (e)  $a = 81^\circ 6' 0'', b = 34^\circ 46' 42'', c = 98^\circ 53' 56''$   
 (f)  $a = 146^\circ 48' 40'', b = 71^\circ 28' 8'', c = 129^\circ 16' 16''$
3. (a)  $A = 118^\circ 44' 10'', B = 29^\circ 38' 9'', C = 68^\circ 7' 32''$   
 (b)  $A = 123^\circ 53' 48'', B = 57^\circ 46' 56'', C = 46^\circ 51' 50''$   
 (c)  $A = 81^\circ 52' 32'', B = 97^\circ 31' 5'', C = 111^\circ 3' 42''$   
 (d)  $A = 34^\circ 59' 19'', B = 150^\circ 13' 15'', C = 33^\circ 11' 39''$   
 (e)  $a = 56^\circ 51' 48'', b = 126^\circ 57' 52'', c = 139^\circ 21' 22''$   
 (f)  $a = 51^\circ 17' 31'', b = 64^\circ 2' 47'', c = 51^\circ 17' 31''$   
 (g)  $a = 97^\circ 44' 19'', b = 53^\circ 49' 25'', c = 104^\circ 25' 9''$   
 (h)  $a = 115^\circ 10', b = 84^\circ 18' 28'', c = 31^\circ 9' 14''$
4. (a)  $a' = 146^\circ 48' 40'', b' = 129^\circ 16' 16'', c' = 71^\circ 28' 8''$

## §43. Page 83

1. (a)  $b = 42^\circ 20' 12''$ ,  $A = 31^\circ 39' 54''$ ,  $C = 114^\circ 35' 50''$   
 (b)  $a = 85^\circ 26' 28''$ ,  $B = 149^\circ 53' 42''$ ,  $C = 37^\circ 54' 6''$   
 (c)  $A = 39^\circ 13' 54''$ ,  $B = 63^\circ 26' 6''$ ,  $c = 156^\circ 42' 58''$   
 (d)  $a = 165^\circ 29' 53''$ ,  $b = 154^\circ 17' 43''$ ,  $C = 93^\circ 19' 34''$   
 (f)  $a = 50^\circ 11' 37''$ ,  $B = 77^\circ 29' 48''$ ,  $c = 153^\circ 40' 13''$
2. (a)  $49^\circ 28'$       (b)  $69^\circ 35'$       (c)  $15^\circ 20'$       (d)  $104^\circ 19'$
3. (a)  $a = 57^\circ 56' 56''$ ,  $b = 137^\circ 20' 32''$ ,  $C = 94^\circ 48' 13''$   
 (b)  $b = 100^\circ 47' 46''$ ,  $A = 96^\circ 2' 12''$ ,  $C = 125^\circ 43' 44''$   
 (c)  $c = 104^\circ 12' 55''$ ,  $A = 63^\circ 48' 26''$ ,  $B = 51^\circ 46' 38''$   
 (d)  $c = 108^\circ 39' 11''$ ,  $A = 64^\circ 48' 54''$ ,  $B = 40^\circ 23' 16''$   
 (e)  $c = 156^\circ 18' 49''$ ,  $A = 29^\circ 42' 0''$ ,  $B = 41^\circ 2' 38''$   
 (f)  $a = 23^\circ 57' 11''$ ,  $b = 118^\circ 2' 13''$ ,  $C = 102^\circ 5' 46''$
4. (a)  $c = 9^\circ 5' 14''$ ,  $A = 56^\circ 30' 0''$ ,  $B = 115^\circ 33' 56''$   
 (b)  $c = 73^\circ 41' 2''$ ,  $A = 130^\circ 25' 0''$ ,  $B = 128^\circ 26' 27''$

## §44. Pages 85, 86

1.  $c_1 = 104^\circ 19' 10''$ ,  $A_1 = 52^\circ 19' 33''$ ,  $C_1 = 124^\circ 42' 2''$   
 $c_2 = 18^\circ 10' 14''$ ,  $A_2 = 127^\circ 40' 27''$ ,  $C_2 = 15^\circ 20' 32''$
2.  $b = 15^\circ 18' 34''$ ,  $c = 38^\circ 59' 34''$ ,  $C = 98^\circ 40' 56''$
3.  $b_1 = 55^\circ 25' 2''$ ,  $c_1 = 81^\circ 27' 26''$ ,  $C_1 = 119^\circ 22' 28''$   
 $b_2 = 124^\circ 34' 58''$ ,  $c_2 = 162^\circ 34' 27''$ ,  $C_2 = 164^\circ 41' 55''$
4.  $b_1 = 81^\circ 15' 15''$ ,  $c_1 = 110^\circ 10' 50''$ ,  $C_1 = 119^\circ 43' 48''$   
 $b_2 = 98^\circ 44' 45''$ ,  $c_2 = 138^\circ 45' 26''$ ,  $C_2 = 142^\circ 24' 59''$
5. Impossible
6.  $c = 88^\circ 57' 44''$ .  $A = 51^\circ 44' 11''$ ,  $B = 139^\circ 29' 35''$

## §45. Pages 86, 87

1.  $A = 126^\circ 18' 42''$ ,  $B = 119^\circ 42' 8''$ ,  $C = 111^\circ 51' 42''$
2.  $c = 89^\circ 37' 43''$ ,  $A = 29^\circ 42' 0''$ ,  $B = 138^\circ 57' 22''$
3.  $a = 123^\circ 34' 46''$ ,  $b = 75^\circ 56' 32''$ ,  $c = 105^\circ 0' 18''$
4.  $b = 88^\circ 12' 19''$ ,  $C = 78^\circ 15' 46''$ ,  $a = 152^\circ 43' 49''$
5.  $a = 114^\circ 26' 50''$ ,  $c = 82^\circ 33' 31''$ ,  $C = 79^\circ 10' 30''$
6.  $c = 153^\circ 38' 40''$ ,  $A = 29^\circ 42' 34''$ ,  $B = 42^\circ 37' 18''$
7.  $a_1 = 42^\circ 37' 18''$ ,  $c_1 = 129^\circ 41' 5''$ ,  $C_1 = 89^\circ 54' 19''$   
 $a_2 = 137^\circ 22' 42''$ ,  $c_2 = 19^\circ 58' 36''$ ,  $C_2 = 26^\circ 21' 18''$
8.  $A = 59^\circ 29' 42''$ ,  $B = 62^\circ 49' 42''$ ,  $C = 65^\circ 50' 48''$
9.  $a = 110^\circ 30' 23''$ ,  $b = 36^\circ 47' 37''$ ,  $C = 135^\circ 12' 15''$
10.  $a = 51^\circ 17' 31''$ ,  $b = 64^\circ 2' 47''$ ,  $c = 51^\circ 17' 31''$

## §46. Page 89

1.  $c = 120^\circ 10' 52''$ ,  $A = 65^\circ 13' 4''$ ,  $B = 49^\circ 27' 53''$
2.  $a = 69^\circ 34' 44''$ ,  $B = 135^\circ 5' 14''$ ,  $C = 50^\circ 29' 54''$
3.  $c = 104^\circ 12' 52''$ ,  $B = 51^\circ 46' 38''$ ,  $A = 63^\circ 48' 24''$
4.  $b = 100^\circ 47' 46''$ ,  $A = 96^\circ 2' 12''$ ,  $C = 125^\circ 43' 46''$

5.  $c = 108^{\circ}39'11''$ ,  $B = 40^{\circ}23'17''$ ,  $A = 64^{\circ}48'55''$
6.  $a = 65^{\circ}28'34''$ ,  $B = 148^{\circ}14'43''$ ,  $C = 44^{\circ}9'3''$
7.  $a = 145^{\circ}24'53''$ ,  $b = 139^{\circ}45'58''$ ,  $C = 49^{\circ}56'16''$
8.  $a = 23^{\circ}57'9''$ ,  $c = 118^{\circ}2'15''$ ,  $B = 102^{\circ}5'52''$
10.  $c = 135^{\circ}49'19''$ ,  $b = 146^{\circ}37'15''$ ,  $A = 105^{\circ}8'17''$
11.  $a = 40^{\circ}1'5''$ ,  $b = 38^{\circ}31'5''$ ,  $C = 130^{\circ}3'48''$

## §47. Page 91

- |  |                            |
|--|----------------------------|
| 1. $a = 112^{\circ}10'4''$                           | 3. $c = 88^{\circ}57'41''$ |
| 2. $c = 73^{\circ}41'0''$                            | 4. $c = 37^{\circ}3'52''$  |
| 5. $A = 51^{\circ}44'7''$ , $B = 139^{\circ}29'36''$ |                            |

## §48. Page 92

1.  $A = 68^{\circ}33'42''$ ,  $B = 130^{\circ}48'18''$ ,  $C = 94^{\circ}0'48''$
3. Impossible
4.  $a = 165^{\circ}2'6''$ ,  $b = 163^{\circ}49'24''$ ,  $c = 11^{\circ}25'6''$
5.  $A = 65^{\circ}49'48''$ ,  $B = 56^{\circ}32'48''$ ,  $C = 116^{\circ}56'48''$
6. No solution. Examine the polar triangle.

## §49. Pages 92, 93

1.  $A = 63^{\circ}48'35''$ ,  $B = 51^{\circ}46'12''$ ,  $c = 104^{\circ}13'27''$
2.  $B = 95^{\circ}38'4''$ ,  $C = 97^{\circ}26'29''$ ,  $a = 64^{\circ}23'15''$
3.  $a = 40^{\circ}1'5''$ ,  $b = 38^{\circ}31'3''$ ,  $C = 130^{\circ}3'50''$
4.  $B_1 = 42^{\circ}37'17''$ ,  $C_1 = 160^{\circ}1'24''$ ,  $c_1 = 153^{\circ}38'42''$   
 $B_2 = 137^{\circ}22'42''$ ,  $C_2 = 50^{\circ}18'55''$ ,  $c_2 = 90^{\circ}5'41''$
5.  $B = 65^{\circ}33'10''$ ,  $C = 97^{\circ}26'29''$ ,  $c = 100^{\circ}49'30''$
6.  $b = 41^{\circ}52'35''$ ,  $c = 41^{\circ}35'4''$ ,  $C = 60^{\circ}42'46''$
7.  $A = 21^{\circ}1'2''$ ,  $B = 8^{\circ}38'46''$ ,  $C = 155^{\circ}31'36''$
8.  $a = 87^{\circ}20'28''$ ,  $b = 76^{\circ}44'2''$ ,  $c = 93^{\circ}55'31''$
9.  $44^{\circ}23'16''$  N.
10.  $L = 22^{\circ}57'36''$  S.,  $\lambda = 166^{\circ}48'4''$  E.
11.  $L = 43^{\circ}24'17''$  N.,  $\lambda = 100^{\circ}24'17''$  E.
12.  $L = 41^{\circ}3'50''$  N.,  $\lambda = 168^{\circ}19'20''$  W.
13.  $C = 224^{\circ}8'45''$ ,  $D = 5832$  miles
14.  $A = 110^{\circ}51'5''$ ,  $B = 48^{\circ}56'16''$ ,  $C = 38^{\circ}26'56''$

## §52. Pages 98 to 100

5.  $C_n = 311^{\circ}3'38''$ ,  $D = 6386.7$  miles
6.  $C_n = 211^{\circ}53'27''$
7.  $D = 6779.9$  miles
8.  $C_n = 230^{\circ}26'57''$
9.  $C_n = 86^{\circ}18'15''$ ,  $D = 5213.7$  miles  
 $L_v = 34^{\circ}32'27''$  N.,  $\lambda_v = 168^{\circ}1'41''$  W.
10.  $C_n = 224^{\circ}8'48''$ ,  $D = 5832$  miles
11.  $L = 44^{\circ}55'14''$



12. (a)  $43^{\circ}9' \text{ W.}$  (d)  $20^{\circ}31'28'' \text{ N.}$   
 (b)  $35^{\circ}53' \text{ N.}$  (e)  $C_n = 31^{\circ}56'17''$  or  $211^{\circ}56'17''$ , 6988.9 miles  
 (c)  $32^{\circ}34'36'' \text{ W.}$  (f) 2870.4 miles
13.  $C_1 = 298^{\circ}16'48''$ ,  $C_2 = 225^{\circ}58'34''$ ,  $D = 6052.4$  miles

## §55. Pages 104, 105

- |  |  |  |
|--|--|--|
| 3. $Z_n = 208^{\circ}12'00'$<br>$h = 59^{\circ}10'22''$  | 7. $Z_n = 312^{\circ}14'54''$<br>$h = 31^{\circ}13'24''$ | 11. $h = 22^{\circ}42'25''$<br>12. $h = 64^{\circ}13'52''$ |
| 4. $Z_n = 203^{\circ}46'46''$<br>$h = 21^{\circ}42'43''$ | 8. $Z_n = 145^{\circ}3'31''$<br>$h = 35^{\circ}33'10''$  | 13. $h = 31^{\circ}13'25''$<br>14. $h = 55^{\circ}36'22''$ |
| 5. $Z_n = 44^{\circ}49'41''$<br>$h = 51^{\circ}46'36''$  | 9. $Z_n = 125^{\circ}18'40''$<br>$h = 45^{\circ}53'20''$ | 15. $h = 51^{\circ}39'30''$<br>16. $h = 59^{\circ}10'15''$ |
| 6. $Z_n = 73^{\circ}11'42''$<br>$h = 64^{\circ}13'50''$  | 10. $Z_n = 85^{\circ}59'36''$<br>$h = 36^{\circ}40'18''$ | 18. $h = 2^{\circ}11'50''$                                 |

## §56. Page 107

1.  $A = E. 29^{\circ}28'6'' \text{ S.}$  2.  $4^h 37^m 48^s \text{ A.M.}$
3. Summer: sunrise at  $4^h 37^m 48^s \text{ A.M.}$ , sunset at  $7^h 22^m 12^s \text{ P.M.}$   
 Winter: sunrise at  $7^h 22^m 12^s \text{ A.M.}$ , sunset at  $4^h 37^m 48^s \text{ P.M.}$
4. (a) March 21: sunrise at  $6^h 0^m 0^s \text{ A.M.}$ , sunset at  $6^h 0^m 0^s \text{ P.M.}$   
 December 21: sunrise at  $10^h 19^m 7^s \text{ A.M.}$ , sunset at  $1^h 40^m 53^s \text{ P.M.}$   
 June 21: sunrise at  $1^h 40^m 53^s \text{ A.M.}$ , sunset at  $10^h 19^m 7^s \text{ P.M.}$
- (b) March 21:  $A = 0^{\circ}0'0''$  at sunrise;  $A = 0^{\circ}0'0''$  at sunset  
 December 21:  $A = E. 66^{\circ}59'30'' \text{ S.}$  at sunrise;  $A = W. 66^{\circ}59'30'' \text{ S.}$  at sunset  
 June 21:  $A = E. 66^{\circ}59'30'' \text{ N.}$  at sunrise;  $A = W. 66^{\circ}59'30'' \text{ N.}$  at sunset
- (c) Length of longest day:  $20^h 38^m 14^s$   
 Length of shortest day:  $3^h 21^m 46^s$
6. (a)  $10^{\circ} \text{ N.}$  (d)  $10^{\circ} \text{ S.}$   
 (b)  $10^{\circ} \text{ S.}$  (e) 30.25 ft.  
 (c)  $h = 13^{\circ}27'$ ,  $h = 33^{\circ}27'$

## §57. Page 109

2. (a)  $t = 7^h 8^m 2^s \text{ A.M.}$ ,  $Z_n = 79^{\circ}26'13''$   
 (b)  $t = 7^h 10^m 41^s \text{ A.M.}$ ,  $Z_n = 84^{\circ}58'52''$   
 (c)  $t = 6^h 50^m 25^s \text{ A.M.}$ ,  $Z_n = 81^{\circ}31'5''$
3.  $t = 8^h 23^m 50^s \text{ A.M.}$ ,  $Z_n = 100^{\circ}44'48''$
4.  $t = 9^h 10^m 46^s \text{ A.M.}$ ,  $Z_n = 125^{\circ}46'0''$
5.  $t = 4^h 37^m 46^s \text{ P.M.}$ ,  $Z_n = 272^{\circ}43'40''$
6.  $t = 3^h 5^m 18^s \text{ P.M.}$ ,  $Z_n = 261^{\circ}6'0''$

## §58. Pages 111, 112

1.  $60^{\circ} \text{ E.}$  5.  $\lambda_2 = ST_1 - ST_2 + \lambda_1$
2.  $15^h 42^m 30^s$  6.  $18^h 19^m 40^s$
3. (a)  $16^h 22^m$ ; (b)  $3^h 38^m$  7.  $23^h 45^m 22^s$
4.  $9^h 48^m 40^s$

## §59. Page 113

- |                                      |                                     |
|--------------------------------------|-------------------------------------|
| 1. $\lambda = 176^\circ 23' 15''$ W. | 4. $\lambda = 60^\circ 29' 0''$ W.  |
| 2. $\lambda = 12^\circ 9' 15''$ E.   | 5. $\lambda = 111^\circ 7' 30''$ W. |
| 3. $\lambda = 124^\circ 23' 45''$ W. | 6. $\lambda = 116^\circ 0' 15''$ W. |

## §60. Page 115

- |                          |                          |                           |
|--------------------------|--------------------------|---------------------------|
| 1. $L = 0^\circ$         | 7. $L = 33^\circ 50'$ N. | 12. $L = 37^\circ 33'$ N. |
| 2. $L = 30^\circ$ N.     | 8. $L = 12^\circ 24'$ S. | 13. $L = 74^\circ 22'$ N. |
| 3. $L = 50^\circ$ N.     | 9. $L = 8^\circ 41'$ S.  | 14. $L = 37^\circ 24'$ S. |
| 4. $L = 4^\circ 6'$ N.   | 10. $L = 0^\circ$        | 15. $L = 45^\circ 32'$ N. |
| 5. $L = 72^\circ 40'$ S. | 11. $L = 7^\circ 11'$ N. | 16. Impossible            |
| 6. $L = 46^\circ 58'$ N. |                          |                           |

## §61. Page 116

- |  |   |
|--|---|
| 1. (a) $L_1 = 13^\circ 26' 28''$ S.<br>$L_2 = 61^\circ 21' 31''$ N.  | (b) $L_1 = 58^\circ 21' 19''$ S.<br>$L_2 = 42^\circ 22' 21''$ N.  |
| 2. (a) $L_1 = 25^\circ 41' 32''$ N.<br>$Z_1 = 255^\circ 0' 0''$<br>$L_2 = 8^\circ 41' 32''$ N.<br>$Z_2 = 285^\circ 0' 0''$   | (c) $L_1 = 10^\circ 15' 58''$ N.<br>$L_2 = 24^\circ 58' 58''$ N.<br>$Z_1 = 77^\circ 29' 28''$<br>$Z_2 = 102^\circ 30' 32''$ |
| (b) $L_1 = 13^\circ 07' 20''$ S.<br>$L_2 = 72^\circ 55' 50''$ N.<br>$Z_1 = 321^\circ 33' 20''$<br>$Z_2 = 218^\circ 26' 40''$ | (d) $L = 44^\circ 22' 51''$ N.<br>$Z = 170^\circ 4' 0''$  |

## §62. Pages 116 to 120

- $Z_n = 237^\circ 53' 17''$
- $h = 13^\circ 48' 1''$ ,  $Z_n = 125^\circ 26' 9''$
- $L_1 = 26^\circ 53' 48''$  N.,  $L_2 = 71^\circ 19' 0''$  N.,  $Z_1 =$  N.  $45^\circ 0' 0''$  W.,  
 $Z_2 =$  N.  $135^\circ 0' 0''$  W.
- $L_1 = 25^\circ 42' 1''$  S.,  $L_2 = 8^\circ 41' 1''$  S.,  $Z_1 =$  S  $105^\circ 0' 0''$  E.,  
 $Z_2 =$  S  $75^\circ 0' 0''$  E.
- (a)  $L_1 = 3^\circ 14' 46''$  S.,  $L_2 = 43^\circ 23' 16''$  S.,  $Z_1 =$  S  $25^\circ 15' 29''$  E.,  
 $Z_2 =$  S  $154^\circ 44' 31''$  E.  
(b)  $L_1 = 11^\circ 29' 32''$  S.,  $L_2 = 62^\circ 39' 40''$  N.,  $Z_1 =$  N  $41^\circ 1' 54''$  E.,  
 $Z_2 =$  N.  $138^\circ 58' 5''$  E.
- (a)  $t = 4^h 27^m 46^s$  P.M.,  $Z_n = 272^\circ 43' 40''$   
(b)  $t = 10^h 7^m 44^s$  A.M.,  $Z_n = 34^\circ 56' 36''$
- Comes within 7.6 nautical miles of the Chicago position
- $D = 3355.2$  miles,  $C_n = 86^\circ 48' 48''$
- $D = 6748.6$  miles,  $C_n = 82^\circ 4' 28''$ ,  $L_v = 28^\circ 29' 44''$  S.,  
 $\lambda_v = 136^\circ 13' 45''$  E.
- $D = 4461.7$  miles,  $C_n = 302^\circ 13' 45''$
- $D = 6430.6$  miles,  $C_n = 300^\circ 40' 2''$
- $L = 43^\circ 25' 37''$  N., 1329.5 miles north of Honolulu
- $169^\circ 7' 4''$  W.

15.  $L = 66^{\circ}2'58''$  N.,  $\lambda = 167^{\circ}46'15''$  E.  
 16. (a)  $L = 57^{\circ}21'21''$  N.,  $\lambda = 17^{\circ}33'33''$  W.  
 (b)  $L = 44^{\circ}37'18''$  N.,  $\lambda = 68^{\circ}20'35''$  W.  
 17.  $152^{\circ}23'$   
 18.  $99^{\circ}57'30''$   
 19.  $d = 32^{\circ}40'36''$  S.  
 20.  $3^{\text{h}} 26^{\text{m}} 0^{\text{s}}$  E.  
 21.  $55^{\circ}45'$  N.  
 22. (a)  $4^{\text{h}} 50^{\text{m}} 59^{\text{s}}$  A.M.,  $7^{\text{h}} 9^{\text{m}} 1^{\text{s}}$  P.M.  
 (b)  $5^{\text{h}} 47^{\text{m}} 56^{\text{s}}$  A.M.,  $6^{\text{h}} 12^{\text{m}} 4^{\text{s}}$  P.M.  
 (c)  $5^{\text{h}} 50^{\text{m}}$  A.M.,  $6^{\text{h}} 10^{\text{m}}$  P.M.  
 (d)  $6^{\text{h}} 12^{\text{m}}$  A.M.,  $5^{\text{h}} 48^{\text{m}}$  P.M.  
 23. (a)  $18^{\text{h}} 28^{\text{m}} 24^{\text{s}}$ ; (b)  $5^{\text{h}} 31^{\text{m}} 36^{\text{s}}$   
 24.  $t = 4^{\text{h}} 29^{\text{m}} 19^{\text{s}}$  E.,  $A = \text{E. } 33^{\circ}35'3''$  N.  
 25. (a)  $2^{\text{h}} 4^{\text{m}} 28^{\text{s}}$ ,  $5^{\text{h}} 6^{\text{m}} 40^{\text{s}}$ ,  $14^{\text{h}} 44^{\text{m}} 25^{\text{s}}$ ,  $2^{\text{h}} 4^{\text{m}} 28^{\text{s}}$   
 (b)  $1^{\text{h}} 41^{\text{m}} 5^{\text{s}}$ ,  $11^{\text{h}} 22^{\text{m}} 15^{\text{s}}$ ,  $9^{\text{h}} 15^{\text{m}} 35^{\text{s}}$ ,  $1^{\text{h}} 41^{\text{m}} 5^{\text{s}}$   
 (c)  $1^{\text{h}} 33^{\text{m}} 42^{\text{s}}$ ,  $8^{\text{h}} 52^{\text{m}} 37^{\text{s}}$ ,  $12^{\text{h}} 0^{\text{m}} 0^{\text{s}}$ ,  $1^{\text{h}} 33^{\text{m}} 42^{\text{s}}$   
 26. (a)  $46^{\circ}58'$  N. (c)  $19^{\circ}40'$  S. (e)  $4^{\circ}6'$  N.  
 (b)  $41^{\circ}42'$  N. (d)  $72^{\circ}40'$  S. (f)  $9^{\circ}30'$  S.  
 27. For visible lower culmination,  $L$ ,  $d$ , and bearing must all be of the same name, with  $L + d > 90^{\circ}$  and at a lower culmination  $h < d$ .  
 28. (a)  $38^{\circ}30'$  N. (c)  $74^{\circ}22'$  N.  
 (b)  $75^{\circ}53'$  S. (d)  $37^{\circ}24'$  S.  
 29. (a)  $7^{\text{h}} 43^{\text{m}} 15^{\text{s}}$  (c) S.  $57^{\circ}14'39''$  E.  
 (b) 6.91  
 30.  $3^{\text{h}} 59^{\text{m}} 23^{\text{s}}$  P.M. 32. (a)  $93^{\circ}19'45''$  E.  
 31.  $2^{\text{h}} 58^{\text{m}} 44^{\text{s}}$  P.M. (b)  $9^{\circ}2'27''$  E.  
 33. The shadow stretches from foot of pole S  $71^{\circ}22'$  W.  
 34.  $Z_n = 75^{\circ}11'$  37.  $6^{\text{h}} 58^{\text{m}}$  A.M.,  $5^{\text{h}} 2^{\text{m}}$  P.M.  
 35. 13.8 ft. 38. 89.7 miles, 341.36 miles  
 36.  $120^{\circ}$  39.  $17^{\circ}14'40''$

## §67. Pages 125, 126

1. (a) 7 miles away on bearing  $75^{\circ}30'$  2. (a)  $L = 36^{\circ}59'$  N.  
 (b) 5 " " " "  $35^{\circ}$   $\lambda = 75^{\circ}43'$  W.  
 (c) 3 " " " "  $82^{\circ}30'$  (b)  $L = 37^{\circ}07'$  N.  
 (d) 4 " toward on "  $50^{\circ}50'$   $\lambda = 75^{\circ}36'$  W.  
 (e) 9 " " " "  $65^{\circ}40'$  (c)  $L = 37^{\circ}9'$  N.  
 (f) 6 " " " "  $147^{\circ}30'$   $\lambda = 75^{\circ}30'$  W.  
 (g) 5 " " " "  $285^{\circ}20'$  (d)  $L = 36^{\circ}52'$  N.  
 (h) 6 " away " "  $205^{\circ}30'$   $\lambda = 75^{\circ}29'$  W.  
 (i) 6 " " " "  $345^{\circ}10'$  3.  $L = 37^{\circ}2'$  N.  
 (j) 5 " toward " "  $210^{\circ}$   $\lambda = 75^{\circ}22'$  W.  
 4.  $h_c = 36^{\circ}40'18''$ ,  $Z_n = 85^{\circ}59'36''$   
 5.  $L = 37^{\circ}19'$  N.,  $\lambda = 75^{\circ}22'$  W.













**FIVE-PLACE LOGARITHMIC AND  
TRIGONOMETRIC TABLES**

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# FIVE-PLACE LOGARITHMIC AND TRIGONOMETRIC TABLES

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## PREFACE

A table of logarithms should be accurate, it should be easy to understand, and it should be as easy to use as possible. The authors, in the tables offered here, have attempted to make improvements along these three lines.

The tables used in trigonometry and its applications have been checked many times and have been carefully read against other tables. If, in spite of this thoroughness in compilation, errors are discovered, the authors would appreciate having them pointed out.

Frequently students fail to understand the process of linear interpolation. It is explained in this book by means of a simple diagram which gives the idea almost at a glance.

The table of logarithms of trigonometric functions (Table II), the most important one for trigonometry, has a number of new features. The proportional parts are tabulated for each second from  $0''$  to  $60''$ , and bold-faced numbers have been so used as to avoid ambiguity. Whenever there is a choice of two numbers one of which is written in bold face, the bold-faced number is always chosen. The simplicity of operation introduced by this plan gives a gain both in speed and in accuracy. In the table proper all six functions are tabulated, and bold-faced numbers are used in such a way as to enable the user to locate approximate position by using them only. It is believed that the gains due to these innovations are decidedly worth while.

LYMAN M. KELLS.  
WILLIS F. KERN.  
JAMES R. BLAND.

ANNAPOLIS, MD.,  
*July, 1935.*



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# FIVE-PLACE LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE I  
COMMON LOGARITHMS OF NUMBERS

**1. Introduction.\*** *The power  $L$  to which a given number  $b$  must be raised to produce a number  $N$  is called the logarithm of  $N$  to the base  $b$ . This relation expressed in symbols is*

$$b^L = N.$$

It appears at once that  $b$  must not be unity and it must not be negative. In the following set of tables, 10 is used as base. This system is called the *common system* or the *Briggs system*. Another important system, called the *natural system*, has  $e$  as base, where  $e = 2.71828$  accurate to six figures.

**2. Characteristic and mantissa.** The common logarithm of any real, positive number may be written as an integer, positive or negative, plus a positive decimal fraction. The integral part is called the *characteristic* and the decimal part the *mantissa*. The characteristic may be written by using the following rules:

**Rule 1.** *The characteristic of the common logarithm of a number greater than 1 is obtained by subtracting 1 from the number of digits to the left of the decimal point.*

**Rule 2.** *The characteristic of the common logarithm of a positive number less than 1 is negative and its magnitude is obtained by adding 1 to the number of zeros immediately following the decimal point.*

If the characteristic of a number is  $-n$  ( $n$  positive), it should be written in the form  $(10 - n) - 10$ . To obtain directly the logarithm of a number less than 1, subtract from 9 the number of zeros immediately following the decimal point, and write the result before the mantissa and  $-10$  after it.

The method of finding the mantissa of the logarithm of a number will be explained in the succeeding articles.

\* Since the theory of logarithms is treated completely in algebra and in trigonometry, only the actual manipulation of the tables is explained here.

## EXERCISES

Verify the characteristic of the logarithm of each of the numbers  $N$  written below.

$N$	$\log N$	$N$	$\log N$
1. 6.830	0.83442.	8. 58.73	1.76886.
2. 68.30	1.83442.	9. 0.6740	9.82866 - 10.
3. 6830	3.83442.	10. 0.007500	7.87506 - 10.
4. 683,000	5.83442.	11. $6.870 \times 10^5$	5.83696.
5. 0.7860	9.89542 - 10.	12. $5.860 \times 10^{-4}$	6.76790 - 10.
6. 0.007860	7.89542 - 10.	13. $3.990 \times 10^{-6}$	4.60097 - 10.
7. 0.0007860	6.89542 - 10.	14. $7.330 \times 10^2$	2.86510.

**3. To find the mantissa. Special case.** The mantissa, or decimal part of the logarithm of a number, depends only on the sequence of the digits and not on the position of the decimal point. Table I lists the mantissas, accurate to five decimal places, of the logarithms of all integers from 1 to 10,000.

The change in the mantissas of the logarithms is so slow that the first two figures do not change for several lines of the table. Consequently the appropriate first two figures are printed in the first column before the first full row to which they apply. Also the appropriate first two figures appear at the left of the first line of mantissas on each page. An asterisk in any row indicates that the first two figures are to be found at the left of the next row.

To find the mantissa of the logarithm of a number locate the first three digits of this number in the left-hand column headed  $N$  and the fourth digit in the row at the top of the page. Then the mantissa of the given number containing four significant figures is in the row whose first three figures are the first three significant figures of the given number, and in the column headed by the fourth. Thus to find the logarithm of 76.64 find 766 in the column headed  $N$ , follow the corresponding row to the entry in the column headed by 4. This entry 88446 represents the mantissa required. Hence we have

$$\log 76.64 = 1.88446. \quad \text{Ans.}$$

## EXERCISES

Verify the logarithms in the exercise of §2.

**4. Interpolation.** When a number contains a fifth significant figure, we find the logarithm corresponding to the first four figures as in §3 and then add an increment obtained by a process called interpolation. This process is based on the assumption that *for relatively small changes in the number  $N$  the changes in  $\log N$  are proportional to the changes in  $N$ .* The following example will serve to illustrate the process of interpolation.

The expression *tabular difference* will be used frequently in what follows. The tabular difference, when used in connection with a table,

means the result of subtracting the lesser of two successive entries from the greater.

**Example.** Find  $\log 235.47$ .

**Solution.** We first find the logarithms in the following form and then compute the difference indicated:

$$\left. \begin{array}{l} \log 235.40 \\ \log 235.47 \\ \log 235.50 \end{array} \right\} \begin{array}{l} 7 \\ 10 \\ 18 \end{array} \left. \begin{array}{l} = 2.37181 \\ = ? \\ = 2.37199 \end{array} \right\} d \quad 18 \text{ (tabular difference*)}$$

By the principle of proportional parts, we have

$$\frac{7}{10} = \frac{d}{18}, \quad \text{or} \quad d = \frac{7}{10}(18) = 12.6 = 13 \text{ (nearly).}$$

Adding 0.00013 to 2.37181, we obtain

$$\log 235.47 = 2.37194. \quad \text{Ans.}$$

The increment 12.6 was rounded off to 13 because we are not justified in writing more than five decimal places in the mantissa.

The essence of this procedure is embodied in the following statement. To find the logarithm of a number composed of five significant figures, first find the logarithm corresponding to the first four figures and to it add one-tenth of the tabular difference multiplied by the fifth digit.

To shorten the process of interpolation,  $10^5$  times each tabular difference occurring in the table has been multiplied by 0.1, 0.2, . . . 0.9, and the results have been tabulated on the right-hand sides of the pages on which these differences occur. The abbreviation Prop. Parts written at the top of the page over these small tables abbreviates the words *proportional parts*. To interpolate in the example just solved, locate the Prop. Parts table headed 18 and find opposite 7 in its left-hand column the entry 12.6 (=13 nearly). In general, this difference should not be computed but should be obtained from the number opposite the fifth digit in the appropriate table of proportional parts.

### EXERCISES

Verify the following logarithms:

- |                                    |   |
|------------------------------------|---|
| 1. $\log 7012.6 = 3.84588$ .       | 8. $\log 0.056321 = 8.75067 - 10$ .       |
| 2. $\log 54.725 = 1.73819$ .       | 9. $\log 4,574,000 = 6.66030$ .           |
| 3. $\log 0.87364 = 9.94133 - 10$ . | 10. $\log 568.91 = 2.75504$ .             |
| 4. $\log 3.7245 = 0.57107$ .       | 11. $\log 4.3965 \times 10^5 = 5.64311$ . |
| 5. $\log 0.00065931 = 6.81909$ .   | 12. $\log 10.905 = 1.03763$ .             |
| 6. $\log 25.819 = 1.41194$ .       | 13. $\log 0.0025725 = 7.41036$ .          |
| 7. $\log 2.3454 = 0.37022$ .       | 14. $\log 0.000032026 = 5.50550 - 10$ .   |

**5. To find the number corresponding to a given logarithm.** If  $\log N = L$ , the number  $N$  is called the *antilogarithm* of  $L$ . The sequence of

\* For convenience the decimal point has been omitted.

digits of a number  $N$  corresponding to a given logarithm  $L$  is found from its mantissa, and the decimal point is then placed in accordance with the rules of §2.

**Example.** Given  $\log N = 1.60334$ , find  $N$ .

**Solution.** The mantissa .60334 lies between the entries .60325 and .60336 of Table I. Using the table and computing the differences indicated, we write the following form:

$$\left. \begin{array}{l} 1.60325 \\ 1.60334 \\ 1.60336 \end{array} \right\} \begin{array}{l} 9 \\ 11 \\ 10 \end{array} \left. \begin{array}{l} = \log 40.110 \\ = \log N \\ = \log 40.120 \end{array} \right\} \begin{array}{l} x \\ \\ 10 \end{array}$$

Assuming that changes in the logarithm are proportional to the corresponding changes in the number, we write

$$\frac{9}{11} = \frac{x}{10}, \quad \text{or} \quad x = 10 \left( \frac{9}{11} \right) = 8 \text{ (nearly).}$$

Hence

$$N = 40.118. \quad \text{Ans.}$$

The essence of the process of interpolation is indicated in the foregoing procedure. However, in practice, the student should always interpolate by using the table of proportional parts. The fifth figure 8 should have been obtained from the table of proportional parts. In the small Prop. Parts table corresponding to the tabular difference 11, we read the fifth figure 8 in the left-hand column opposite the entry 8.8, the entry nearest to 9.

### EXERCISES

Verify the following antilogarithms:

- |                                     |                                       |
|-------------------------------------|---------------------------------------|
| 1. $3.57351 = \log 3745.5$ .        | 8. $4.76224 = \log 57842$ .           |
| 2. $2.82315 = \log 665.50$ .        | 9. $6.51738 - 10 = \log 0.00032914$ . |
| 3. $0.12112 = \log 1.3217$ .        | 10. $1.49715 = \log 31.416$ .         |
| 4. $1.92594 = \log 84.321$ .        | 11. $4.21691 - 10 = \log 16478$ .     |
| 5. $9.47954 - 10 = \log 0.30167$ .  | 12. $5.09873 = \log 125520$ .         |
| 6. $8.65636 - 10 = \log 0.045327$ . | 13. $9.27951 - 10 = \log 0.19033$ .   |
| 7. $0.37976 = \log 2.3975$ .        | 14. $7.88000 - 10 = \log 0.0075858$ . |

## TABLE II

### LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

**6. Table of logarithms of trigonometric functions.** Table II gives the logarithms of the sines, cosines, tangents, cotangents, secants, and cosecants of angles at intervals of  $1'$  from  $0^\circ$  to  $90^\circ$ . The names of the functions written at the top of any page apply to angles having the number of degrees written at the top of the page, and the function names written at the bottom apply to angles having the number of degrees written at the bottom. The left-hand or the right-hand minute column applies according as the number of degrees in the angle is written on the left side or on the right side of the block of numbers under consideration.

For example, to find  $\log \sin 32^\circ 46'$ , we find the page at the top of which  $32^\circ$  appears, find the row containing 46 in the left-hand minute column, and read 73337 in this row and in the column headed  $l \sin$ . Hence  $\log \sin 32^\circ 46' = 9.73337 - 10$ . The number 9 was found at the head of the  $l \sin$  column and the number  $-10$  is to be applied to every logarithm in the table. Again, to find  $\log \tan 142^\circ 36'$ , find the page at the top of which  $142^\circ$  appears, find the row containing 36 in the right-hand minute column, and read 88341 in this row and in the column headed  $l \tan$ . Hence  $\log \tan 142^\circ 36' = (-) 9.88341 - 10$ . The minus sign in parentheses before the log indicates that a negative number is under consideration. The characteristic was obtained as in the first example.

### EXERCISES

Verify the following:

1.  $\log \sin 37^\circ 27' = 9.78395 - 10$ .
2.  $\log \tan 36^\circ 41' = 9.87211 - 10$ .
3.  $\log \cot 28^\circ 16' = 0.26946$ .
4.  $\log \cos 62^\circ 20' = 9.66682 - 10$ .
5.  $\log \csc 69^\circ 54' = 0.02729$ .
6.  $\log \sin 131^\circ 10' = 9.87668 - 10$ .
7.  $\log \tan 142^\circ 27' = (-) 9.88577 - 10$ .
8.  $\log \sec 134^\circ 47' = (-) 0.15216$ .
9.  $\log \cos 45^\circ 47' = 9.84347 - 10$ .
10.  $\log \csc 135^\circ 13' = (-) 0.15216$ .
11.  $\log \cot 132^\circ 0' = (-) 9.95444 - 10$ .

**7. Given the angle, to find the logarithm of a trigonometric function.** The principles involved here are the same as those involved in finding

logarithms and antilogarithms of numbers. Interpolation for seconds is accomplished by direct interpolation or by using the columns headed *d 1'* and the columns headed proportional parts. The following example will illustrate the procedure.

**Example.** Find  $\log \tan 65^\circ 42' 17''$ .

**Solution.** Using the table to find logarithms and computing differences, we write the following form:

$$\begin{array}{rcl} \log \tan 65^\circ 42' 00'' & \left. \vphantom{\log \tan 65^\circ 42' 00''} \right\} 17'' & = 0.34533 \\ \log \tan 65^\circ 42' 17'' & \left. \vphantom{\log \tan 65^\circ 42' 17''} \right\} 60'' & ? \\ \log \tan 65^\circ 43' 00'' & & = 0.34566 \end{array} \left. \vphantom{\log \tan 65^\circ 42' 00''} \right\} x \left. \vphantom{\log \tan 65^\circ 42' 00''} \right\} 33$$

Hence assuming that, for small changes, change of logarithm is proportional to change of angle, we have

$$\frac{x}{33} = \frac{17}{60}, \quad \text{or} \quad x = 33 \left( \frac{17}{60} \right) = 9.35 = 9 \text{ (nearly).}$$

Therefore

$$\log \tan 65^\circ 42' 17'' = 0.34533 + 0.00009 = 0.34542. \quad \text{Ans.}$$

The essence of the process of interpolation is indicated in the foregoing procedure. However, in practice, the student should always interpolate by using the columns headed *d 1'* and the proportional parts column.

Each entry in the column headed *d 1'* gives the difference of the logarithms between which it is spaced in each of the adjacent columns. In each column headed by *proportional parts* appears  $\frac{1}{60}, \frac{2}{60}, \frac{3}{60} \dots$  of the number heading the column. Hence the difference 9 to be applied in the case of the foregoing example is found in the proportional parts column headed by 33 (the tabular difference for  $1'$  written between 0.34533 and 0.34566) and in the row with the 17 of the seconds column. Again, to find  $\log \cot 10^\circ 28' 36''$ , we find the entry 73345 for  $\log \cot 10^\circ 28'$ , note the appropriate number 71 in the adjacent column headed *d 1'*, enter the proportional parts column headed by 71, read in this column 43 opposite the 36 of the seconds column; subtract 43 from 73345, and write  $\log \cot 10^\circ 28' 36'' = 0.73302$ .

It is worthy of note that *the changes of logarithms due to the seconds of an angle* must be added or subtracted according as the value of the function for angles near the one under consideration is increasing or decreasing with increasing angle.

### EXERCISES

Verify the following:

1.  $\log \sin 35^\circ 17' 8'' = 9.76166 - 10$ .
2.  $\log \cos 48^\circ 24' 21'' = 9.82207 - 10$ .
3.  $\log \sec 142^\circ 37' 15'' = (-) 0.09984$ .

4.  $\log \csc 56^\circ 21' 57'' = 0.07956$ .
5.  $\log \cot 23^\circ 16' 50'' = 0.36626$ .
6.  $\log \csc 128^\circ 47' 52'' = 0.10826$ .
7.  $\log \tan -69^\circ 38' 54'' = (-) 0.43070$ .
8.  $\log \sin 197^\circ 36' 57'' = 9.48092 - 10$ .
9.  $\log \sin 137^\circ 45' 22'' = 9.82756 - 10$ .
10.  $\log \cos 137^\circ 45' 22'' = (-) 9.86940 - 10$ .
11.  $\log \sin 209^\circ 32' 50'' = 9.69297 - 10$ .
12.  $\log \cos 330^\circ 27' 10'' = 9.93949 - 10$ .

**8. Given the logarithm of a trigonometric function, to find the angle.**

The following example will indicate the procedure necessary to find the angle when the logarithm of a trigonometric function of the angle is given:

**Example.** Find  $\theta$  if  $\log \cos \theta$  is  $9.85391 - 10$ .

**Solution.** Using the table to find logarithms and computing differences, we write the following form:

$$\left. \begin{array}{l} \log \cos 44^\circ 24' 00'' \\ \log \cos 44^\circ 24' ?'' \\ \log \cos 44^\circ 25' 00'' \end{array} \right\} \begin{array}{l} x \\ 60'' \\ \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} = 9.85399 \\ = 9.85391 \\ = 9.85386 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 8 \\ 13 \end{array}$$

Hence

$$\frac{x}{60} = \frac{8}{13}, \quad \text{or} \quad x = \frac{8}{13}(60) = 37'' \text{ (nearly),}$$

and

$$\theta = 44^\circ 24' 37''. \quad \text{Ans.}$$

The essence of the process of interpolation is indicated in the foregoing procedure. In practice, however, the columns headed  $d$  1' and the proportional parts columns should be used in interpolation. Thus, to find  $\theta$  in the example just considered, we first find  $44^\circ 24'$  and difference 8 as above, then read 13 in the column headed  $d$  1' adjacent to and slightly below the entry 85399, enter the corresponding proportional parts column, opposite the bold-faced one of the five 8's tabulated read  $37''$  in the seconds column, and then write  $\theta = 44^\circ 24' 37''$ .

When finding the number of seconds in an angle corresponding to a given logarithm of a trigonometric function, the student may find several identical entries in the proportional parts column involved. In this case, and in any case where there is a choice between two or more entries one of which is printed in **bold face**, always give preference to the **bold-faced** entry.

### EXERCISES

Find the value of  $\theta$  less than  $360^\circ$  in the following:

1.  $\log \sin \theta = 9.96162 - 10$ . Ans.  $66^\circ 16' 0''$  and  $113^\circ 44' 0''$ .
2.  $\log \cos \theta = 9.99537 - 10$ . Ans.  $8^\circ 21' 0''$  and  $351^\circ 39' 0''$ .
3.  $\log \cot \theta = 0.52368$ . Ans.  $16^\circ 40' 13''$  and  $196^\circ 40' 13''$ .





3.78381. Therefore  $\alpha = 6078.7''$ . Since  $1^\circ 41' = 6060''$ ,  $6078.7'' = 1^\circ 41' 19''$ .

## EXERCISES

Verify the following:

- |   |   |
|---|---|
| 1. $\log \sin 0^\circ 44' 13'' = 8.10930 - 10$ .  | 6. $\log \cot 89^\circ 3' 11'' = 8.21824 - 10$ .  |
| 2. $\log \cos 89^\circ 21' 31'' = 8.04899 - 10$ . | 7. $\log \cos 88^\circ 41' 20'' = 8.35948 - 10$ . |
| 3. $\log \tan 0^\circ 32' 23'' = 7.97406 - 10$ .  | 8. $\log \sin 0^\circ 59' 8'' = 8.23554 - 10$ .   |
| 4. $\log \cot 0^\circ 25' 56'' = 2.12241$ .       | 9. $\log \tan 1^\circ 29' 10'' = 8.41403 - 10$ .  |
| 5. $\log \tan 1^\circ 10' 9'' = 8.30981 - 10$ .   | 10. $\log \sec 88^\circ 16' 10'' = 1.52000$ .     |
- Verify the following:
11.  $\log \cos \theta = 8.32967 - 10$ ;  $\theta = 88^\circ 46' 33''$  and  $271^\circ 13' 27''$ .  
 12.  $\log \tan \theta = 8.11584 - 10$ ;  $\theta = 0^\circ 44' 53''$  and  $180^\circ 44' 53''$ .  
 13.  $\log \sin \theta = 8.23468 - 10$ ;  $\theta = 0^\circ 59' 1''$  and  $179^\circ 0' 59''$ .

# TABLE III

## NATURAL TRIGONOMETRIC FUNCTIONS

**10. Table of natural values of trigonometric functions.** Table II contains the numerical values of the sines, cosines, tangents, and cotangents of angles from  $0^\circ$  to  $90^\circ$  at intervals of  $1'$ . In the case of an angle in the range from  $0^\circ$  to  $45^\circ$ , the number of degrees in the angle and the names of the functions are found at the top of the page and the left-hand minute column applies; in the case of angles in the range from  $45^\circ$  to  $90^\circ$ , the number of degrees in the angle and the names of the functions are found at the bottom of the page and the right-hand minute column applies. Interpolation must be carried out without the aid of difference columns or tables of proportional parts.

The following examples illustrate the method of using the tables.

**Example 1.** Find  $\sin 68^\circ 28'$ .

*Solution.* We first find the page at the bottom of which  $68^\circ$  appears and then find the row of the  $68^\circ$  block containing  $28'$  in the right-hand minute column. In this row and in the column having  $\sin$  at its foot we find 020 to which we must prefix 0.93 to obtain  $\sin 68^\circ 28' = 0.93020$ .

**Example 2.** Find  $\sin 38^\circ 38' 27''$ .

*Solution.* Using the tables and computing differences, we find the values exhibited in the following form:

$$\begin{array}{rcl} \sin 38^\circ 38' 00'' & \left. \vphantom{\sin 38^\circ 38' 00''} \right\} 27'' & = 0.62433 \\ \sin 38^\circ 38' 27'' & \left. \vphantom{\sin 38^\circ 38' 27''} \right\} 60'' & = ? \\ \sin 38^\circ 39' 00'' & \left. \vphantom{\sin 38^\circ 39' 00''} \right\} & = 0.62456 \end{array} \left\{ x \right\} 23$$

Hence

$$\frac{x}{23} = \frac{27}{60}, \quad \text{or} \quad x = \left( \frac{27}{60} \right) 23 = 10 \text{ (nearly).}$$

Therefore

$$\sin 38^\circ 38' 27'' = 0.62433 + 0.00010 = 0.62443. \quad \text{Ans.}$$

**Example 3.** If  $\cot \theta = 0.37806$ , find  $\theta$ .

*Solution.* Using the tables and computing differences, we find the values exhibited in the following form:

$$\begin{array}{rcl} \cot 69^\circ 17' 00'' & \left. \vphantom{\cot 69^\circ 17' 00''} \right\} x & = 0.37820 \\ \cot \quad ? & \left. \vphantom{\cot \quad ?} \right\} 60 & = 0.37806 \\ \cot 69^\circ 18' 00'' & \left. \vphantom{\cot 69^\circ 18' 00''} \right\} & = 0.37787 \end{array} \left\{ 14 \right\} 33$$

Hence

$$\frac{x}{60} = \frac{14}{33}, \quad \text{or} \quad x = \frac{14}{33}(60) = 25'' \text{ (nearly), and } \theta = 69^\circ 17' 25''. \quad \text{Ans.}$$

Since  $\cot \theta$  is positive in the third quadrant, we may also write an answer  $180^\circ + 69^\circ 17' 25'' = 249^\circ 17' 25''$ . *Ans.*

### EXERCISES

Verify the following:

- |   |   |
|---|---|
| 1. $\sin 53^\circ 42' 0'' = 0.80593$ .    | 5. $\cos 83^\circ 17' 38'' = 0.11678$ . |
| 2. $\cos 31^\circ 53' 9'' = 0.84911$ .    | 6. $\sin 87^\circ 37' 25'' = 0.99914$ . |
| 3. $\tan 156^\circ 42' 13'' = -0.43059$ . | 7. $\cot 13^\circ 14' 52'' = 4.2475$ .  |
| 4. $\cot 27^\circ 51' 17'' = 1.8923$ .    | 8. $\tan 83^\circ 40' 30'' = 9.0218$ .  |

Find the values of  $\theta$  less than  $360^\circ$  in the following:

- |                                |  |
|--------------------------------|--|
| 9. $\sin \theta = 0.89742$ .   | <i>Ans.</i> $63^\circ 49' 12''$ and $116^\circ 10' 48''$ . |
| 10. $\cos \theta = 0.43750$ .  | <i>Ans.</i> $64^\circ 3' 20''$ and $295^\circ 56' 40''$ .  |
| 11. $\tan \theta = -0.92834$ . | <i>Ans.</i> $137^\circ 7' 41''$ and $317^\circ 7' 41''$ .  |
| 12. $\cot \theta = 1.8923$ .   | <i>Ans.</i> $27^\circ 51' 17''$ and $207^\circ 51' 17''$ . |
| 13. $\cos \theta = 0.95140$ .  | <i>Ans.</i> $17^\circ 56' 14''$ and $342^\circ 3' 46''$ .  |
| 14. $\sin \theta = 0.13552$ .  | <i>Ans.</i> $7^\circ 47' 19''$ and $172^\circ 12' 41''$ .  |

sent out from the parent plant produces both roots and new shoots after which the runner may die, thus severing the daughter plant from the parent. The young plants which form at the rooting nodes of the runner may be cut off and set out. Stolons form roots naturally, but rooting may be hastened by covering them with soil. It will be readily observed that the layer is in reality an artificial stolon. (See Fig. 110.)

**Exercise 104. Suckering and propagation by runners.** Observe in the field the suckers of such plants as mentioned in the foregoing paragraph. Cut

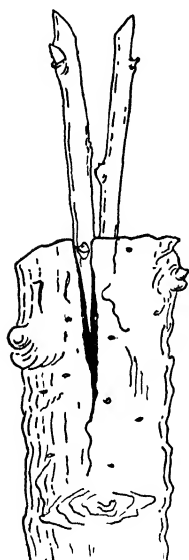


FIG. 115.—Cleft grafting. At right, two views of the scion, and at left, the scions in position in the cleft of the stock.

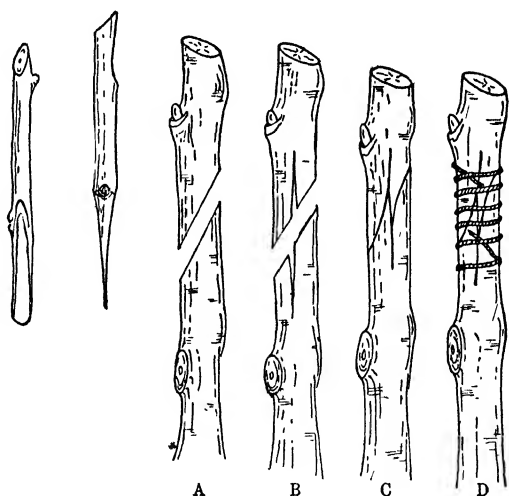


FIG. 116.—Steps in tongue or whip grafting.

off a portion of a root which bears a sucker, and transplant. Also observe in a strawberry bed how the plants naturally propagate themselves by runners.

**Propagation by grafting.** This is a very old horticultural practice, and is in common use in propagating fruit trees. The fruit grower, in order that he may be certain as to the variety

TABLE I

FIVE-PLACE TABLE OF COMMON LOGARITHMS OF NUMBERS

From 1 to 10,000



TABLE I  
FIVE-PLACE TABLE OF COMMON LOGARITHMS OF NUMBERS

From 1 to 10,000

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
<b>0</b>	—	<b>20</b>	1.30 103	<b>40</b>	1.60 206	<b>60</b>	1.77 815	<b>80</b>	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
<b>10</b>	1.00 000	<b>30</b>	1.47 712	<b>50</b>	1.69 897	<b>70</b>	1.84 510	<b>90</b>	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
<b>20</b>	1.30 103	<b>40</b>	1.60 206	<b>60</b>	1.77 815	<b>80</b>	1.90 309	<b>100</b>	2.00 000

TABLE I

0-50

N.	L. 0	1	2	3	4	5	6	7	8	9
<b>0</b>		00 000	30 103	47 712	60 206	69 897	77 815	84 510	90 309	95 424
1	00 000	04 139	07 918	11 394	14 613	17 609	20 412	23 045	25 527	27 875
2	30 103	32 222	34 242	36 173	38 021	39 794	41 497	43 136	44 716	46 240
3	47 712	49 136	50 515	51 851	53 148	54 407	55 630	56 820	57 978	59 106
4	60 206	61 278	62 325	63 347	64 345	65 321	66 276	67 210	68 124	69 020
5	69 897	70 757	71 600	72 428	73 239	74 036	74 819	75 587	76 343	77 085
6	77 815	78 533	79 239	79 934	80 618	81 291	81 954	82 607	83 251	83 885
7	84 510	85 126	85 733	86 332	86 923	87 506	88 081	88 649	89 209	89 763
8	90 309	90 849	91 381	91 908	92 428	92 942	93 450	93 952	94 448	94 939
9	95 424	95 904	96 379	96 848	97 313	97 772	98 227	98 677	99 123	99 564
<b>10</b>	00 000	00 432	00 860	01 284	01 703	02 119	02 531	02 938	03 342	03 743
11	04 139	04 532	04 922	05 308	05 690	06 070	06 446	06 819	07 188	07 555
12	07 918	08 279	08 636	08 991	09 342	09 691	10 037	10 380	10 721	11 059
13	11 394	11 727	12 057	12 385	12 710	13 033	13 354	13 672	13 988	14 301
14	14 613	14 922	15 229	15 534	15 836	16 137	16 435	16 732	17 026	17 319
15	17 609	17 898	18 184	18 469	18 752	19 033	19 312	19 590	19 866	20 140
16	20 412	20 683	20 952	21 219	21 484	21 748	22 011	22 272	22 531	22 789
17	23 045	23 300	23 553	23 805	24 055	24 304	24 551	24 797	25 042	25 285
18	25 527	25 768	26 007	26 245	26 482	26 717	26 951	27 184	27 416	27 646
19	27 875	28 103	28 330	28 556	28 780	29 003	29 226	29 447	29 667	29 885
<b>20</b>	30 103	30 320	30 535	30 750	30 963	31 175	31 387	31 597	31 806	32 015
21	32 222	32 428	32 634	32 838	33 041	33 244	33 445	33 646	33 846	34 044
22	34 242	34 439	34 635	34 830	35 025	35 218	35 411	35 603	35 793	35 984
23	36 173	36 361	36 549	36 736	36 922	37 107	37 291	37 475	37 658	37 840
24	38 021	38 202	38 382	38 561	38 739	38 917	39 094	39 270	39 445	39 620
25	39 794	39 967	40 140	40 312	40 483	40 654	40 824	40 993	41 162	41 330
26	41 497	41 664	41 830	41 996	42 160	42 325	42 488	42 651	42 813	42 975
27	43 136	43 297	43 457	43 616	43 775	43 933	44 091	44 248	44 404	44 560
28	44 716	44 871	45 025	45 179	45 332	45 484	45 637	45 788	45 939	46 090
29	46 240	46 389	46 538	46 687	46 835	46 982	47 129	47 276	47 422	47 567
<b>30</b>	47 712	47 857	48 001	48 144	48 287	48 430	48 572	48 714	48 855	48 996
31	49 136	49 276	49 415	49 554	49 693	49 831	49 969	50 106	50 243	50 379
32	50 515	50 651	50 786	50 920	51 055	51 188	51 322	51 455	51 587	51 720
33	51 851	51 983	52 114	52 244	52 375	52 504	52 634	52 763	52 892	53 020
34	53 148	53 275	53 403	53 529	53 656	53 782	53 908	54 033	54 158	54 283
35	54 407	54 531	54 654	54 777	54 900	55 023	55 145	55 267	55 388	55 509
36	55 630	55 751	55 871	55 991	56 110	56 229	56 348	56 467	56 585	56 703
37	56 820	56 937	57 054	57 171	57 287	57 403	57 519	57 634	57 749	57 864
38	57 978	58 092	58 206	58 320	58 433	58 546	58 659	58 771	58 883	58 995
39	59 106	59 218	59 329	59 439	59 550	59 660	59 770	59 879	59 988	60 097
<b>40</b>	60 206	60 314	60 423	60 531	60 638	60 746	60 853	60 959	61 066	61 172
41	61 278	61 384	61 490	61 595	61 700	61 805	61 909	62 014	62 118	62 221
42	62 325	62 428	62 531	62 634	62 737	62 839	62 941	63 043	63 144	63 246
43	63 347	63 448	63 548	63 649	63 749	63 849	63 949	64 048	64 147	64 246
44	64 345	64 444	64 542	64 640	64 738	64 836	64 933	65 031	65 128	65 225
45	65 321	65 418	65 514	65 610	65 706	65 801	65 896	65 992	66 087	66 181
46	66 276	66 370	66 464	66 558	66 652	66 745	66 839	66 932	67 025	67 117
47	67 210	67 302	67 394	67 486	67 578	67 669	67 761	67 852	67 943	68 034
48	68 124	68 215	68 305	68 395	68 485	68 574	68 664	68 753	68 842	68 931
49	69 020	69 108	69 197	69 285	69 373	69 461	69 548	69 636	69 723	69 810
<b>50</b>	69 897	69 984	70 070	70 157	70 243	70 329	70 415	70 501	70 586	70 672
N.	L. 0	1	2	3	4	5	6	7	8	9



TABLE I

## 50-100

N.	L. 0	1	2	3	4	5	6	7	8	9
<b>50</b>	69 897	69 984	70 070	70 157	70 243	70 329	70 415	70 501	70 586	70 672
51	70 757	70 842	70 927	71 012	71 096	71 181	71 265	71 349	71 433	71 517
52	71 600	71 684	71 767	71 850	71 933	72 016	72 099	72 181	72 263	72 346
53	72 428	72 509	72 591	72 673	72 754	72 835	72 916	72 997	73 078	73 159
54	73 239	73 320	73 400	73 480	73 560	73 640	73 719	73 799	73 878	73 957
55	74 036	74 115	74 194	74 273	74 351	74 429	74 507	74 586	74 663	74 741
56	74 819	74 896	74 974	75 051	75 128	75 205	75 282	75 358	75 435	75 511
57	75 587	75 664	75 740	75 815	75 891	75 967	76 042	76 118	76 193	76 268
58	76 343	76 418	76 492	76 567	76 641	76 716	76 790	76 864	76 938	77 012
59	77 085	77 159	77 232	77 305	77 379	77 452	77 525	77 597	77 670	77 743
<b>60</b>	77 815	77 887	77 960	78 032	78 104	78 176	78 247	78 319	78 390	78 462
61	78 533	78 604	78 675	78 746	78 817	78 888	78 958	79 029	79 099	79 169
62	79 239	79 309	79 379	79 449	79 518	79 588	79 657	79 727	79 796	79 865
63	79 934	80 003	80 072	80 140	80 209	80 277	80 346	80 414	80 482	80 550
64	80 618	80 686	80 754	80 821	80 889	80 956	81 023	81 090	81 158	81 224
65	81 291	81 358	81 425	81 491	81 558	81 624	81 690	81 757	81 823	81 889
66	81 954	82 020	82 086	82 151	82 217	82 282	82 347	82 413	82 478	82 543
67	82 607	82 672	82 737	82 802	82 866	82 930	82 995	83 059	83 123	83 187
68	83 251	83 315	83 378	83 442	83 506	83 569	83 632	83 696	83 759	83 822
69	83 885	83 948	84 011	84 073	84 136	84 198	84 261	84 323	84 386	84 448
<b>70</b>	84 510	84 572	84 634	84 696	84 757	84 819	84 880	84 942	85 003	85 065
71	85 126	85 187	85 248	85 309	85 370	85 431	85 491	85 552	85 612	85 673
72	85 733	85 794	85 854	85 914	85 974	86 034	86 094	86 153	86 213	86 273
73	86 332	86 392	86 451	86 510	86 570	86 629	86 688	86 747	86 806	86 864
74	86 923	86 982	87 040	87 099	87 157	87 216	87 274	87 332	87 390	87 448
75	87 506	87 564	87 622	87 679	87 737	87 795	87 852	87 910	87 967	88 024
76	88 081	88 138	88 195	88 252	88 309	88 366	88 423	88 480	88 536	88 593
77	88 649	88 705	88 762	88 818	88 874	88 930	88 986	89 042	89 098	89 154
78	89 209	89 265	89 321	89 376	89 432	89 487	89 542	89 597	89 653	89 708
79	89 763	89 818	89 873	89 927	89 982	90 037	90 091	90 146	90 200	90 255
<b>80</b>	90 309	90 363	90 417	90 472	90 526	90 580	90 634	90 687	90 741	90 795
81	90 849	90 902	90 956	91 009	91 062	91 116	91 169	91 222	91 275	91 328
82	91 381	91 434	91 487	91 540	91 593	91 645	91 698	91 751	91 803	91 855
83	91 908	91 960	92 012	92 065	92 117	92 169	92 221	92 273	92 324	92 376
84	92 428	92 480	92 531	92 583	92 634	92 686	92 737	92 788	92 840	92 891
85	92 942	92 993	93 044	93 095	93 146	93 197	93 247	93 298	93 349	93 399
86	93 450	93 500	93 551	93 601	93 651	93 702	93 752	93 802	93 852	93 902
87	93 952	94 002	94 052	94 101	94 151	94 201	94 250	94 300	94 349	94 399
88	94 448	94 498	94 547	94 596	94 645	94 694	94 743	94 792	94 841	94 890
89	94 939	94 988	95 036	95 085	95 134	95 182	95 231	95 279	95 328	95 376
<b>90</b>	95 424	95 472	95 521	95 569	95 617	95 665	95 713	95 761	95 809	95 856
91	95 904	95 952	95 999	96 047	96 095	96 142	96 190	96 237	96 284	96 332
92	96 379	96 426	96 473	96 520	96 567	96 614	96 661	96 708	96 755	96 802
93	96 848	96 895	96 942	96 988	97 035	97 081	97 128	97 174	97 220	97 267
94	97 313	97 359	97 405	97 451	97 497	97 543	97 589	97 635	97 681	97 727
95	97 772	97 818	97 864	97 909	97 955	98 000	98 046	98 091	98 137	98 182
96	98 227	98 272	98 318	98 363	98 408	98 453	98 498	98 543	98 588	98 632
97	98 677	98 722	98 767	98 811	98 856	98 900	98 945	98 989	99 034	99 078
98	99 123	99 167	99 211	99 255	99 300	99 344	99 388	99 432	99 476	99 520
99	99 564	99 607	99 651	99 695	99 739	99 782	99 826	99 870	99 913	99 957
<b>100</b>	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
N.	L. 0	1	2	3	4	5	6	7	8	9

TABLE I

100-150

N.	L.	o	r	2	3	4	5	6	7	8	9	Prop. Parts			
100	00	000	043	087	130	173	217	260	303	346	389				
101		432	475	518	561	604	647	689	732	775	817		44	43	42
102		860	903	945	988	*030	*072	*115	*157	*199	*242	1	4.4	4.3	4.2
103	01	284	326	368	410	452	494	536	578	620	662	2	8.8	8.6	8.4
104		703	745	787	828	870	912	953	995	*036	*078	3	13.2	12.9	12.6
105	02	119	160	202	243	284	325	366	407	449	490	4	17.6	17.2	16.8
106		531	572	612	653	694	735	776	816	857	898	5	22.0	21.5	21.0
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302	6	26.4	25.8	25.2
108	03	342	383	423	463	503	543	583	623	663	703	7	30.8	30.1	29.4
109		743	782	822	862	902	941	981	*021	*060	*100	8	35.2	34.4	33.6
110	04	139	179	218	258	297	336	376	415	454	493	9	39.6	38.7	37.8
111		532	571	610	650	689	727	766	805	844	883		41	40	39
112		922	961	999	*038	*077	*115	*154	*192	*231	*269	1	4.1	4.0	3.9
113	05	308	346	385	423	461	500	538	576	614	652	2	8.2	8.0	7.8
114		690	729	767	805	843	881	918	956	994	*032	3	12.3	12.0	11.7
115	06	070	108	145	183	221	258	296	333	371	408	4	16.4	16.0	15.6
116		446	483	521	558	595	633	670	707	744	781	5	20.5	20.0	19.5
117		819	856	893	930	967	*004	*041	*078	*115	*151	6	24.6	24.0	23.4
118	07	188	225	262	298	335	372	408	445	482	518	7	28.7	28.0	27.3
119		555	591	628	664	700	737	773	809	846	882	8	32.8	32.0	31.2
120		918	954	990	*027	*063	*099	*135	*171	*207	*243	9	36.9	36.0	35.1
121	08	279	314	350	386	422	458	493	529	565	600		38	37	36
122		636	672	707	743	778	814	849	884	920	955	1	3.8	3.7	3.6
123	09	*026	*061	*096	*132	*167	*202	*237	*272	*307	*342	2	7.6	7.4	7.2
124		342	377	412	447	482	517	552	587	621	656	3	11.4	11.1	10.8
125		691	726	760	795	830	864	899	934	968	*003	4	15.2	14.8	14.4
126	10	037	072	106	140	175	209	243	278	312	346	5	19.0	18.5	18.0
127		380	415	449	483	517	551	585	619	653	687	6	22.8	22.2	21.6
128		721	755	789	823	857	890	924	958	992	*025	7	26.6	25.9	25.2
129	11	059	093	126	160	193	227	261	294	327	361	8	30.4	29.6	28.8
130		394	428	461	494	528	561	594	628	661	694	9	34.2	33.3	32.4
131		727	760	793	826	860	893	926	959	992	*024		35	34	33
132	12	057	090	123	156	189	222	254	287	320	352	1	3.5	3.4	3.3
133		385	418	450	483	516	548	581	613	646	678	2	7.0	6.8	6.6
134		710	743	775	808	840	872	905	937	969	*001	3	10.5	10.2	9.9
135	13	033	066	098	130	162	194	226	258	290	322	4	14.0	13.6	13.2
136		354	386	418	450	481	513	545	577	609	640	5	17.5	17.0	16.5
137		672	704	735	767	799	830	862	893	925	956	6	21.0	20.4	19.8
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270	7	24.5	23.8	23.1
139	14	301	333	364	395	426	457	489	520	551	582	8	28.0	27.2	26.4
140		613	644	675	706	737	768	799	829	860	891	9	31.5	30.6	29.7
141		922	953	983	*014	*045	*076	*106	*137	*168	*198		32	31	30
142	15	229	259	290	320	351	381	412	442	473	503	1	3.2	3.1	3.0
143		534	564	594	625	655	685	715	746	776	806	2	6.4	6.2	6.0
144		836	866	897	927	957	987	*017	*047	*077	*107	3	9.6	9.3	9.0
145	16	137	167	197	227	256	286	316	346	376	406	4	12.8	12.4	12.0
146		435	465	495	524	554	584	613	643	673	702	5	16.0	15.5	15.0
147		732	761	791	820	850	879	909	938	967	997	6	19.2	18.6	18.0
148	17	026	056	085	114	143	173	202	231	260	289	7	22.4	21.7	21.0
149		319	348	377	406	435	464	493	522	551	580	8	25.6	24.8	24.0
150		609	638	667	696	725	754	782	811	840	869	9	28.8	27.9	27.0
N.	L.	o	r	2	3	4	5	6	7	8	9	Prop. Parts			

TABLE I

150-200

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts	
150	17	609	638	667	696	725	754	782	811	840	869		
151		898	926	955	984	*013	*041	*070	*099	*127	*156	29	28
152	18	184	213	241	270	298	327	355	384	412	441	1	2.9 2.8
153		469	498	526	554	583	611	639	667	696	724	2	5.8 5.6
154		752	780	808	837	865	893	921	949	977	*005	3	8.7 8.4
155	19	033	061	089	117	145	173	201	229	257	285	4	11.6 11.2
156		312	340	368	396	424	451	479	507	535	562	5	14.5 14.0
157		590	618	645	673	700	728	756	783	811	838	6	17.4 16.8
158		866	893	921	948	976	*003	*030	*058	*085	*112	7	20.3 19.6
159	20	140	167	194	222	249	276	303	330	358	385	8	23.2 22.4
160		412	439	466	493	520	548	575	602	629	656	9	26.1 25.2
161		683	710	737	763	790	817	844	871	898	925		27 26
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192	1	2.7 2.6
163	21	219	245	272	299	325	352	378	405	431	458	2	5.4 5.2
164		484	511	537	564	590	617	643	669	696	722	3	8.1 7.8
165		748	775	801	827	854	880	906	932	958	985	4	10.8 10.4
166	22	011	037	063	089	115	141	167	194	220	246	5	13.5 13.0
167		272	298	324	350	376	401	427	453	479	505	6	16.2 15.6
168		531	557	583	608	634	660	686	712	737	763	7	18.9 18.2
169		789	814	840	866	891	917	943	968	994	*019	8	21.6 20.8
170	23	045	070	096	121	147	172	198	223	249	274	9	24.3 23.4
171		300	325	350	376	401	426	452	477	502	528		25
172		553	578	603	629	654	679	704	729	754	779	1	2.5
173		805	830	855	880	905	930	955	980	*005	*030	2	5.0
174	24	055	080	105	130	155	180	204	229	254	279	3	7.5
175		304	329	353	378	403	428	452	477	502	527	4	10.0
176		551	576	601	625	650	674	699	724	748	773	5	12.5
177		797	822	846	871	895	920	944	969	993	*018	6	15.0
178	25	042	066	091	115	139	164	188	212	237	261	7	17.5
179		285	310	334	358	382	406	431	455	479	503	8	20.0
180		527	551	575	600	624	648	672	696	720	744	9	22.5
181		768	792	816	840	864	888	912	935	959	983		24 23
182	26	007	031	055	079	102	126	150	174	198	221	1	2.4 2.3
183		245	269	293	316	340	364	387	411	435	458	2	4.8 4.6
184		482	505	529	553	576	600	623	647	670	694	3	7.2 6.9
185		717	741	764	788	811	834	858	881	905	928	4	9.6 9.2
186		951	975	998	*021	*045	*068	*091	*114	*138	*161	5	12.0 11.5
187	27	184	207	231	254	277	300	323	346	370	393	6	14.4 13.8
188		416	439	462	485	508	531	554	577	600	623	7	16.8 16.1
189		646	669	692	715	738	761	784	807	830	852	8	19.2 18.4
190		875	898	921	944	967	989	*012	*035	*058	*081	9	21.6 20.7
191	28	103	126	149	171	194	217	240	262	285	307		22 21
192		330	353	375	398	421	443	466	488	511	533	1	2.2 2.1
193		556	578	601	623	646	668	691	713	735	758	2	4.4 4.2
194		780	803	825	847	870	892	914	937	959	981	3	6.6 6.3
195	29	003	026	048	070	092	115	137	159	181	203	4	8.8 8.4
196		226	248	270	292	314	336	358	380	403	425	5	11.0 10.5
197		447	469	491	513	535	557	579	601	623	645	6	13.2 12.6
198		667	688	710	732	754	776	798	820	842	863	7	15.4 14.7
199		885	907	929	951	973	994	*016	*038	*060	*081	8	17.6 16.8
200	30	103	125	146	168	190	211	233	255	276	298	9	19.8 18.9
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts	

TABLE I

200-250

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
200	30	103	125	146	168	190	211	233	255	276	298	22 21
201		320	341	363	384	406	428	449	471	492	514	1 2.2 2.1
202		535	557	578	600	621	643	664	685	707	728	2 4.4 4.2
203		750	771	792	814	835	856	878	899	920	942	3 6.6 6.3
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154	4 8.8 8.4
205	31	175	197	218	239	260	281	302	323	345	366	5 11.0 10.5
206		387	408	429	450	471	492	513	534	555	576	6 13.2 12.6
207		597	618	639	660	681	702	723	744	765	785	7 15.4 14.7
208		806	827	848	869	890	911	931	952	973	994	8 17.6 16.8
209	32	015	035	056	077	098	118	139	160	181	201	9 19.8 18.9
210		222	243	263	284	305	325	346	366	387	408	20
211		428	449	469	490	510	531	552	572	593	613	1 2.0
212		634	654	675	695	715	736	756	777	797	818	2 4.0
213		838	858	879	899	919	940	960	980	*001	*021	3 6.0
214	33	041	062	082	102	122	143	163	183	203	224	4 8.0
215		244	264	284	304	325	345	365	385	405	425	5 10.0
216		445	465	486	506	526	546	566	586	606	626	6 12.0
217		646	666	686	706	726	746	766	786	806	826	7 14.0
218		846	866	885	905	925	945	965	985	*005	*025	8 16.0
219	34	044	064	084	104	124	143	163	183	203	223	9 18.0
220		242	262	282	301	321	341	361	380	400	420	19
221		439	459	479	498	518	537	557	577	596	616	1 1.9
222		635	655	674	694	713	733	753	772	792	811	2 3.8
223		830	850	869	889	908	928	947	967	986	*005	3 5.7
224	35	025	044	064	083	102	122	141	160	180	199	4 7.6
225		218	238	257	276	295	315	334	353	372	392	5 9.5
226		411	430	449	468	488	507	526	545	564	583	6 11.4
227		603	622	641	660	679	698	717	736	755	774	7 13.3
228		793	813	832	851	870	889	908	927	946	965	8 15.2
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154	9 17.1
230	36	173	192	211	229	248	267	286	305	324	342	18
231		361	380	399	418	436	455	474	493	511	530	1 1.8
232		549	568	586	605	624	642	661	680	698	717	2 3.6
233		736	754	773	791	810	829	847	866	884	903	3 5.4
234		922	940	959	977	996	*014	*033	*051	*070	*088	4 7.2
235	37	107	125	144	162	181	199	218	236	254	273	5 9.0
236		291	310	328	346	365	383	401	420	438	457	6 10.8
237		475	493	511	530	548	566	585	603	621	639	7 12.6
238		658	676	694	712	731	749	767	785	803	822	8 14.4
239		840	858	876	894	912	931	949	967	985	*003	9 16.2
240	38	021	039	057	075	093	112	130	148	166	184	17
241		202	220	238	256	274	292	310	328	346	364	1 1.7
242		382	399	417	435	453	471	489	507	525	543	2 3.4
243		561	578	596	614	632	650	668	686	703	721	3 5.1
244		739	757	775	792	810	828	846	863	881	899	4 6.8
245		917	934	952	970	987	*005	*023	*041	*058	*076	5 8.5
246	39	094	111	129	146	164	182	199	217	235	252	6 10.2
247		270	287	305	322	340	358	375	393	410	428	7 11.9
248		445	463	480	498	515	533	550	568	585	602	8 13.6
249		620	637	655	672	690	707	724	742	759	777	9 15.3
250		794	811	829	846	863	881	898	915	933	950	
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

250-300

N.	L. o	1	2	3	4	5	6	7	8	9	Prop. Parts
250	39	794	811	829	846	863	881	898	915	930	18
251		967	985	*002	*019	*037	*054	*071	*088	*106	1
252	40	140	157	175	192	209	226	243	261	278	2
253		312	329	346	364	381	398	415	432	449	3
254		483	500	518	535	552	569	586	603	620	4
255		654	671	688	705	722	739	756	773	790	5
256		824	841	858	875	892	909	926	943	960	6
257		993	*010	*027	*044	*061	*078	*095	*111	*128	7
258	41	162	179	196	212	229	246	263	280	296	8
259		330	347	363	380	397	414	430	447	464	9
260		497	514	531	547	564	581	597	614	631	17
261		664	681	697	714	731	747	764	780	797	1
262		830	847	863	880	896	913	929	946	963	2
263		996	*012	*029	*045	*062	*078	*095	*111	*127	3
264	42	160	177	193	210	226	243	259	275	292	4
265		325	341	357	374	390	406	423	439	455	5
266		488	504	521	537	553	570	586	602	619	6
267		651	667	684	700	716	732	749	765	781	7
268		813	830	846	862	878	894	911	927	943	8
269		975	991	*008	*024	*040	*056	*072	*088	*104	9
270	43	136	152	169	185	201	217	233	249	265	log e = 0.43429
271		297	313	329	345	361	377	393	409	425	1
272		457	473	489	505	521	537	553	569	584	2
273		616	632	648	664	680	696	712	727	743	3
274		775	791	807	823	838	854	870	886	902	4
275		933	949	965	981	996	*012	*028	*044	*059	5
276	44	091	107	122	138	154	170	185	201	217	6
277		248	264	279	295	311	326	342	358	373	7
278		404	420	436	451	467	483	498	514	529	8
279		560	576	592	607	623	638	654	669	685	9
280		716	731	747	762	778	793	809	824	840	15
281		871	886	902	917	932	948	963	979	994	1
282	45	025	040	056	071	086	102	117	133	148	2
283		179	194	209	225	240	255	271	286	301	3
284		332	347	362	378	393	408	423	439	454	4
285		484	500	515	530	545	561	576	591	606	5
286		637	652	667	682	697	712	728	743	758	6
287		788	803	818	834	849	864	879	894	909	7
288		939	954	969	984	*000	*015	*030	*045	*060	8
289	46	090	105	120	135	150	165	180	195	210	9
290		240	255	270	285	300	315	330	345	359	14
291		389	404	419	434	449	464	479	494	509	1
292		538	553	568	583	598	613	627	642	657	2
293		687	702	716	731	746	761	776	790	805	3
294		835	850	864	879	894	909	923	938	953	4
295		982	997	*012	*026	*041	*056	*070	*085	*100	5
296	47	129	144	159	173	188	202	217	232	246	6
297		276	290	305	319	334	349	363	378	392	7
298		422	436	451	465	480	494	509	524	538	8
299		567	582	596	611	625	640	654	669	683	9
300		712	727	741	756	770	784	799	813	828	12.6
N.	L. o	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

300-350

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
300	47	712	727	741	756	770	784	799	813	828	842	
301		857	871	885	900	914	929	943	958	972	986	
302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	244	259	273	15
304		287	302	316	330	344	359	373	387	401	416	1 1.5
305		430	444	458	473	487	501	515	530	544	558	2 3.0
306		572	586	601	615	629	643	657	671	686	700	3 4.5
307		714	728	742	756	770	785	799	813	827	841	4 6.0
308		855	869	883	897	911	926	940	954	968	982	5 7.5
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9.0
310	49	136	150	164	178	192	206	220	234	248	262	7 10.5
311		276	290	304	318	332	346	360	374	388	402	8 12.0
312		415	429	443	457	471	485	499	513	527	541	9 13.5
313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	log $\pi = 0.49715$
315		831	845	859	872	886	900	914	927	941	955	14
316		969	982	996	*010	*024	*037	*051	*065	*079	*092	1 1.4
317	50	106	120	133	147	161	174	188	202	215	229	2 2.8
318		243	256	270	284	297	311	325	338	352	365	3 4.2
319		379	393	406	420	433	447	461	474	488	501	4 5.6
320		515	529	542	556	569	583	596	610	623	637	5 7.0
321		651	664	678	691	705	718	732	745	759	772	6 8.4
322		786	799	813	826	840	853	866	880	893	907	7 9.8
323		920	934	947	961	974	987	*001	*014	*028	*041	8 11.2
324	51	055	068	081	095	108	121	135	148	162	175	9 12.6
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	13
328		587	601	614	627	640	654	667	680	693	706	1 1.3
329		720	733	746	759	772	786	799	812	825	838	2 2.6
330		851	865	878	891	904	917	930	943	957	970	3 3.9
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101	4 5.2
332	52	114	127	140	153	166	179	192	205	218	231	5 6.5
333		244	257	270	284	297	310	323	336	349	362	6 7.8
334		375	388	401	414	427	440	453	466	479	492	7 9.1
335		504	517	530	543	556	569	582	595	608	621	8 10.4
336		634	647	660	673	686	699	711	724	737	750	9 11.7
337		763	776	789	802	815	827	840	853	866	879	
338		892	905	917	930	943	956	969	982	994	*007	
339	53	020	033	046	058	071	084	097	110	122	135	12
340		148	161	173	186	199	212	224	237	250	263	1 1.2
341		275	288	301	314	326	339	352	364	377	390	2 2.4
342		403	415	428	441	453	466	479	491	504	517	3 3.6
343		529	542	555	567	580	593	605	618	631	643	4 4.8
344		656	668	681	694	706	719	732	744	757	769	5 6.0
345		782	794	807	820	832	845	857	870	882	895	6 7.2
346		908	920	933	945	958	970	983	995	*008	*020	7 8.4
347	54	033	045	058	070	083	095	108	120	133	145	8 9.6
348		158	170	183	195	208	220	233	245	258	270	9 10.8
349		283	295	307	320	332	345	357	370	382	394	
350		407	419	432	444	456	469	481	494	506	518	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

350-400

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
350	54	407	419	432	444	456	469	481	494	506	518	
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354		900	913	925	937	949	962	974	986	998	*011	13
355	55	023	035	047	060	072	084	096	108	121	133	1
356		145	157	169	182	194	206	218	230	242	255	2
357		267	279	291	303	315	328	340	352	364	376	3
358		388	400	413	425	437	449	461	473	485	497	4
359		509	522	534	546	558	570	582	594	606	618	5
360		630	642	654	666	678	691	703	715	727	739	6
361		751	763	775	787	799	811	823	835	847	859	7
362		871	883	895	907	919	931	943	955	967	979	8
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9
364	56	110	122	134	146	158	170	182	194	205	217	10.4
365		229	241	253	265	277	289	301	312	324	336	11.7
366		348	360	372	384	396	407	419	431	443	455	12
367		467	478	490	502	514	526	538	549	561	573	1
368		585	597	608	620	632	644	656	667	679	691	2
369		703	714	726	738	750	761	773	785	797	808	3
370		820	832	844	855	867	879	891	902	914	926	4
371		937	949	961	972	984	996	*008	*019	*031	*043	5
372	57	054	066	078	089	101	113	*124	136	148	159	6
373		171	183	194	206	217	229	241	252	264	276	7
374		287	299	310	322	334	345	357	368	380	392	8
375		403	415	426	438	449	461	473	484	496	507	9
376		519	530	542	553	565	576	588	600	611	623	10.8
377		634	646	657	669	680	692	703	715	726	738	11
378		749	761	772	784	795	807	818	830	841	852	1
379		864	875	887	898	910	921	933	944	955	967	2
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081	3
381	58	092	104	115	127	138	149	161	172	184	195	4
382		206	218	229	240	252	263	274	286	297	309	5
383		320	331	343	354	365	377	388	399	410	422	6
384		433	444	456	467	478	490	501	512	524	535	7
385		546	557	569	580	591	602	614	625	636	647	8
386		659	670	681	692	704	715	726	737	749	760	9
387		771	782	794	805	816	827	838	850	861	872	
388		883	894	906	917	928	939	950	961	973	984	
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095	10
390	59	106	118	129	140	151	162	173	184	195	207	1
391		218	229	240	251	262	273	284	295	306	318	2
392		329	340	351	362	373	384	395	406	417	428	3
393		439	450	461	472	483	494	506	517	528	539	4
394		550	561	572	583	594	605	616	627	638	649	5
395		660	671	682	693	704	715	726	737	748	759	6
396		770	780	791	802	813	824	835	846	857	868	7
397		879	890	901	912	923	934	945	956	966	977	8
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086	9
399	60	097	108	119	130	141	152	163	173	184	195	10.0
400		206	217	228	239	249	260	271	282	293	304	9.0
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

400-450

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
400	60	206	217	228	239	249	260	271	282	293	304	
401		314	325	336	347	358	369	379	390	401	412	
402		423	433	444	455	466	477	487	498	509	520	
403		531	541	552	563	574	584	595	606	617	627	
404		638	649	660	670	681	692	703	713	724	735	
405		746	756	767	778	788	799	810	821	831	842	
406		853	863	874	885	895	906	917	927	938	949	
407		959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61	066	077	087	098	109	119	130	140	151	162	
409		172	183	194	204	215	225	236	247	257	268	
410		278	289	300	310	321	331	342	352	363	374	
411		384	395	405	416	426	437	448	458	469	479	
412		490	500	511	521	532	542	553	563	574	584	
413		595	606	616	627	637	648	658	669	679	690	
414		700	711	721	731	742	752	763	773	784	794	
415		805	815	826	836	847	857	868	878	888	899	
416		909	920	930	941	951	962	972	982	993	*003	
417	62	014	024	034	045	055	066	076	086	097	107	
418		118	128	138	149	159	170	180	190	201	211	
419		221	232	242	252	263	273	284	294	304	315	
420		325	335	346	356	366	377	387	397	408	418	
421		428	439	449	459	469	480	490	500	511	521	
422		531	542	552	562	572	583	593	603	613	624	
423		634	644	655	665	675	685	696	706	716	726	
424		737	747	757	767	778	788	798	808	818	829	
425		839	849	859	870	880	890	900	910	921	931	
426		941	951	961	972	982	992	*002	*012	*022	*033	
427	63	043	053	063	073	083	094	104	114	124	134	
428		144	155	165	175	185	195	205	215	225	236	
429		246	256	266	276	286	296	306	317	327	337	
430		347	357	367	377	387	397	407	417	428	438	
431		448	458	468	478	488	498	508	518	528	538	
432		548	558	568	579	589	599	609	619	629	639	
433		649	659	669	679	689	699	709	719	729	739	
434		749	759	769	779	789	799	809	819	829	839	
435		849	859	869	879	889	899	909	919	929	939	
436		949	959	969	979	988	998	*008	*018	*028	*038	
437	64	048	058	068	078	088	098	108	118	128	137	
438		147	157	167	177	187	197	207	217	227	237	
439		246	256	266	276	286	296	306	316	326	335	
440		345	355	365	375	385	395	404	414	424	434	
441		444	454	464	473	483	493	503	513	523	532	
442		542	552	562	572	582	591	601	611	621	631	
443		640	650	660	670	680	689	699	709	719	729	
444		738	748	758	768	777	787	797	807	816	826	
445		836	846	856	865	875	885	895	904	914	924	
446		933	943	953	963	972	982	992	*002	*011	*021	
447	65	031	040	050	060	070	079	089	099	108	118	
448		128	137	147	157	167	176	186	196	205	215	
449		225	234	244	254	263	273	283	292	302	312	
450		321	331	341	350	360	369	379	389	398	408	
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts



TABLE I

450-500

N.	L.	o	r	2	3	4	5	6	7	8	9	Prop. Parts
450	65	321	331	341	350	360	369	379	389	398	408	
451		418	427	437	447	456	466	475	485	495	504	
452		514	523	533	543	552	562	571	581	591	600	
453		610	619	629	639	648	658	667	677	686	696	
454		706	715	725	734	744	753	763	772	782	792	
455		801	811	820	830	839	849	858	868	877	887	10
456		896	906	916	925	935	944	954	963	973	982	1
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077	2
458	66	087	096	106	115	124	134	143	153	162	172	3
459		181	191	200	210	219	229	238	247	257	266	4
460		276	285	295	304	314	323	332	342	351	361	5
461		370	380	389	398	408	417	427	436	445	455	6
462		464	474	483	492	502	511	521	530	539	549	7
463		558	567	577	586	596	605	614	624	633	642	8
464		652	661	671	680	689	699	708	717	727	736	9
465		745	755	764	773	783	792	801	811	820	829	
466		839	848	857	867	876	885	894	904	913	922	
467		932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	089	099	108	
469		117	127	136	145	154	164	173	182	191	201	
470		210	219	228	237	247	256	265	274	284	293	9
471		302	311	321	330	339	348	357	367	376	385	1
472		394	403	413	422	431	440	449	459	468	477	2
473		486	495	504	514	523	532	541	550	560	569	3
474		578	587	596	605	614	624	633	642	651	660	4
475		669	679	688	697	706	715	724	733	742	752	5
476		761	770	779	788	797	806	815	825	834	843	6
477		852	861	870	879	888	897	906	916	925	934	7
478		943	952	961	970	979	988	997	*006	*015	*024	8
479	68	034	043	052	061	070	079	088	097	106	115	9
480		124	133	142	151	160	169	178	187	196	205	
481		215	224	233	242	251	260	269	278	287	296	
482		305	314	323	332	341	350	359	368	377	386	
483		395	404	413	422	431	440	449	458	467	476	
484		485	494	502	511	520	529	538	547	556	565	
485		574	583	592	601	610	619	628	637	646	655	
486		664	673	681	690	699	708	717	726	735	744	8
487		753	762	771	780	789	797	806	815	824	833	1
488		842	851	860	869	878	886	895	904	913	922	2
489		931	940	949	958	966	975	984	993	*002	*011	3
490	69	020	028	037	046	055	064	073	082	090	099	4
491		108	117	126	135	144	152	161	170	179	188	5
492		197	205	214	223	232	241	249	258	267	276	6
493		285	294	302	311	320	329	338	346	355	364	7
494		373	381	390	399	408	417	425	434	443	452	8
495		461	469	478	487	496	504	513	522	531	539	9
496		548	557	566	574	583	592	601	609	618	627	
497		636	644	653	662	671	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	801	
499		810	819	827	836	845	854	862	871	880	888	
500		897	906	914	923	932	940	949	958	966	975	
N.	L.	o	r	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

500-550

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
500	69	897	906	914	923	932	940	949	958	966	975	
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	9
506		415	424	432	441	449	458	467	475	484	492	1 0.9
507		501	509	518	526	535	544	552	561	569	578	2 1.8
508		586	595	603	612	621	629	638	646	655	663	3 2.7
509		672	680	689	697	706	714	723	731	740	749	4 3.6
510		757	766	774	783	791	800	808	817	825	834	5 4.5
511		842	851	859	868	876	885	893	902	910	919	6 5.4
512		927	935	944	952	961	969	978	986	995	*003	7 6.3
513	71	012	020	029	037	046	054	063	071	079	088	8 7.2
514		096	105	113	122	130	139	147	155	164	172	9 8.1
515		181	189	198	206	214	223	231	240	248	257	
516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
518		433	441	450	458	466	475	483	492	500	508	
519		517	525	533	542	550	559	567	575	584	592	
520		600	609	617	625	634	642	650	659	667	675	8
521		684	692	700	709	717	725	734	742	750	759	1 0.8
522		767	775	784	792	800	809	817	825	834	842	2 1.6
523		850	858	867	875	883	892	900	908	917	925	3 2.4
524		933	941	950	958	966	975	983	991	999	*008	4 3.2
525	72	016	024	032	041	049	057	066	074	082	090	5 4.0
526		099	107	115	123	132	140	148	156	165	173	6 4.8
527		181	189	198	206	214	222	230	239	247	255	7 5.6
528		263	272	280	288	296	304	313	321	329	337	8 6.4
529		346	354	362	370	378	387	395	403	411	419	9 7.2
530		428	436	444	452	460	469	477	485	493	501	
531		509	518	526	534	542	550	558	567	575	583	
532		591	599	607	616	624	632	640	648	656	665	
533		673	681	689	697	705	713	722	730	738	746	
534		754	762	770	779	787	795	803	811	819	827	
535		835	843	852	860	868	876	884	892	900	908	
536		916	925	933	941	949	957	965	973	981	989	7
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070	1 0.7
538	73	078	086	094	102	111	119	127	135	143	151	2 1.4
539		159	167	175	183	191	199	207	215	223	231	3 2.1
540		239	247	255	263	272	280	288	296	304	312	4 2.8
541		320	328	336	344	352	360	368	376	384	392	5 3.5
542		400	408	416	424	432	440	448	456	464	472	6 4.2
543		480	488	496	504	512	520	528	536	544	552	7 4.9
544		560	568	576	584	592	600	608	616	624	632	8 5.6
545		640	648	656	664	672	679	687	695	703	711	9 6.3
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	933	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	
550	74	036	044	052	060	068	076	084	092	099	107	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

550-600

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
550	74	036	044	052	060	068	076	084	092	099	107	
551		115	123	131	139	147	155	162	170	178	186	
552		194	202	210	218	225	233	241	249	257	265	
553		273	280	288	296	304	312	320	327	335	343	
554		351	359	367	374	382	390	398	406	414	421	
555		429	437	445	453	461	468	476	484	492	500	
556		507	515	523	531	539	547	554	562	570	578	
557		586	593	601	609	617	624	632	640	648	656	
558		663	671	679	687	695	702	710	718	726	733	
559		741	749	757	764	772	780	788	796	803	811	
560		819	827	834	842	850	858	865	873	881	889	
561		896	904	912	920	927	935	943	950	958	966	
562		974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75	051	059	066	074	082	089	097	105	113	120	
564		128	136	143	151	159	166	174	182	189	197	
565		205	213	220	228	236	243	251	259	266	274	
566		282	289	297	305	312	320	328	335	343	351	
567		358	366	374	381	389	397	404	412	420	427	
568		435	442	450	458	465	473	481	488	496	504	
569		511	519	526	534	542	549	557	565	572	580	
570		587	595	603	610	618	626	633	641	648	656	
571		664	671	679	686	694	702	709	717	724	732	
572		740	747	755	762	770	778	785	793	800	808	
573		815	823	831	838	846	853	861	868	876	884	
574		891	899	906	914	921	929	937	944	952	959	
575		967	974	982	989	997	*005	*012	*020	*027	*035	
576	76	042	050	057	065	072	080	087	095	103	110	
577		118	125	133	140	148	155	163	170	178	185	
578		193	200	208	215	223	230	238	245	253	260	
579		268	275	283	290	298	305	313	320	328	335	
580		343	350	358	365	373	380	388	395	403	410	
581		418	425	433	440	448	455	462	470	477	485	
582		492	500	507	515	522	530	537	545	552	559	
583		567	574	582	589	597	604	612	619	626	634	
584		641	649	656	664	671	678	686	693	701	708	
585		716	723	730	738	745	753	760	768	775	782	
586		790	797	805	812	819	827	834	842	849	856	
587		864	871	879	886	893	901	908	916	923	930	
588		938	945	953	960	967	975	982	989	997	*004	
589	77	012	019	026	034	041	048	056	063	070	078	
590		085	093	100	107	115	122	129	137	144	151	
591		159	166	173	181	188	195	203	210	217	225	
592		232	240	247	254	262	269	276	283	291	298	
593		305	313	320	327	335	342	349	357	364	371	
594		379	386	393	401	408	415	422	430	437	444	
595		452	459	466	474	481	488	495	503	510	517	
596		525	532	539	546	554	561	568	576	583	590	
597		597	605	612	619	627	634	641	648	656	663	
598		670	677	685	692	699	706	714	721	728	735	
599		743	750	757	764	772	779	786	793	801	808	
600		815	822	830	837	844	851	859	866	873	880	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

8  
0.8  
1.6  
2.4  
3.2  
4.0  
4.8  
5.6  
6.4  
7.2

7  
0.7  
1.4  
2.1  
2.8  
3.5  
4.2  
4.9  
5.6  
6.3

TABLE I

600-650

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
600	77	815	822	830	837	844	851	859	866	873	880	
601		887	895	902	909	916	924	931	938	945	952	
602		960	967	974	981	988	996	*003	*010	*017	*025	
603	78	032	039	046	053	061	068	075	082	089	097	
604		104	111	118	125	132	140	147	154	161	168	
605		176	183	190	197	204	211	219	226	233	240	8
606		247	254	262	269	276	283	290	297	305	312	1 0.8
607		319	326	333	340	347	355	362	369	376	383	2 1.6
608		390	398	405	412	419	426	433	440	447	455	3 2.4
609		462	469	476	483	490	497	504	512	519	526	4 3.2
610		533	540	547	554	561	569	576	583	590	597	5 4.0
611		604	611	618	625	633	640	647	654	661	668	6 4.8
612		675	682	689	696	704	711	718	725	732	739	7 5.6
613		746	753	760	767	774	781	789	796	803	810	8 6.4
614		817	824	831	838	845	852	859	866	873	880	9 7.2
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	7
621		309	316	323	330	337	344	351	358	365	372	1 0.7
622		379	386	393	400	407	414	421	428	435	442	2 1.4
623		449	456	463	470	477	484	491	498	505	511	3 2.1
624		518	525	532	539	546	553	560	567	574	581	4 2.8
625		588	595	602	609	616	623	630	637	644	650	5 3.5
626		657	664	671	678	685	692	699	706	713	720	6 4.2
627		727	734	741	748	754	761	768	775	782	789	7 4.9
628		796	803	810	817	824	831	837	844	851	858	8 5.6
629		865	872	879	886	893	900	906	913	920	927	9 6.3
630		934	941	948	955	962	969	975	982	989	996	
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	085	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	6
637		414	421	428	434	441	448	455	462	468	475	1 0.6
638		482	489	496	502	509	516	523	530	536	543	2 1.2
639		550	557	564	570	577	584	591	598	604	611	3 1.8
640		618	625	632	638	645	652	659	665	672	679	4 2.4
641		686	693	699	706	713	720	726	733	740	747	5 3.0
642		754	760	767	774	781	787	794	801	808	814	6 3.6
643		821	828	835	841	848	855	862	868	875	882	7 4.2
644		889	895	902	909	916	922	929	936	943	949	8 4.8
645		956	963	969	976	983	990	996	*003	*010	*017	9 5.4
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	238	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

650-700

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
650	81	291	298	305	311	318	325	331	338	345	351	<div>7</div> <div>1 0.7</div> <div>2 1.4</div> <div>3 2.1</div> <div>4 2.8</div> <div>5 3.5</div> <div>6 4.2</div> <div>7 4.9</div> <div>8 5.6</div> <div>9 6.3</div>
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	<div>6</div> <div>1 0.6</div> <div>2 1.2</div> <div>3 1.8</div> <div>4 2.4</div> <div>5 3.0</div> <div>6 3.6</div> <div>7 4.2</div> <div>8 4.8</div> <div>9 5.4</div>
661	82	020	027	033	040	046	053	060	066	073	079	
662		086	092	099	105	112	119	125	132	138	145	
663		151	158	164	171	178	184	191	197	204	210	
664		217	223	230	236	243	249	256	263	269	276	
665		282	289	295	302	308	315	321	328	334	341	
666		347	354	360	367	373	380	387	393	400	406	
667		413	419	426	432	439	445	452	458	465	471	
668		478	484	491	497	504	510	517	523	530	536	
669		543	549	556	562	569	575	582	588	595	601	
670		607	614	620	627	633	640	646	653	659	666	<div>6</div> <div>1 0.6</div> <div>2 1.2</div> <div>3 1.8</div> <div>4 2.4</div> <div>5 3.0</div> <div>6 3.6</div> <div>7 4.2</div> <div>8 4.8</div> <div>9 5.4</div>
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	<div>6</div> <div>1 0.6</div> <div>2 1.2</div> <div>3 1.8</div> <div>4 2.4</div> <div>5 3.0</div> <div>6 3.6</div> <div>7 4.2</div> <div>8 4.8</div> <div>9 5.4</div>
681		315	321	327	334	340	347	353	359	366	372	
682		378	385	391	398	404	410	417	423	429	436	
683		442	448	455	461	467	474	480	487	493	499	
684		506	512	518	525	531	537	544	550	556	563	
685		569	575	582	588	594	601	607	613	620	626	
686		632	639	645	651	658	664	670	677	683	689	
687		696	702	708	715	721	727	734	740	746	753	
688		759	765	771	778	784	790	797	803	809	816	
689		822	828	835	841	847	853	860	866	872	879	
690		885	891	897	904	910	916	923	929	935	942	<div>6</div> <div>1 0.6</div> <div>2 1.2</div> <div>3 1.8</div> <div>4 2.4</div> <div>5 3.0</div> <div>6 3.6</div> <div>7 4.2</div> <div>8 4.8</div> <div>9 5.4</div>
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

700-750

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	1 0.7
709		065	071	077	083	089	095	101	107	114	120	2 1.4
710		126	132	138	144	150	156	163	169	175	181	3 2.1
711		187	193	199	205	211	217	224	230	236	242	4 2.8
712		248	254	260	266	272	278	285	291	297	303	5 3.5
713		309	315	321	327	333	339	345	352	358	364	6 4.2
714		370	376	382	388	394	400	406	412	418	425	7 4.9
715		431	437	443	449	455	461	467	473	479	485	8 5.6
716		491	497	503	509	516	522	528	534	540	546	9 6.3
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	1 0.6
723		914	920	926	932	938	944	950	956	962	968	2 1.2
724		974	980	986	992	998	*004	*010	*016	*022	*028	3 1.8
725	86	034	040	046	052	058	064	070	076	082	088	4 2.4
726		094	100	106	112	118	124	130	136	141	147	5 3.0
727		153	159	165	171	177	183	189	195	201	207	6 3.6
728		213	219	225	231	237	243	249	255	261	267	7 4.2
729		273	279	285	291	297	303	308	314	320	326	8 4.8
730		332	338	344	350	356	362	368	374	380	386	9 5.4
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	1 0.5
739		864	870	876	882	888	894	900	906	911	917	2 1.0
740		923	929	935	941	947	953	958	964	970	976	3 1.5
741		982	988	994	999	*005	*011	*017	*023	*029	*035	4 2.0
742	87	040	046	052	058	064	070	075	081	087	093	5 2.5
743		099	105	111	116	122	128	134	140	146	151	6 3.0
744		157	163	169	175	181	186	192	198	204	210	7 3.5
745		216	221	227	233	239	245	251	256	262	268	8 4.0
746		274	280	286	291	297	303	309	315	320	326	9 4.5
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

750-800

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
750	87	506	512	518	523	529	535	541	547	552	558	
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		679	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	058	064	070	076	
760		081	087	093	098	104	110	116	121	127	133	
761		138	144	150	156	161	167	173	178	184	190	6
762		195	201	207	213	218	224	230	235	241	247	1 0.6
763		252	258	264	270	275	281	287	292	298	304	2 1.2
764		309	315	321	326	332	338	343	349	355	360	3 1.8
765		366	372	377	383	389	395	400	406	412	417	4 2.4
766		423	429	434	440	446	451	457	463	468	474	5 3.0
767		480	485	491	497	502	508	513	519	525	530	6 3.6
768		536	542	547	553	559	564	570	576	581	587	7 4.2
769		593	598	604	610	615	621	627	632	638	643	8 4.8
770		649	655	660	666	672	677	683	689	694	700	9 5.4
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	
781		265	271	276	282	287	293	298	304	310	315	5
782		321	326	332	337	343	348	354	360	365	371	1 0.5
783		376	382	387	393	398	404	409	415	421	426	2 1.0
784		432	437	443	448	454	459	465	470	476	481	3 1.5
785		487	492	498	504	509	515	520	526	531	537	4 2.0
786		542	548	553	559	564	570	575	581	586	592	5 2.5
787		597	603	609	614	620	625	631	636	642	647	6 3.0
788		653	658	664	669	675	680	686	691	697	702	7 3.5
789		708	713	719	724	730	735	741	746	752	757	8 4.0
790		763	768	774	779	785	790	796	801	807	812	9 4.5
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	
N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

800-850

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
800	90	309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	
811		902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	6
813	91	009	014	020	025	030	036	041	046	052	057	1 0.6
814		062	068	073	078	084	089	094	100	105	110	2 1.2
815		116	121	126	132	137	142	148	153	158	164	3 1.8
816		169	174	180	185	190	196	201	206	212	217	4 2.4
817		222	228	233	238	243	249	254	259	265	270	5 3.0
818		275	281	286	291	297	302	307	312	318	323	6 3.6
819		328	334	339	344	350	355	360	365	371	376	7 4.2
820		381	387	392	397	403	408	413	418	424	429	8 4.8
821		434	440	445	450	455	461	466	471	477	482	9 5.4
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	
831		960	965	971	976	981	986	991	997	*002	*007	5
832	92	012	018	023	028	033	038	044	049	054	059	1 0.5
833		065	070	075	080	085	091	096	101	106	111	2 1.0
834		117	122	127	132	137	143	148	153	158	163	3 1.5
835		169	174	179	184	189	195	200	205	210	215	4 2.0
836		221	226	231	236	241	247	252	257	262	267	5 2.5
837		273	278	283	288	293	298	304	309	314	319	6 3.0
838		324	330	335	340	345	350	355	361	366	371	7 3.5
839		376	381	387	392	397	402	407	412	418	423	8 4.0
840		428	433	438	443	449	454	459	464	469	474	9 4.5
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts



TABLE I

850-900

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
850	92	942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	6
856		247	252	258	263	268	273	278	283	288	293	1 0.6
857		298	303	308	313	318	323	328	334	339	344	2 1.2
858		349	354	359	364	369	374	379	384	389	394	3 1.8
859		399	404	409	414	420	425	430	435	440	445	4 2.4
860		450	455	460	465	470	475	480	485	490	495	5 3.0
861		500	505	510	515	520	526	531	536	541	546	6 3.6
862		551	556	561	566	571	576	581	586	591	596	7 4.2
863		601	606	611	616	621	626	631	636	641	646	8 4.8
864		651	656	661	666	671	676	682	687	692	697	9 5.4
865		702	707	712	717	722	727	732	737	742	747	
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	5
871	94	002	007	012	017	022	027	032	037	042	047	1 0.5
872		052	057	062	067	072	077	082	086	091	096	2 1.0
873		101	106	111	116	121	126	131	136	141	146	3 1.5
874		151	156	161	166	171	176	181	186	191	196	4 2.0
875		201	206	211	216	221	226	231	236	240	245	5 2.5
876		250	255	260	265	270	275	280	285	290	295	6 3.0
877		300	305	310	315	320	325	330	335	340	345	7 3.5
878		349	354	359	364	369	374	379	384	389	394	8 4.0
879		399	404	409	414	419	424	429	433	438	443	9 4.5
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	4
887		792	797	802	807	812	817	822	827	832	836	1 0.4
888		841	846	851	856	861	866	871	876	880	885	2 0.8
889		890	895	900	905	910	915	919	924	929	934	3 1.2
890		939	944	949	954	959	963	968	973	978	983	4 1.6
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	5 2.0
892	95	036	041	046	051	056	061	066	071	075	080	6 2.4
893		085	090	095	100	105	109	114	119	124	129	7 2.8
894		134	139	143	148	153	158	163	168	173	177	8 3.2
895		182	187	192	197	202	207	211	216	221	226	9 3.6
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

900-950

N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts
900	95	424	429	434	439	444	448	453	458	463	468	<div>5</div> <div>1 0.5</div> <div>2 1.0</div> <div>3 1.5</div> <div>4 2.0</div> <div>5 2.5</div> <div>6 3.0</div> <div>7 3.5</div> <div>8 4.0</div> <div>9 4.5</div>
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	<div>4</div> <div>1 0.4</div> <div>2 0.8</div> <div>3 1.2</div> <div>4 1.6</div> <div>5 2.0</div> <div>6 2.4</div> <div>7 2.8</div> <div>8 3.2</div> <div>9 3.6</div>
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96	047	052	057	061	066	071	076	080	085	090	
914		095	099	104	109	114	118	123	128	133	137	
915		142	147	152	156	161	166	171	175	180	185	
916		190	194	199	204	209	213	218	223	227	232	
917		237	242	246	251	256	261	265	270	275	280	
918		284	289	294	298	303	308	313	317	322	327	
919		332	336	341	346	350	355	360	365	369	374	
920		379	384	388	393	398	402	407	412	417	421	<div>4</div> <div>1 0.4</div> <div>2 0.8</div> <div>3 1.2</div> <div>4 1.6</div> <div>5 2.0</div> <div>6 2.4</div> <div>7 2.8</div> <div>8 3.2</div> <div>9 3.6</div>
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	<div>4</div> <div>1 0.4</div> <div>2 0.8</div> <div>3 1.2</div> <div>4 1.6</div> <div>5 2.0</div> <div>6 2.4</div> <div>7 2.8</div> <div>8 3.2</div> <div>9 3.6</div>
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97	035	039	044	049	053	058	063	067	072	077	
935		081	086	090	095	100	104	109	114	118	123	
936		128	132	137	142	146	151	155	160	165	169	
937		174	179	183	188	192	197	202	206	211	216	
938		220	225	230	234	239	243	248	253	257	262	
939		267	271	276	280	285	290	294	299	304	308	
940		313	317	322	327	331	336	340	345	350	354	<div>4</div> <div>1 0.4</div> <div>2 0.8</div> <div>3 1.2</div> <div>4 1.6</div> <div>5 2.0</div> <div>6 2.4</div> <div>7 2.8</div> <div>8 3.2</div> <div>9 3.6</div>
941		359	364	368	373	377	382	387	391	396	400	
942		405	410	414	419	424	428	433	437	442	447	
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	749	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	
N.	L.	0	1	2	3	4	5	6	7	8	9	Prop. Parts

TABLE I

950-1000

N.	L.	o	1	2	3	4	5	6	7	8	9	Prop. Parts
950	97	772	777	782	786	791	795	800	804	809	813	<div>5</div> <div>0.5</div> <div>1.0</div> <div>1.5</div> <div>2.0</div> <div>2.5</div> <div>3.0</div> <div>3.5</div> <div>4.0</div> <div>4.5</div>
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	<div>4</div> <div>0.4</div> <div>0.8</div> <div>1.2</div> <div>1.6</div> <div>2.0</div> <div>2.4</div> <div>2.8</div> <div>3.2</div> <div>3.6</div>
961		272	277	281	286	290	295	299	304	308	313	
962		318	322	327	331	336	340	345	349	354	358	
963		363	367	372	376	381	385	390	394	399	403	
964		408	412	417	421	426	430	435	439	444	448	
965		453	457	462	466	471	475	480	484	489	493	
966		498	502	507	511	516	520	525	529	534	538	
967		543	547	552	556	561	565	570	574	579	583	
968		588	592	597	601	605	610	614	619	623	628	
969		632	637	641	646	650	655	659	664	668	673	
970		677	682	686	691	695	700	704	709	713	717	<div>4</div> <div>0.4</div> <div>0.8</div> <div>1.2</div> <div>1.6</div> <div>2.0</div> <div>2.4</div> <div>2.8</div> <div>3.2</div> <div>3.6</div>
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	<div>4</div> <div>0.4</div> <div>0.8</div> <div>1.2</div> <div>1.6</div> <div>2.0</div> <div>2.4</div> <div>2.8</div> <div>3.2</div> <div>3.6</div>
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	
986		388	392	396	401	405	410	414	419	423	427	
987		432	436	441	445	449	454	458	463	467	471	
988		476	480	484	489	493	498	502	506	511	515	
989		520	524	528	533	537	542	546	550	555	559	
990		564	568	572	577	581	585	590	594	599	603	<div>4</div> <div>0.4</div> <div>0.8</div> <div>1.2</div> <div>1.6</div> <div>2.0</div> <div>2.4</div> <div>2.8</div> <div>3.2</div> <div>3.6</div>
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	039	Prop. Parts



TABLE II  
LOGARITHMS OF TRIGONOMETRIC FUNCTIONS



"	'	<i>l</i> sin	log <i>S</i>	<i>l</i> csc	<i>l</i> tan	log <i>T</i>	<i>l</i> cot	<i>l</i> sec	<i>l</i> cos	'
0	<b>0</b>	Inf. neg.	—	Infinit.	Inf. neg.	—	Infinit.	10.00000	10.00000	<b>60</b>
60	1	6.46373	5.31443	13.53627	6.46373	5.31443	13.53627	00000	00000	59
120	2	76476	5.31443	23524	76476	5.31443	23524	00000	00000	58
180	3	94085	5.31443	05915	94085	5.31443	05915	00000	00000	57
240	4	7.06579	5.31443	12.93421	7.06579	5.31442	12.93421	00000	00000	56
300	<b>5</b>	7.16270	5.31443	12.83730	7.16270	5.31442	12.83730	10.00000	10.00000	<b>55</b>
360	6	24188	5.31443	75812	24188	5.31442	75812	00000	00000	54
420	7	30882	5.31443	69118	30882	5.31442	69118	00000	00000	53
480	8	36682	5.31443	63318	36682	5.31442	63318	00000	00000	52
540	9	41797	5.31443	58203	41797	5.31442	58203	00000	00000	51
600	<b>10</b>	7.46373	5.31443	12.53627	7.46373	5.31442	12.53627	10.00000	10.00000	<b>50</b>
660	11	50512	5.31443	49488	50512	5.31442	49488	00000	00000	49
720	12	54291	5.31443	45709	54291	5.31442	45709	00000	00000	48
780	13	57767	5.31443	42233	57767	5.31442	42233	00000	00000	47
840	14	60985	5.31443	39015	60986	5.31442	39014	00000	00000	46
900	<b>15</b>	7.63982	5.31443	12.36018	7.63982	5.31442	12.36018	10.00000	10.00000	<b>45</b>
960	16	66784	5.31443	33216	66785	5.31442	33215	00000	00000	44
1020	17	69417	5.31443	30583	69418	5.31442	30582	00001	9.99999	43
1080	18	71900	5.31443	28100	71900	5.31442	28100	00001	99999	42
1140	19	74248	5.31443	25752	74248	5.31442	25752	00001	99999	41
1200	<b>20</b>	7.76475	5.31443	12.23525	7.76476	5.31442	12.23524	10.00001	9.99999	<b>40</b>
1260	21	78594	5.31443	21406	78595	5.31442	21405	00001	99999	39
1320	22	80615	5.31443	19385	80615	5.31442	19385	00001	99999	38
1380	23	82545	5.31443	17455	82546	5.31442	17454	00001	99999	37
1440	24	84393	5.31443	15607	84394	5.31442	15606	00001	99999	36
1500	<b>25</b>	7.86166	5.31443	12.13834	7.86167	5.31442	12.13833	10.00001	9.99999	<b>35</b>
1560	26	87870	5.31443	12130	87871	5.31442	12129	00001	99999	34
1620	27	89509	5.31443	10491	89510	5.31442	10490	00001	99999	33
1680	28	91088	5.31443	08912	91089	5.31442	08911	00001	99999	32
1740	29	92612	5.31443	07388	92613	5.31441	07387	00002	99999	31
1800	<b>30</b>	7.94084	5.31443	12.05916	7.94086	5.31441	12.05914	10.00002	9.99998	<b>30</b>
1860	31	95508	5.31443	04492	95510	5.31441	04490	00002	99998	29
1920	32	96887	5.31443	03113	96889	5.31441	03111	00002	99998	28
1980	33	98223	5.31443	01777	98225	5.31441	01775	00002	99998	27
2040	34	99520	5.31443	00480	99522	5.31441	00478	00002	99998	26
2100	<b>35</b>	8.00779	5.31443	11.99221	8.00781	5.31441	11.99219	10.00002	9.99998	<b>25</b>
2160	36	02002	5.31443	97998	02004	5.31441	97996	00002	99998	24
2220	37	03192	5.31443	96808	03194	5.31441	96806	00003	99997	23
2280	38	04350	5.31443	95650	04353	5.31441	95647	00003	99997	22
2340	39	05478	5.31443	94522	05481	5.31441	94519	00003	99997	21
2400	<b>40</b>	8.06578	5.31443	11.93422	8.06581	5.31441	11.93419	10.00003	9.99997	<b>20</b>
2460	41	07650	5.31444	92350	07653	5.31440	92347	00003	99997	19
2520	42	08696	5.31444	91304	08700	5.31440	91300	00003	99997	18
2580	43	09718	5.31444	90282	09722	5.31440	90278	00003	99997	17
2640	44	10717	5.31444	89283	10720	5.31440	89280	00004	99996	16
2700	<b>45</b>	8.11693	5.31444	11.88307	8.11696	5.31440	11.88304	10.00004	9.99996	<b>15</b>
2760	46	12647	5.31444	87353	12651	5.31440	87349	00004	99996	14
2820	47	13581	5.31444	86419	13585	5.31440	86415	00004	99996	13
2880	48	14495	5.31444	85505	14500	5.31440	85500	00004	99996	12
2940	49	15391	5.31444	84609	15395	5.31440	84605	00004	99996	11
3000	<b>50</b>	8.16268	5.31444	11.83732	8.16273	5.31439	11.83727	10.00005	9.99995	<b>10</b>
3060	51	17128	5.31444	82872	17133	5.31439	82867	00005	99995	9
3120	52	17971	5.31444	82029	17976	5.31439	82024	00005	99995	8
3180	53	18798	5.31444	81202	18804	5.31439	81196	00005	99995	7
3240	54	19610	5.31444	80390	19616	5.31439	80384	00005	99995	6
3300	<b>55</b>	8.20407	5.31444	11.79593	8.20413	5.31439	11.79587	10.00006	9.99994	<b>5</b>
3360	56	21189	5.31444	78811	21195	5.31439	78805	00006	99994	4
3420	57	21958	5.31445	78042	21964	5.31439	78036	00006	99994	3
3480	58	22713	5.31445	77287	22720	5.31438	77280	00006	99994	2
3540	59	23456	5.31445	76544	23462	5.31438	76538	00006	99994	1
3600	<b>60</b>	24186	5.31445	75814	24192	5.31438	75808	00007	99993	<b>0</b>
	'	<i>l</i> cos		<i>l</i> sec	<i>l</i> cot		<i>l</i> tan	<i>l</i> csc	<i>l</i> sin	'

1°

TABLE II

178°

"	'	<i>l</i> sin	log <i>S</i>	<i>l</i> csc	<i>l</i> tan	log <i>T</i>	<i>l</i> cot	<i>l</i> sec	<i>l</i> cos	'
3600	0	8.24186	5.31445	11.75814	8.24192	5.31438	11.75808	10.00007	9.99993	60
3660	1	24903	5.31445	75097	24910	5.31438	75090	00007	9.99993	59
3720	2	25609	5.31445	74391	25616	5.31438	74384	00007	9.99993	58
3780	3	26304	5.31445	73696	26312	5.31438	73688	00007	9.99993	57
3840	4	26988	5.31445	73012	26996	5.31437	73004	00008	9.99992	56
3900	5	8.27661	5.31445	11.72339	8.27669	5.31437	11.72331	10.00008	9.99992	55
3960	6	28324	5.31445	71676	28332	5.31437	71668	00008	9.99992	54
4020	7	28977	5.31445	71023	28986	5.31437	71014	00008	9.99992	53
4080	8	29621	5.31445	70379	29629	5.31437	70371	00008	9.99992	52
4140	9	30255	5.31445	69745	30263	5.31437	69737	00009	9.99991	51
4200	10	8.30879	5.31446	11.69121	8.30888	5.31437	11.69112	10.00009	9.99991	50
4260	11	31495	5.31446	68505	31505	5.31436	68495	00009	9.99991	49
4320	12	32103	5.31446	67897	32112	5.31436	67888	00010	9.99990	48
4380	13	32702	5.31446	67298	32711	5.31436	67289	00010	9.99990	47
4440	14	33292	5.31446	66708	33302	5.31436	66698	00010	9.99990	46
4500	15	8.33875	5.31446	11.66125	8.33886	5.31436	11.66114	10.00010	9.99990	45
4560	16	34450	5.31446	65550	34461	5.31435	65539	00011	9.99989	44
4620	17	35018	5.31446	64982	35029	5.31435	64971	00011	9.99989	43
4680	18	35578	5.31446	64422	35590	5.31435	64410	00011	9.99989	42
4740	19	36131	5.31446	63869	36143	5.31435	63857	00011	9.99989	41
4800	20	8.36678	5.31446	11.63322	8.36689	5.31435	11.63311	10.00012	9.99988	40
4860	21	37217	5.31447	62783	37229	5.31434	62771	00012	9.99988	39
4920	22	37750	5.31447	62250	37762	5.31434	62238	00012	9.99988	38
4980	23	38276	5.31447	61724	38289	5.31434	61711	00013	9.99987	37
5040	24	38796	5.31447	61204	38809	5.31434	61191	00013	9.99987	36
5100	25	8.39310	5.31447	11.60690	8.39323	5.31434	11.60677	10.00013	9.99987	35
5160	26	39818	5.31447	60182	39832	5.31433	60168	00014	9.99986	34
5220	27	40320	5.31447	59680	40334	5.31433	59666	00014	9.99986	33
5280	28	40816	5.31447	59184	40830	5.31433	59170	00014	9.99986	32
5340	29	41307	5.31447	58693	41321	5.31433	58679	00015	9.99985	31
5400	30	8.41792	5.31447	11.58208	8.41807	5.31433	11.58193	10.00015	9.99985	30
5460	31	42272	5.31448	57728	42287	5.31432	57713	00015	9.99985	29
5520	32	42746	5.31448	57254	42762	5.31432	57238	00016	9.99984	28
5580	33	43216	5.31448	56784	43232	5.31432	56768	00016	9.99984	27
5640	34	43680	5.31448	56320	43696	5.31432	56304	00016	9.99984	26
5700	35	8.44139	5.31448	11.55861	8.44156	5.31431	11.55844	10.00017	9.99983	25
5760	36	44594	5.31448	55406	44611	5.31431	55389	00017	9.99983	24
5820	37	45044	5.31448	54956	45061	5.31431	54939	00017	9.99983	23
5880	38	45489	5.31448	54511	45507	5.31431	54493	00018	9.99982	22
5940	39	45930	5.31449	54070	45948	5.31431	54052	00018	9.99982	21
6000	40	8.46366	5.31449	11.53634	8.46385	5.31430	11.53615	10.00018	9.99982	20
6060	41	46799	5.31449	53201	46817	5.31430	53183	00019	9.99981	19
6120	42	47226	5.31449	52774	47245	5.31430	52755	00019	9.99981	18
6180	43	47650	5.31449	52350	47669	5.31430	52331	00019	9.99981	17
6240	44	48069	5.31449	51931	48089	5.31429	51911	00020	9.99980	16
6300	45	8.48485	5.31449	11.51515	8.48505	5.31429	11.51495	10.00020	9.99980	15
6360	46	48896	5.31449	51104	48917	5.31429	51083	00021	9.99979	14
6420	47	49304	5.31450	50696	49325	5.31428	50675	00021	9.99979	13
6480	48	49708	5.31450	50292	49729	5.31428	50271	00021	9.99979	12
6540	49	50108	5.31450	49892	50130	5.31428	49870	00022	9.99978	11
6600	50	8.50504	5.31450	11.49496	8.50527	5.31428	11.49473	10.00022	9.99978	10
6660	51	50897	5.31450	49103	50920	5.31427	49080	00023	9.99977	9
6720	52	51287	5.31450	48713	51310	5.31427	48690	00023	9.99977	8
6780	53	51673	5.31450	48327	51696	5.31427	48304	00023	9.99977	7
6840	54	52055	5.31450	47945	52079	5.31427	47921	00024	9.99976	6
6900	55	8.52434	5.31451	11.47566	8.52459	5.31426	11.47541	10.00024	9.99976	5
6960	56	52810	5.31451	47190	52835	5.31426	47165	00025	9.99975	4
7020	57	53183	5.31451	46817	53208	5.31426	46792	00025	9.99975	3
7080	58	53552	5.31451	46448	53578	5.31425	46422	00026	9.99974	2
7140	59	53919	5.31451	46081	53945	5.31425	46055	00026	9.99974	1
7200	60	54282	5.31451	45718	54308	5.31425	45692	00026	9.99974	0
'		<i>l</i> cos		<i>l</i> sec	<i>l</i> cot		<i>l</i> tan	<i>l</i> csc		'

91°

88°



"	'	$l \sin$	$\log S$	$l \csc$	$l \tan$	$\log T$	$l \cot$	$l \sec$	d 1'	$l \cos$	'
7200	0	8.54282	5.31451	11.45718	8.54308	5.31425	11.45692	10.00026	1	9.99974	60
7260	1	54642	5.31451	45358	54669	5.31425	45331	00027	1	99973	59
7320	2	54999	5.31452	45001	55027	5.31424	44973	00027	1	99973	58
7380	3	55354	5.31452	44646	55382	5.31424	44618	00028	1	99972	57
7440	4	55705	5.31452	44295	55734	5.31424	44266	00028	1	99972	56
7500	5	8.56054	5.31452	11.43946	8.56083	5.31423	11.43917	10.00029	9	99971	55
7560	6	56400	5.31452	43600	56429	5.31423	43571	00029	1	99971	54
7620	7	56743	5.31452	43257	56773	5.31423	43227	00030	1	99970	53
7680	8	57084	5.31453	42916	57114	5.31422	42886	00030	1	99970	52
7740	9	57421	5.31453	42579	57452	5.31422	42548	00031	1	99969	51
7800	10	8.57757	5.31453	11.42243	8.57788	5.31422	11.42212	10.00031	9	99969	50
7860	11	58089	5.31453	41911	58121	5.31421	41879	00032	1	99968	49
7920	12	58419	5.31453	41581	58451	5.31421	41549	00032	1	99968	48
7980	13	58747	5.31453	41253	58779	5.31421	41221	00033	1	99967	47
8040	14	59072	5.31454	40928	59105	5.31421	40895	00033	1	99967	46
8100	15	8.59395	5.31454	11.40605	8.59428	5.31420	11.40572	10.00033	9	99967	45
8160	16	59715	5.31454	40285	59749	5.31420	40251	00034	1	99966	44
8220	17	60033	5.31454	39967	60068	5.31420	39932	00034	1	99966	43
8280	18	60349	5.31454	39651	60384	5.31419	39616	00035	1	99965	42
8340	19	60662	5.31454	39338	60698	5.31419	39302	00036	1	99964	41
8400	20	8.60973	5.31455	11.39027	8.61009	5.31418	11.38991	10.00036	9	99964	40
8460	21	61282	5.31455	38718	61319	5.31418	38681	00037	1	99963	39
8520	22	61589	5.31455	38411	61626	5.31418	38374	00037	1	99963	38
8580	23	61894	5.31455	38106	61931	5.31417	38069	00038	1	99962	37
8640	24	62196	5.31455	37804	62234	5.31417	37766	00038	1	99962	36
8700	25	8.62497	5.31455	11.37503	8.62535	5.31417	11.37465	10.00039	9	99961	35
8760	26	62795	5.31456	37205	62834	5.31416	37166	00039	1	99961	34
8820	27	63091	5.31456	36909	63131	5.31416	36869	00040	1	99960	33
8880	28	63385	5.31456	36615	63426	5.31416	36574	00040	1	99960	32
8940	29	63678	5.31456	36322	63718	5.31415	36282	00041	1	99959	31
9000	30	8.63968	5.31456	11.36032	8.64009	5.31415	11.35991	10.00041	9	99959	30
9060	31	64256	5.31456	35744	64298	5.31415	35702	00042	1	99958	29
9120	32	64543	5.31457	35457	64585	5.31414	35415	00042	1	99958	28
9180	33	64827	5.31457	35173	64870	5.31414	35130	00043	1	99957	27
9240	34	65110	5.31457	34890	65154	5.31413	34846	00044	1	99956	26
9300	35	8.65391	5.31457	11.34609	8.65435	5.31413	11.34565	10.00044	9	99956	25
9360	36	65670	5.31457	34330	65715	5.31413	34285	00045	1	99955	24
9420	37	65947	5.31458	34053	65993	5.31412	34007	00045	1	99955	23
9480	38	66223	5.31458	33777	66269	5.31412	33731	00046	1	99954	22
9540	39	66497	5.31458	33503	66543	5.31412	33457	00046	1	99954	21
9600	40	8.66769	5.31458	11.33231	8.66816	5.31411	11.33184	10.00047	9	99953	20
9660	41	67039	5.31458	32961	67087	5.31411	32913	00048	1	99952	19
9720	42	67308	5.31459	32692	67356	5.31410	32644	00048	1	99952	18
9780	43	67575	5.31459	32425	67624	5.31410	32376	00049	1	99951	17
9840	44	67841	5.31459	32159	67890	5.31410	32110	00049	1	99951	16
9900	45	8.68104	5.31459	11.31896	8.68154	5.31409	11.31846	10.00050	9	99950	15
9960	46	68367	5.31459	31633	68417	5.31409	31583	00051	1	99949	14
10020	47	68627	5.31460	31373	68678	5.31408	31322	00051	1	99949	13
10080	48	68886	5.31460	31114	68938	5.31408	31062	00052	1	99948	12
10140	49	69144	5.31460	30856	69196	5.31408	30804	00052	1	99948	11
10200	50	8.69400	5.31460	11.30600	8.69453	5.31407	11.30547	10.00053	9	99947	10
10260	51	69654	5.31460	30346	69708	5.31407	30292	00054	1	99946	9
10320	52	69907	5.31461	30093	69962	5.31406	30038	00054	1	99946	8
10380	53	70159	5.31461	29841	70214	5.31406	29786	00055	1	99945	7
10440	54	70409	5.31461	29591	70465	5.31405	29535	00056	1	99944	6
10500	55	8.70658	5.31461	11.29342	8.70714	5.31405	11.29286	10.00056	9	99944	5
10560	56	70905	5.31461	29095	70962	5.31405	29038	00057	1	99943	4
10620	57	71151	5.31462	28849	71208	5.31404	28792	00058	1	99942	3
10680	58	71395	5.31462	28605	71453	5.31404	28547	00058	1	99942	2
10740	59	71638	5.31462	28362	71697	5.31403	28303	00059	1	99941	1
10800	60	71880	5.31462	28120	71940	5.31403	28060	00060	1	99940	0
'		$l \cos$		$l \sec$	$l \cot$		$l \tan$	$l \csc$	d 1'	$l \sin$	'

3°

TABLE II

176°

°	l sin		d	l csc		l tan	d	l cot		l sec	d	l cos		°	Proportional Parts							
	8.	1'		11.	1'	8.	1'	11.	10.	1'		9.	1'		241	239	237	235	234	232	229	
0	71880	240		28120		71940	241	28060	00060			99940	60	0	0	0	0	0	0	0	0	
1	72120	239		27880		72181	239	27819	060			940	59	1	4	4	4	4	4	4	4	
2	359	238		641		420	239	580	061			939	58	2	8	8	8	8	8	8	8	
3	597	237		403		659	237	341	062			938	57	3	12	12	12	12	12	12	11	
4	834	235		166		896	236	104	062			938	56	4	16	16	16	16	16	15	15	
5	73069	234		26931		73132	234	26868	063			937	55	5	20	20	20	20	19	19	19	
6	303	232		697		366	234	634	064			936	54	6	24	24	24	24	23	23	23	
7	535	232		465		600	232	400	064			936	53	7	28	28	28	27	27	27	27	
8	767	232		233		832	232	168	065			935	52	8	32	32	32	31	31	31	31	
9	997	230		003		74063	231	25937	066			934	51	9	36	36	36	35	35	35	34	
10	74226	229		25774		292	229	708	066			934	50	10	40	40	40	39	39	39	38	
11	454	228		546		521	229	479	067			933	49	11	44	44	43	43	43	43	42	
12	680	226		320		748	226	252	068			932	48	12	48	48	47	47	47	46	46	
13	906	226		094		974	226	026	068			932	47	13	52	52	51	51	51	50	50	
14	75130	224		24870		75199	225	24801	069			931	46	14	56	56	55	55	55	54	53	
15	353	222		647		423	222	577	070			930	45	15	60	60	59	59	59	58	57	
16	575	220		425		645	222	355	071			929	44	16	64	64	63	63	62	62	61	
17	795	220		205		867	220	133	071			929	43	17	68	68	67	67	66	66	65	
18	76015	219		23985		76087	219	23913	072			928	42	18	72	72	71	70	70	70	69	
19	234	217		766		306	219	694	073			927	41	19	76	76	75	74	74	73	73	
20	451	216		549		525	217	475	074			926	40	20	80	80	79	78	78	77	76	
21	667	216		333		742	216	258	074			926	39	21	84	84	83	82	82	81	80	
22	883	214		117		958	215	042	075			925	38	22	88	88	87	86	86	85	84	
23	77097	213		22903		77173	214	22827	076			924	37	23	92	92	91	90	90	89	88	
24	310	212		690		387	214	613	077			923	36	24	96	96	95	94	94	93	92	
25	522	211		478		600	211	400	077			923	35	25	100	100	99	98	97	97	95	
26	733	210		267		811	211	189	078			922	34	26	104	104	103	102	101	101	99	
27	943	209		057		78022	210	21978	079			921	33	27	108	108	107	106	105	104	103	
28	78152	208		21848		232	210	768	080			920	32	28	112	112	111	110	109	108	107	
29	360	208		640		441	209	559	080			920	31	29	116	116	115	114	113	112	111	
30	78568	206		21432		78649	206	21351	00081			99919	30	30	120	120	118	118	117	116	114	
31	774	205		226		855	206	145	082			918	29	31	125	123	122	121	121	120	118	
32	979	204		021		79061	205	20939	083			917	28	32	129	127	126	125	125	124	122	
33	79183	203		20817		266	204	734	083			917	27	33	133	131	130	129	129	128	126	
34	386	202		614		470	203	530	084			916	26	34	137	135	134	133	133	131	130	
35	588	201		412		673	202	327	085			915	25	35	141	139	138	137	137	135	134	
36	789	201		211		875	201	125	086			914	24	36	145	143	142	141	140	139	137	
37	990	199		010		80076	201	19924	087			913	23	37	149	147	146	145	144	143	141	
38	80189	199		19811		277	199	723	087			913	22	38	153	151	150	149	148	147	145	
39	388	197		612		476	198	524	088			912	21	39	157	155	154	153	152	151	149	
40	585	197		415		674	198	326	089			911	20	40	161	159	158	157	156	155	153	
41	782	196		218		872	198	128	090			910	19	41	165	163	162	161	160	159	156	
42	978	195		022		81068	196	18932	091			909	18	42	169	167	166	164	164	162	160	
43	81173	194		18827		264	195	736	091			909	17	43	173	171	170	168	168	166	164	
44	367	193		633		459	194	541	092			908	16	44	177	175	174	172	172	170	168	
45	560	192		440		653	193	347	093			907	15	45	181	179	178	176	175	174	172	
46	752	192		248		846	192	154	094			906	14	46	185	183	182	180	179	178	176	
47	944	190		056		82038	192	17962	095			905	13	47	189	187	186	184	183	182	179	
48	82134	190		17866		230	190	770	096			904	12	48	193	191	190	188	187	186	183	
49	324	189		676		420	190	580	096			904	11	49	197	195	194	192	191	189	187	
50	513	188		487		610	189	390	097			903	10	50	201	199	198	196	195	193	191	
51	701	187		299		799	188	201	098			902	9	51	205	203	201	200	199	197	195	
52	888	187		112		987	188	013	099			901	8	52	209	207	205	204	203	201	198	
53	83075	186		16925		83175	186	16825	100			900	7	53	213	211	209	208	207	205	202	
54	261	185		739		361	186	639	101			899	6	54	217	215	213	212	211	209	206	
55	446	184		554		547	185	453	102			898	5	55	221	219	217	215	215	213	210	
56	630	183		370		732	184	268	102			898	4	56	225	223	221	219	218	217	214	
57	813	183		187		916	184	084	103			897	3	57	229	227	225	223	222	220	218	
58	996	181		004		84100	182	15900	104			896	2	58	233	231	229	227	226	224	221	
59	84177	181		15823		282	182	718	105			895	1	59	237	235	233	231	230	228	225	
60	84358	181		15642		84464	182	15536	00106			99894	0	60	241	239	237	235	234	232	229	
	8.	1'		11.		8.	1'	11.	10.			9.			241	239	237	235	234	232	229	
	l cos	l		l sec		l cot	d	l tan	l csc	d		l sin										

93°

86°

TABLE II

"	Proportional Parts																			
	227	225	223	220	217	215	213	211	208	206	203	201	199	197	195	193	192	189	187	185
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	4	4	4	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3
2	8	8	7	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6	6
3	11	11	11	11	11	11	11	11	11	10	10	10	10	10	10	10	10	9	9	9
4	15	15	15	15	14	14	14	14	14	14	14	13	13	13	13	13	13	12	12	12
5	19	19	19	18	18	18	18	18	17	17	17	17	17	17	16	16	16	16	15	15
6	23	22	22	22	22	22	21	21	21	21	20	20	20	20	20	20	19	19	18	18
7	26	26	26	26	25	25	25	25	24	24	24	23	23	23	23	23	22	22	22	21
8	30	30	30	29	29	29	29	28	28	28	27	27	27	27	26	26	26	25	25	24
9	34	34	33	33	33	32	32	32	31	31	30	30	30	30	29	29	29	28	28	27
10	38	38	37	37	36	36	36	35	35	34	34	34	33	33	32	32	32	31	31	30
11	42	41	41	40	40	39	39	39	38	38	37	37	36	36	35	35	35	34	34	33
12	45	45	45	44	43	43	43	42	42	41	41	40	40	39	39	39	38	38	37	37
13	49	49	48	48	47	47	46	46	45	45	44	44	43	43	42	42	41	41	40	39
14	53	52	52	51	51	50	50	49	49	48	47	47	46	46	45	45	44	44	43	42
15	57	56	56	55	54	53	53	52	51	51	50	50	49	49	48	48	47	47	46	45
16	61	60	59	59	58	57	57	56	55	55	54	53	53	52	51	51	50	50	49	48
17	64	64	63	62	61	61	60	60	59	58	58	57	56	56	55	55	54	54	53	52
18	68	68	67	66	65	64	64	63	62	62	61	60	60	59	58	58	57	56	56	55
19	72	71	71	70	69	68	67	67	66	65	64	63	62	62	61	61	60	59	59	58
20	76	75	74	73	72	72	71	70	69	68	67	66	66	65	64	64	63	62	62	61
21	79	79	78	77	76	75	74	73	72	71	70	70	69	68	67	66	66	65	65	64
22	83	82	82	81	80	79	78	77	76	76	74	74	73	72	72	71	70	69	69	68
23	87	86	85	84	83	82	82	81	80	79	78	77	76	76	75	74	74	72	71	70
24	91	90	89	88	87	86	85	84	83	82	81	80	80	79	78	77	76	75	74	73
25	95	94	93	92	90	89	88	87	86	85	84	83	82	81	80	80	79	78	77	76
26	98	98	97	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79
27	102	101	100	99	98	97	96	95	94	93	91	90	90	89	88	87	86	85	84	83
28	106	105	104	103	101	100	99	98	97	96	95	94	93	92	91	90	88	87	86	85
29	110	109	108	106	105	104	103	102	101	100	98	97	96	95	94	93	93	91	90	89
30	114	112	112	110	108	108	106	106	104	103	102	100	100	98	98	96	96	94	94	92
31	117	116	115	114	112	111	110	109	107	106	105	104	103	102	101	100	99	98	97	96
32	121	120	119	117	116	115	114	113	111	110	108	107	106	105	104	103	102	101	100	99
33	125	124	123	121	119	118	117	116	114	113	112	111	109	108	107	106	104	103	102	101
34	129	128	126	125	123	122	121	120	118	117	115	114	113	112	110	109	109	107	106	105
35	132	131	130	128	127	125	124	123	121	120	118	117	116	115	114	113	112	110	109	108
36	136	135	134	132	130	129	128	127	125	124	122	121	119	118	117	116	115	113	112	111
37	140	139	138	136	134	133	131	130	128	127	125	124	123	121	120	119	118	117	115	114
38	144	142	141	139	137	136	135	134	132	130	129	127	126	125	124	122	122	120	118	117
39	148	146	145	143	141	140	138	137	135	134	132	131	129	128	127	125	125	123	122	120
40	151	150	149	147	145	143	142	141	139	137	135	134	133	131	130	129	128	126	125	123
41	155	154	152	150	148	147	146	144	142	141	139	137	136	135	133	132	131	129	128	126
42	159	158	156	154	152	150	149	148	146	144	142	141	139	138	136	135	134	132	131	130
43	163	161	160	158	156	154	153	151	149	148	145	144	143	141	140	138	138	135	134	133
44	166	165	164	161	159	158	156	155	153	151	149	147	146	144	143	142	141	139	137	136
45	170	169	167	165	163	161	160	158	156	155	152	151	149	148	146	145	144	142	140	139
46	174	172	171	169	166	165	163	162	159	158	156	154	153	151	150	148	147	145	143	142
47	178	176	175	172	170	168	167	165	163	161	159	157	156	154	153	151	150	148	146	145
48	182	180	178	176	174	172	170	169	166	165	162	161	159	158	156	154	151	150	148	146
49	185	184	182	180	177	176	174	172	170	168	166	164	163	161	159	158	157	154	153	151
50	189	188	186	183	181	179	178	176	173	172	169	168	166	164	162	161	160	158	156	154
51	193	191	190	187	184	183	181	179	177	175	173	171	169	167	166	164	163	161	159	157
52	197	195	193	191	188	186	185	183	180	179	176	174	172	171	169	167	166	164	162	160
53	201	199	197	194	192	190	188	186	184	182	179	178	176	174	172	170	167	165	163	161
54	204	202	201	198	195	194	192	190	187	185	183	181	179	177	176	174	173	170	168	166
55	208	206	204	202	199	197	195	193	191	189	186	184	182	181	179	177	176	173	171	170
56	212	210	208	205	203	201	199	197	194	192	189	188	186	184	182	180	179	176	175	173
57	216	214	212	209	206	204	202	200	198	196	193	191	189	187	185	183	182	180	178	177
58	219	218	216	213	210	208	206	204	201	199	196	194	192	190	188	187	186	183	181	179
59	223	221	219	216	213	211	209	207	205	203	200	198	196	194	192	190	189	186	184	182
60	227	225	223	220	217	215	213	211	208	206	203	201	199	197	195	193	192	189	187	185
"	227	225	223	220	217	215	213	211	208	206	203	201	199	197	195	193	192	189	187	185

Proportional Parts

4°

TABLE II

175°

										Proportional Parts								
										182	181	179	177	176	175	174		
"	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	"								
0	8.	1'	11.	8.	1'	11.	10.	1'	9.	0	0	0	0	0	0	0	0	
1	84358	181	15642	84464	182	15536	00106	1	99894	60								
2	539	179	461	646	180	354	107	2	89359									
3	718	179	282	826	180	174	108	3	89258									
4	897	177	103	85006	178	14994	109	4	89157									
5	85075	177	14925	185	178	815	109	5	89156									
6	252	177	748	363	177	637	110	6	89055									
7	429	176	571	540	177	460	111	7	88954									
8	605	175	395	717	176	283	112	8	88853									
9	780	175	220	893	176	107	113	9	88752									
10	955	173	045	86069	174	13931	114	10	88651									
11	86128	173	13872	243	174	757	115	11	88550									
12	301	173	699	417	174	583	116	12	88449									
13	474	171	526	591	172	409	117	13	88348									
14	645	171	355	763	172	237	118	14	88247									
15	816	171	184	935	171	065	119	15	88146									
16	987	169	013	87106	171	12894	120	16	88045									
17	87156	169	12844	277	170	723	121	17	87944									
18	325	169	675	447	169	553	121	18	87943									
19	494	169	506	616	169	384	122	19	87842									
20	661	168	339	785	168	215	123	20	87741									
21	829	166	171	953	167	047	124	21	87640									
22	995	166	005	88120	167	11880	125	22	87539									
23	88161	165	11839	287	166	713	126	23	87438									
24	326	164	674	453	166	547	127	24	87337									
25	490	164	510	618	165	382	128	25	87236									
26	654	163	346	783	165	217	129	26	87135									
27	817	163	183	948	163	052	130	27	87034									
28	980	162	020	89111	163	10889	131	28	86933									
29	89142	162	10858	274	163	726	132	29	86832									
30	304	160	696	437	161	563	133	30	86731									
31	89464	161	10536	89598	162	10402	00134	31	99866	30								
32	625	159	375	760	160	240	135	32	86529									
33	784	159	216	920	160	080	136	33	86428									
34	943	159	057	90080	160	09920	137	34	86327									
35	90102	158	09898	240	159	760	138	35	86226									
36	260	157	740	399	158	601	139	36	86125									
37	417	157	583	557	158	443	140	37	86024									
38	574	156	426	715	157	285	141	38	85923									
39	730	155	270	872	157	128	142	39	85822									
40	885	155	115	91029	156	08971	143	40	85721									
41	91040	155	08960	185	155	815	144	41	85620									
42	195	154	805	340	155	660	145	42	85519									
43	349	154	651	495	155	505	146	43	85418									
44	502	153	498	650	155	350	147	44	85317									
45	655	152	345	803	154	197	148	45	85216									
46	807	152	193	957	153	043	149	46	85115									
47	959	151	041	92110	152	07890	150	47	85014									
48	2110	151	07890	262	152	738	152	48	84813									
49	261	150	739	414	151	586	153	49	84712									
50	411	150	589	565	151	435	154	50	84611									
51	561	149	439	716	150	284	155	51	84510									
52	710	149	290	866	150	134	156	52	8449									
53	859	148	141	93016	149	06984	157	53	8438									
54	93007	147	06993	165	148	835	158	54	8427									
55	154	147	846	313	149	687	159	55	8416									
56	301	147	699	462	147	538	160	56	8405									
57	448	146	552	609	147	391	161	57	8394									
58	594	146	406	756	147	244	162	58	8383									
59	740	145	260	903	146	097	163	59	8372									
60	885	145	115	94049	146	05951	164	60	8361									
61	94030	145	05970	94195	145	05805	00166	61	99834	0								
8.	d	11.	8.	d	11.	10.	9.	d	l sin	"	182	181	179	177	176	175	174	
l cos	1'	l sec	l cot	1'	l tan	l csc	l sin	1'	"	"	Proportional Parts							

94°

85°

TABLE II

"	Proportional Parts																					
	173	172	171	169	167	166	165	163	162	160	159	158	157	155	153	152	151	150	149	147	146	145
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	2	2	2
2	6	6	6	6	6	6	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
3	9	9	9	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	7	7	7	7
4	12	11	11	11	11	11	11	11	11	11	11	11	10	10	10	10	10	10	10	10	10	10
5	14	14	14	14	14	14	14	14	13	13	13	13	13	13	13	13	13	12	12	12	12	12
6	17	17	17	17	17	17	16	16	16	16	16	16	16	16	15	15	15	15	15	15	15	14
7	20	20	20	20	19	19	19	19	19	19	19	18	18	18	18	18	18	18	17	17	17	17
8	23	23	23	23	22	22	22	22	22	21	21	21	21	21	20	20	20	20	20	20	19	19
9	26	26	26	25	25	25	25	24	24	24	24	24	24	23	23	23	23	22	22	22	22	22
10	29	29	28	28	28	28	28	27	27	27	26	26	26	26	26	25	25	25	25	24	24	24
11	32	32	31	31	31	30	30	30	30	29	29	29	29	28	28	28	28	28	27	27	27	27
12	35	34	34	34	33	33	33	33	32	32	32	32	32	31	31	31	30	30	30	29	29	29
13	37	37	37	37	36	36	36	35	35	35	34	34	34	34	33	33	33	32	32	32	32	31
14	40	40	40	39	39	39	38	38	38	37	37	37	37	36	36	35	35	35	35	34	34	34
15	43	43	43	42	42	42	41	41	40	40	40	39	39	38	38	38	38	38	37	37	36	36
16	46	46	46	45	45	44	44	43	43	43	42	42	42	41	41	41	40	40	40	39	39	39
17	49	49	48	48	47	47	47	46	46	45	45	45	44	44	43	43	43	42	42	42	41	41
18	52	52	51	51	50	50	50	49	49	48	48	47	47	46	46	46	45	45	45	44	44	44
19	55	54	54	54	53	53	52	52	51	51	50	50	49	48	48	48	48	48	47	47	46	46
20	58	57	57	56	56	55	55	54	54	53	53	53	52	52	51	51	50	50	50	49	49	48
21	61	60	60	59	58	58	58	57	57	56	56	55	55	54	54	53	53	52	52	51	51	51
22	63	63	63	62	61	61	60	60	59	59	58	58	58	57	56	56	55	55	55	54	54	53
23	66	66	66	65	64	64	63	62	62	61	61	61	60	59	59	58	58	58	57	56	56	56
24	69	69	68	68	67	66	66	65	65	64	64	63	63	62	61	61	60	60	60	59	58	58
25	72	72	71	70	70	69	69	68	68	67	66	66	65	65	64	63	63	62	62	61	61	60
26	75	75	74	73	72	72	72	71	70	69	69	68	68	67	66	66	65	65	64	63	63	63
27	78	77	77	76	75	75	74	73	73	72	72	71	71	70	69	68	68	67	66	66	65	65
28	81	80	80	79	78	77	77	76	76	75	74	74	73	72	71	71	70	70	69	68	68	68
29	84	83	83	82	81	80	80	79	78	77	76	76	75	74	73	73	72	72	71	71	70	70
30	86	86	86	84	84	83	82	82	81	80	80	79	78	78	76	76	75	74	74	73	72	72
31	89	89	88	87	86	86	85	84	84	83	82	82	81	80	79	79	78	78	77	76	75	75
32	92	92	91	90	89	89	88	87	86	85	85	84	84	83	82	81	81	80	79	78	78	77
33	95	95	94	93	92	91	91	90	89	88	87	87	86	85	84	84	83	82	82	81	80	80
34	98	97	97	96	95	94	94	92	92	91	90	90	89	88	87	86	86	85	84	83	83	82
35	101	100	100	99	97	97	96	95	94	93	93	92	92	90	89	89	88	88	87	86	85	85
36	104	103	103	101	100	100	99	98	97	96	95	95	94	93	92	91	91	90	89	88	88	87
37	107	106	105	104	103	102	102	101	100	99	98	97	97	96	94	94	93	92	92	91	90	89
38	110	109	108	107	106	105	104	103	103	101	101	100	99	98	97	96	96	95	94	93	92	92
39	112	112	111	110	109	108	107	106	105	104	103	103	102	101	99	99	98	98	97	96	95	94
40	115	115	114	113	111	111	110	109	108	107	106	105	105	103	102	101	101	100	99	98	97	97
41	118	118	117	115	114	113	113	111	111	109	109	108	107	106	105	104	103	102	102	100	100	99
42	121	120	120	118	117	116	116	114	113	112	111	111	110	108	107	106	106	105	104	103	102	102
43	124	123	123	121	120	119	118	117	116	115	114	113	113	111	110	109	108	108	107	105	105	104
44	127	126	125	124	122	122	121	120	119	117	117	116	115	114	112	111	111	110	109	108	107	106
45	130	129	128	127	125	124	124	122	122	120	119	118	118	116	115	114	113	112	112	110	110	109
46	133	132	131	130	128	127	126	125	124	123	122	121	120	119	117	117	116	115	114	113	112	111
47	136	135	134	132	131	130	129	128	127	125	125	124	123	121	122	119	118	118	117	115	114	114
48	138	138	137	135	134	133	132	130	130	128	127	126	126	124	122	122	121	120	119	118	117	116
49	141	140	140	138	136	136	135	133	132	131	130	129	128	127	125	124	123	122	122	120	119	118
50	144	143	142	141	139	138	138	136	135	133	132	132	131	129	128	127	126	125	124	122	122	121
51	147	146	145	144	142	141	140	139	138	136	135	134	133	132	130	129	128	128	127	125	124	123
52	150	149	148	146	145	144	143	141	140	139	138	137	136	134	133	132	131	130	129	127	127	126
53	153	152	151	149	148	147	146	144	143	141	140	140	139	137	136	135	134	133	132	130	129	128
54	156	155	154	152	150	149	148	147	146	144	143	142	141	140	138	137	136	135	134	132	131	130
55	159	158	157	155	153	152	151	149	148	147	146	145	144	142	140	139	138	138	137	135	134	133
56	161	161	160	158	156	155	154	152	151	149	148	147	146	144	143	142	141	140	139	137	136	135
57	164	163	162	161	159	158	157	155	154	152	151	150	149	147	145	144	143	142	142	140	139	138
58	167	166	165	163	161	160	160	158	157	155	154	153	152	150	148	147	146	145	144	142	141	140
59	170	169	168	166	164	163	162	160	159	157	156	155	154	152	150	149	148	148	147	145	144	143
60	173	172	171	169	167	166	165	163	162	160	159	158	157	155	153	152	151	150	149	147	146	145
"	173	172	171	169	167	166	165	163	162	160	159	158	157	155	153	152	151	150	149	147	146	145
	Proportional Parts																					

Proportional Parts

5°

TABLE II

174°

°	l sin		d	l csc		l tan		d	l cot		l sec		d	l cos		°	Proportional Parts						
	8.	1'		11.	8.	1'	11.		10.	1'	9.	145		144	143		142	141	140	139			
0	94030			05970	94195		05805	00166		99834	60	0	0	0	0	0	0	0	0	0			
1	174	144		826	340	145	660	167		833	59	1	2	2	2	2	2	2	2	2			
2	317	144		683	485	145	515	168		832	58	2	5	5	5	5	5	5	5	5			
3	461	143		539	630	145	370	169		831	57	3	7	7	7	7	7	7	7	7			
4	603	143		397	773	144	227	170		830	56	4	10	10	10	9	9	9	9	9			
5	746	141		254	917	143	083	171		829	55	5	12	12	12	12	12	12	12	12			
6	887	142		113	95060	143	04940	172		828	54	6	14	14	14	14	14	14	14	14			
7	95029	141		04971	202	142	798	173		827	53	7	17	17	17	17	16	16	16	16			
8	170	140		830	344	142	656	175		825	52	8	19	19	19	19	19	19	19	19			
9	310	140		690	486	141	514	176		824	51	9	22	22	21	21	21	21	21	21			
10	450	139		550	627	140	373	177		823	50	10	24	24	24	24	24	23	23	23			
11	589	139		411	767	141	233	178		822	49	11	27	26	26	26	26	26	25	25			
12	728	139		272	908	141	092	179		821	48	12	29	29	29	28	28	28	28	28			
13	867	138		133	96047	139	03953	180		820	47	13	31	31	31	31	31	30	30	30			
14	96005	138		03995	187	138	813	181		819	46	14	34	34	33	33	33	33	32	32			
15	143	137		857	325	139	675	183		817	45	15	36	36	36	36	35	35	35	35			
16	280	137		720	464	138	536	184		816	44	16	39	38	38	38	38	37	37	37			
17	417	136		583	602	137	398	185		815	43	17	41	41	41	40	40	40	39	39			
18	553	136		447	739	137	261	186		814	42	18	44	43	43	43	42	42	42	42			
19	689	136		311	877	136	123	187		813	41	19	46	46	45	45	45	44	44	44			
20	825	135		175	97013	137	02987	188		812	40	20	48	48	48	47	47	47	46	46			
21	960	135		040	150	135	850	190		810	39	21	51	50	50	50	49	49	49	49			
22	97095	134		02905	285	136	715	191		809	38	22	53	53	52	52	52	51	51	51			
23	229	134		771	421	135	579	192		808	37	23	56	55	55	54	54	54	53	53			
24	363	133		637	556	135	444	193		807	36	24	58	58	57	57	56	56	56	56			
25	496	133		504	691	134	309	194		806	35	25	61	60	60	59	59	58	58	58			
26	629	133		371	825	134	175	196		804	34	26	63	62	62	61	61	61	60	60			
27	762	132		238	959	133	041	197		803	33	27	65	65	64	64	63	63	63	63			
28	894	132		106	98092	133	01908	198		802	32	28	68	67	67	66	66	65	65	65			
29	98026	131		01974	225	133	775	199		801	31	29	70	70	69	69	68	68	67	67			
30	98157	131		01843	98358	132	01642	00200		99800	30	30	72	72	72	71	70	70	70	70			
31	288	131		712	490	132	510	202		798	29	31	75	74	74	73	73	72	72	72			
32	419	130		581	622	131	378	203		797	28	32	77	77	76	76	75	75	74	74			
33	549	130		451	753	131	247	204		796	27	33	80	79	79	78	78	77	77	76			
34	679	129		321	884	131	116	205		795	26	34	82	82	81	80	80	79	79	79			
35	808	129		192	99015	130	00985	207		793	25	35	85	84	83	83	82	82	81	81			
36	937	129		063	145	130	855	208		792	24	36	87	86	86	85	85	84	83	83			
37	99066	128		00934	275	130	725	209		791	23	37	89	89	88	88	87	86	86	86			
38	194	128		806	405	129	595	210		790	22	38	92	91	91	90	89	89	88	88			
39	322	128		678	534	128	466	212		788	21	39	94	94	93	92	92	91	90	90			
40	450	127		550	662	129	338	213		787	20	40	97	96	95	95	94	93	93	93			
41	577	127		423	791	128	209	214		786	19	41	99	98	98	97	96	96	95	95			
42	704	126		296	919	127	081	215		785	18	42	102	101	100	99	99	98	97	97			
43	830	126		170	00046	127	99954	217		783	17	43	104	103	102	102	101	100	100	100			
44	956	126		044	174	127	826	218		782	16	44	106	106	105	104	103	103	102	102			
45	00082	125		99918	301	126	699	219		781	15	45	109	108	107	106	106	105	104	104			
46	207	125		793	427	126	573	220		780	14	46	111	110	110	109	108	107	107	107			
47	332	124		668	553	126	447	222		778	13	47	114	113	112	111	110	110	109	109			
48	456	125		544	679	126	321	223		777	12	48	116	115	114	114	113	112	111	111			
49	581	123		419	805	125	195	224		776	11	49	118	118	117	116	115	114	114	114			
50	704	124		296	930	125	070	225		775	10	50	121	120	119	118	118	117	116	116			
51	828	124		172	01055	124	98945	227		773	9	51	123	122	122	121	120	119	118	118			
52	951	123		049	179	123	821	228		772	8	52	126	125	124	123	122	121	120	120			
53	01074	122		98926	303	124	697	229		771	7	53	128	127	126	125	125	124	123	123			
54	196	122		804	427	123	573	231		769	6	54	130	130	129	128	127	126	125	125			
55	318	122		682	550	123	450	232		768	5	55	133	132	131	130	129	128	127	127			
56	440	121		560	673	123	327	233		767	4	56	135	134	133	133	132	131	130	130			
57	561	121		439	796	122	204	235		765	3	57	138	137	136	135	134	133	132	132			
58	682	121		318	918	122	082	236		764	2	58	140	139	138	137	136	135	134	134			
59	803	120		197	02040	122	97960	237		763	1	59	143	142	141	140	139	138	137	137			
60	01923			98077	02162		97838	00239		99761	0	60	145	144	143	142	141	140	139	139			
9.	l sin		d	l csc		l tan		d	l cot		l sec		d	l cos		°	Proportional Parts						
8.	1'	11.		8.	1'	11.	10.		1'	9.	145	144		143	142		141	140	139				

95°

84°

TABLE II

"	Proportional Parts																			2	1
	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121	120		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0
2	5	5	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0	0
3	7	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	6	6	0	0
4	9	9	9	9	9	9	9	9	9	9	9	8	8	8	8	8	8	8	8	0	0
5	12	11	11	11	11	11	11	11	11	11	11	11	10	10	10	10	10	10	10	0	0
6	14	14	14	14	13	13	13	13	13	13	13	13	13	12	12	12	12	12	12	0	0
7	16	16	16	16	16	16	15	15	15	15	15	15	15	15	14	14	14	14	14	0	0
8	18	18	18	18	18	18	18	18	18	17	17	17	17	17	17	16	16	16	16	0	0
9	21	21	20	20	20	20	20	20	20	19	19	19	19	19	19	18	18	18	18	0	0
10	23	23	23	22	22	22	22	22	22	21	21	21	21	21	21	20	20	20	20	0	0
11	25	25	25	25	24	24	24	24	24	24	23	23	23	23	23	22	22	22	22	0	0
12	28	27	27	27	27	26	26	26	26	26	25	25	25	25	25	24	24	24	24	0	0
13	30	30	29	29	29	29	29	28	28	28	28	28	27	27	27	26	26	26	26	0	0
14	32	32	32	32	31	31	31	31	30	30	30	30	29	29	29	29	28	28	28	0	0
15	34	34	34	34	34	33	33	33	32	32	32	32	32	31	31	31	30	30	30	0	0
16	37	37	36	36	36	35	35	35	35	34	34	34	34	33	33	33	32	32	32	1	0
17	39	39	39	38	38	38	37	37	37	36	36	36	36	35	35	35	34	34	34	1	0
18	41	41	41	40	40	40	40	39	39	38	38	38	38	37	37	37	36	36	36	1	0
19	44	43	43	43	42	42	42	41	41	41	41	40	40	40	39	39	39	38	38	1	0
20	46	46	45	45	45	44	44	44	43	43	43	42	42	42	41	41	41	40	40	1	0
21	48	48	48	47	47	47	46	46	46	45	45	44	44	44	43	43	43	42	42	1	0
22	51	50	50	50	49	49	48	48	48	47	47	47	46	46	45	45	45	44	44	1	0
23	53	53	52	52	51	51	51	50	50	49	49	49	48	48	48	47	47	46	46	1	0
24	55	55	54	54	54	53	53	52	52	51	51	50	50	50	49	49	49	48	48	1	0
25	58	57	57	56	56	55	55	55	54	54	53	53	52	52	52	51	51	50	50	1	0
26	60	59	59	58	58	58	57	57	56	56	55	55	55	54	54	53	53	52	52	1	0
27	62	62	61	61	60	60	59	59	58	58	57	57	56	56	55	55	54	54	54	1	0
28	64	64	63	63	63	62	62	61	61	60	60	59	58	58	57	57	56	56	56	1	0
29	67	66	66	65	65	64	63	63	62	61	61	60	60	59	59	58	58	58	58	1	0
30	69	68	68	68	67	66	66	66	65	64	64	64	63	62	62	62	61	60	60	1	0
31	71	71	70	70	69	69	68	68	67	66	66	66	65	65	64	64	63	63	62	1	0
32	74	73	73	72	71	71	70	69	69	68	68	67	66	66	66	65	65	64	64	1	0
33	76	75	75	74	74	73	73	72	72	71	70	70	69	69	68	68	67	67	66	1	1
34	78	77	77	76	76	75	75	74	74	73	73	72	71	71	70	69	69	68	68	1	1
35	80	80	79	79	78	78	77	76	76	75	74	74	73	73	72	72	71	71	70	1	1
36	83	82	82	81	80	80	79	79	78	77	77	76	76	75	74	74	73	73	72	1	1
37	85	84	84	83	83	82	81	80	80	79	78	78	77	76	76	75	75	74	74	1	1
38	87	87	86	86	85	84	84	83	82	82	81	80	80	79	79	78	77	77	76	1	1
39	90	89	88	88	87	86	86	85	84	84	83	83	82	81	81	80	79	79	78	1	1
40	92	91	91	90	89	89	88	87	87	86	85	85	84	83	83	82	81	81	80	1	1
41	94	94	93	92	92	91	90	90	89	88	87	87	86	85	85	84	83	83	82	1	1
42	97	96	95	94	94	93	92	92	91	90	90	89	88	88	87	86	85	85	84	1	1
43	99	98	97	97	96	95	95	94	93	92	92	91	90	90	89	88	87	87	86	1	1
44	101	100	100	99	98	98	97	96	95	95	94	93	92	92	91	90	89	89	88	1	1
45	104	103	102	101	100	100	99	98	98	97	96	95	94	94	93	92	92	91	90	2	1
46	106	105	104	104	103	102	101	100	100	99	98	97	97	96	95	94	94	93	92	2	1
47	108	107	107	106	105	104	103	103	102	101	100	99	99	98	97	96	96	95	94	2	1
48	110	110	109	108	107	106	106	105	104	103	102	102	101	100	99	98	98	97	96	2	1
49	113	112	111	110	109	109	108	107	106	105	105	104	103	102	101	100	100	99	98	2	1
50	115	114	113	112	112	111	110	109	108	108	107	106	105	104	103	102	102	101	100	2	1
51	117	116	116	115	114	113	112	111	110	110	109	108	107	106	105	104	103	102	102	2	1
52	120	119	118	117	116	115	114	114	113	112	111	110	109	108	107	107	106	105	104	2	1
53	122	121	120	119	118	117	117	116	115	114	113	112	111	110	109	108	107	106	106	2	1
54	124	123	122	122	121	120	119	118	117	116	115	114	113	112	112	111	110	109	108	2	1
55	126	126	125	124	123	122	121	120	119	118	117	116	116	115	114	113	112	111	110	2	1
56	129	128	127	126	125	124	123	122	121	120	119	119	118	117	116	115	114	113	112	2	1
57	131	130	129	128	127	126	125	124	124	123	122	121	120	119	118	117	116	115	114	2	1
58	133	132	131	130	130	129	128	127	126	125	124	123	122	121	120	119	118	117	116	2	1
59	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121	120	119	118	2	1
60	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121	120	2	1
"	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121	120	2	1

Proportional Parts

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TABLE II

173°

										Proportional Parts					
										121	120	119	118	117	
°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°	121	120	119	118	117
9.	1'	10.	9.	1'	10.	10.	10.	1'	9.						
0	01923		98077	02162		97838	00239		99761	0	0	0	0	0	0
1	02043	120	97957	283	121	717	240	1	760	1	2	2	2	2	2
2	163	120	837	404	121	596	241	2	759	2	4	4	4	4	4
3	283	119	717	525	121	475	243	3	757	3	6	6	6	6	6
4	402	118	598	645	121	355	244	4	756	4	8	8	8	8	8
5	520	119	480	766	119	234	245	5	755	5	10	10	10	10	10
6	639	118	361	885	120	115	247	6	753	6	12	12	12	12	12
7	757	117	243	03005	119	96995	248	7	752	7	14	14	14	14	14
8	874	118	126	124	119	876	249	8	751	8	16	16	16	16	16
9	992	117	008	242	119	758	251	9	749	9	18	18	18	18	18
10	03109		96891	361		639	252		748	10	20	20	20	20	20
11	226	117	774	479	118	521	253	1	747	11	22	22	22	22	21
12	342	116	658	597	117	403	255	2	745	12	24	24	24	24	23
13	458	116	542	714	117	286	256	3	744	13	26	26	26	26	25
14	574	116	426	832	118	168	258	4	742	14	28	28	28	28	27
15	690	115	310	948	117	052	259	5	741	15	30	30	30	29	29
16	805	115	195	04065	116	95935	260	6	740	16	32	32	32	31	31
17	920	114	080	181	116	819	262	7	738	17	34	34	34	33	33
18	04034	113	95966	297	116	703	263	8	737	18	36	36	36	35	35
19	149	113	851	413	115	587	264	9	736	19	38	38	38	37	37
20	262	114	738	528	115	472	266		734	20	40	40	40	39	39
21	376	114	624	643	115	357	267	1	733	21	42	42	42	41	41
22	490	113	510	758	115	242	269	2	731	22	44	44	44	43	43
23	603	113	397	873	115	127	270	3	730	23	46	46	46	45	45
24	715	112	285	987	114	013	272	4	728	24	48	48	48	47	47
25	828	112	172	05101	113	94899	273	5	727	25	50	50	50	49	49
26	940	112	060	214	113	786	274	6	726	26	52	52	52	51	51
27	05052	112	94948	328	114	672	276	7	724	27	54	54	54	53	53
28	164	112	836	441	113	559	277	8	723	28	56	56	56	55	55
29	275	111	725	553	112	447	279	9	721	29	58	58	58	57	57
30	05386		94614	05666		94334	00280		99720	30	60	60	60	59	58
31	497	111	503	778	112	222	282	1	718	31	63	62	61	61	60
32	607	110	393	890	112	110	283	2	717	32	65	64	63	63	62
33	717	110	283	06002	111	93998	284	3	716	33	67	66	65	65	64
34	827	110	173	113	111	887	286	4	714	34	69	68	67	67	66
35	937		063	224		776	287		713	35	71	70	69	69	68
36	06046	109	93954	335	110	665	289	1	711	36	73	72	71	71	70
37	155	109	845	445	111	555	290	2	710	37	75	74	73	73	72
38	264	108	736	556	110	444	292	3	708	38	77	76	75	75	74
39	372	108	628	666	109	334	293	4	707	39	79	78	77	77	76
40	481		519	775		225	295		705	40	81	80	79	79	78
41	589	108	411	885	110	115	296	1	704	41	83	82	81	81	80
42	696	107	304	994	109	006	298	2	702	42	85	84	83	83	82
43	804	108	196	07103	108	92897	299	3	701	43	87	86	85	85	84
44	911	107	089	211	108	789	301	4	699	44	89	88	87	87	86
45	07018		92982	320		680	302		698	45	91	90	89	89	88
46	124	106	876	428	108	572	304	1	696	46	93	92	91	90	90
47	231	106	769	536	107	464	305	2	695	47	95	94	93	92	92
48	337	106	663	643	108	357	307	3	693	48	97	96	95	94	94
49	442	105	558	751	107	249	308	4	692	49	99	98	97	96	96
50	548		452	858		142	310		690	50	101	100	99	98	98
51	653	105	347	964	107	036	311	1	689	51	103	102	101	100	99
52	758	105	242	08071	106	91929	313	2	687	52	105	104	103	102	101
53	863	105	137	177	106	823	314	3	686	53	107	106	105	104	103
54	968	104	032	283	106	717	316	4	684	54	109	108	107	106	105
55	08072		91928	389		611	317		683	55	111	110	109	108	107
56	176	104	824	495	105	505	319	1	681	56	113	112	111	110	109
57	280	104	720	600	105	400	320	2	680	57	115	114	113	112	111
58	383	103	617	705	105	295	322	3	678	58	117	116	115	114	113
59	486	103	514	810	104	190	323	4	677	59	119	118	117	116	115
60	08589		91411	08914		91086	00325		99675	60	121	120	119	118	117
	9.	d	10.	9.	d	10.	10.	d	9.		121	120	119	118	117
	l cos	1'	l sec	l cot	1'	l tan	l csc	1'	l sin		Proportional Parts				

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TABLE II

"	Proportional Parts															2	1
	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0
2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0	0
3	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	0	0
4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	0	0
5	10	10	9	9	9	9	9	9	9	9	9	9	9	9	9	0	0
6	12	12	11	11	11	11	11	11	11	11	11	11	10	10	10	0	0
7	14	13	13	13	13	13	13	13	13	12	12	12	12	12	12	0	0
8	15	15	15	15	15	15	15	15	15	14	14	14	14	14	14	0	0
9	17	17	17	17	17	17	17	16	16	16	16	16	16	16	16	0	0
10	19	19	19	19	19	18	18	18	18	18	18	18	18	17	17	0	0
11	21	21	21	21	21	20	20	20	20	20	20	20	19	19	19	0	0
12	23	23	23	23	22	22	22	22	22	22	21	21	21	21	21	0	0
13	25	25	25	24	24	24	24	24	23	23	23	23	23	23	23	0	0
14	27	27	27	26	26	26	26	25	25	25	25	25	24	24	24	0	0
15	29	29	29	28	28	28	27	27	27	27	27	26	26	26	26	0	0
16	31	31	30	30	30	30	29	29	29	29	28	28	28	28	28	1	0
17	33	33	32	32	32	31	31	31	31	30	30	30	30	29	29	1	0
18	35	34	34	34	34	33	33	33	32	32	32	32	32	31	31	1	0
19	37	36	36	36	35	35	35	35	34	34	34	34	33	33	33	1	0
20	39	38	38	38	37	37	37	36	36	36	35	35	35	35	35	1	0
21	41	40	40	40	39	39	39	38	38	37	37	37	37	36	36	1	0
22	43	42	42	41	41	41	40	40	40	39	39	38	38	38	38	1	0
23	44	44	44	43	43	43	42	42	42	41	41	41	40	40	40	1	0
24	46	46	46	45	45	44	44	44	43	43	42	42	42	42	42	1	0
25	48	48	47	47	47	46	46	45	45	45	44	44	44	43	43	1	0
26	50	50	49	49	49	48	48	47	47	46	46	46	45	45	45	1	0
27	52	52	51	51	50	50	49	49	49	48	48	47	47	47	47	1	0
28	54	54	53	53	52	52	51	51	50	50	49	49	49	49	49	1	0
29	56	56	55	55	54	54	53	53	52	52	51	51	50	50	50	1	0
30	58	58	57	56	56	56	55	54	54	54	53	52	52	52	52	1	0
31	60	59	59	58	58	57	57	56	56	55	55	54	54	54	54	1	1
32	62	61	61	60	60	59	59	58	58	57	57	56	56	55	55	1	1
33	64	63	63	62	62	61	61	60	59	59	58	58	57	57	57	1	1
34	66	65	65	64	63	63	62	62	61	61	60	60	60	59	59	1	1
35	68	67	67	66	65	65	64	64	63	62	62	61	61	61	61	1	1
36	70	69	68	68	67	67	66	65	65	64	64	63	63	62	62	1	1
37	72	71	70	70	69	68	68	67	66	66	65	65	64	64	64	1	1
38	73	73	72	72	71	70	70	69	68	68	67	66	66	66	66	1	1
39	75	75	74	73	73	72	72	71	70	70	69	68	68	68	68	1	1
40	77	77	76	75	75	74	73	73	72	71	71	70	70	69	69	1	1
41	79	79	78	77	77	76	75	74	74	73	72	72	72	71	71	1	1
42	81	80	80	79	78	78	77	76	76	75	74	74	73	73	73	1	1
43	83	82	82	81	80	80	79	78	77	77	76	75	75	75	75	1	1
44	85	84	84	83	82	81	81	80	79	78	78	77	77	76	76	1	1
45	87	86	85	85	84	83	83	82	81	80	79	79	78	78	78	2	1
46	89	88	87	87	86	85	84	84	83	82	81	80	80	80	80	2	1
47	91	90	89	89	88	87	86	85	85	84	83	82	81	81	81	2	1
48	93	92	91	90	90	89	88	87	86	86	85	84	83	83	83	2	1
49	95	94	93	92	91	91	90	89	88	87	87	86	85	85	85	2	1
50	97	96	95	94	93	92	92	91	90	89	88	88	87	87	87	2	1
51	99	98	97	96	95	94	93	93	92	91	90	89	88	88	88	2	1
52	101	100	99	98	97	96	95	94	94	93	92	91	90	90	90	2	1
53	102	102	101	100	99	98	97	96	95	95	94	93	92	92	92	2	1
54	104	104	103	102	101	100	99	98	97	96	95	94	94	94	94	2	1
55	106	105	105	104	103	102	101	100	99	98	97	96	95	95	95	2	1
56	108	107	106	105	105	104	103	102	101	100	99	98	97	97	97	2	1
57	110	109	108	107	106	105	105	104	103	102	101	100	99	99	99	2	1
58	112	111	110	109	108	107	106	105	104	103	102	102	101	101	101	2	1
59	114	113	112	111	110	109	108	107	106	105	104	103	102	102	102	2	1
60	116	115	114	113	112	111	110	109	108	107	106	105	104	104	104	2	1
"	116	115	114	113	112	111	110	109	108	107	106	105	104	104	104	2	1

Proportional Parts

7°

TABLE II

172°

°	l sin 9.	d 1'	l csc 10.	l tan 9.	d 1'	l cot 10.	l sec 10.	d 1'	l cos 9.	°	Proportional Parts			
											105	104	103	102
0	05589	103	91411	08914	105	91086	00325	1	99675	60	0	0	0	0
1	692	103	308	09019	104	90981	326	2	674	59	2	2	2	2
2	795	103	205	123	104	877	328	2	672	58	4	3	3	3
3	897	102	103	227	103	773	330	1	670	57	5	5	5	5
4	999	102	001	330	104	670	331	2	669	56	7	7	7	7
5	09101	101	90899	434	103	566	333	1	667	55	9	9	9	9
6	202	101	798	537	103	463	334	2	666	54	10	10	10	10
7	304	102	696	640	102	360	336	1	664	53	12	12	12	12
8	405	101	595	742	103	258	337	2	663	52	14	14	14	14
9	506	101	494	845	102	155	339	2	661	51	16	16	15	15
10	606	101	394	947	102	053	341	1	659	50	18	17	17	17
11	707	100	293	10049	101	89951	342	2	658	49	19	19	19	19
12	807	100	193	150	102	850	344	1	656	48	21	21	21	20
13	907	99	093	252	102	748	345	2	655	47	23	23	22	22
14	10006	99	89994	353	101	647	347	2	653	46	24	24	24	24
15	106	99	894	454	101	546	349	1	651	45	26	26	26	25
16	205	99	795	555	101	445	350	2	650	44	28	28	27	27
17	304	98	696	656	100	344	352	1	648	43	30	29	29	29
18	402	98	598	756	100	244	353	2	647	42	32	31	31	31
19	501	98	499	856	100	144	355	2	645	41	33	33	33	32
20	599	98	401	956	100	044	357	1	643	40	35	35	34	34
21	697	98	303	11056	99	88944	358	2	642	39	37	36	36	36
22	795	98	205	155	99	845	360	2	640	38	38	38	38	37
23	893	97	107	254	99	746	362	1	638	37	40	40	39	39
24	990	97	010	353	99	647	363	2	637	36	42	42	41	41
25	11087	97	88913	452	99	548	365	2	635	35	44	43	43	43
26	184	97	816	551	98	449	367	1	633	34	46	45	45	44
27	281	96	719	649	98	351	368	2	632	33	47	47	46	46
28	377	97	623	747	98	253	370	1	630	32	49	49	48	48
29	474	97	526	845	98	155	371	2	629	31	51	50	50	49
30	11570	96	88430	11943	97	88057	00373	2	99627	30	52	52	52	51
31	666	95	334	12040	98	87960	375	1	625	29	54	54	53	53
32	761	95	239	138	98	862	376	2	624	28	56	55	55	54
33	857	95	143	235	97	765	378	2	622	27	58	57	57	56
34	952	95	048	332	97	668	380	2	620	26	60	59	58	58
35	12047	95	87953	428	96	572	382	1	618	25	61	61	60	59
36	142	94	858	525	96	475	383	2	617	24	63	62	62	61
37	236	94	764	621	96	379	385	2	615	23	65	64	64	63
38	331	95	669	717	96	283	387	2	613	22	66	66	65	65
39	425	94	575	813	96	187	388	1	612	21	68	68	67	66
40	519	93	481	909	95	091	390	2	610	20	70	69	69	68
41	612	93	388	13004	95	86996	392	1	608	19	72	71	70	70
42	706	94	294	099	95	901	393	2	607	18	74	73	72	71
43	799	93	201	194	95	806	395	2	605	17	75	75	74	73
44	892	93	108	289	95	711	397	2	603	16	77	76	76	75
45	985	93	015	384	94	616	399	1	601	15	79	78	77	77
46	13078	93	86922	478	95	522	400	2	600	14	80	80	79	78
47	171	92	829	573	94	427	402	2	598	13	82	81	81	80
48	263	92	737	667	94	333	404	1	596	12	84	83	82	82
49	355	92	645	761	93	239	405	2	595	11	86	85	84	83
50	447	92	553	854	94	146	407	2	593	10	88	87	86	85
51	539	91	461	948	94	052	409	2	591	9	89	88	88	87
52	630	92	370	14041	93	88959	411	1	589	8	91	90	89	88
53	722	92	278	134	93	866	412	2	588	7	93	92	91	90
54	813	91	187	227	93	773	414	2	586	6	94	94	93	92
55	904	90	096	320	92	680	416	2	584	5	96	95	94	93
56	994	90	006	412	92	588	418	1	582	4	98	97	96	95
57	14085	90	85915	504	93	496	419	2	581	3	100	99	98	97
58	175	90	825	597	92	403	421	2	579	2	102	101	100	99
59	266	90	734	688	91	312	423	2	577	1	103	102	101	100
60	14356	90	85644	14780	92	85220	00425	2	99575	0	105	104	103	102
°	l sin 9.	d 1'	l sec 10.	l cot 9.	d 1'	l tan 10.	l csc 10.	d 1'	l cos 9.	°	Proportional Parts			
°	l sin 9.	d 1'	l sec 10.	l cot 9.	d 1'	l tan 10.	l csc 10.	d 1'	l cos 9.	°	105	104	103	102

97°

82°

TABLE II

"	Proportional Parts													
	101	100	99	98	97	96	95	94	93	92	91	90	2	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	2	2	2	2	2	2	2	2	2	2	1	0	0
2	3	3	3	3	3	3	3	3	3	3	3	3	0	0
3	5	5	5	5	5	5	5	5	5	5	5	5	0	0
4	7	7	7	7	6	6	6	6	6	6	6	6	0	0
5	8	8	8	8	8	8	8	8	8	8	8	7	0	0
6	10	10	10	10	10	10	10	9	9	9	9	9	0	0
7	12	12	12	11	11	11	11	11	11	11	11	11	0	0
8	13	13	13	13	13	13	13	13	12	12	12	12	0	0
9	15	15	15	15	15	14	14	14	14	14	14	13	0	0
10	17	17	16	16	16	16	16	16	16	15	15	15	0	0
11	19	18	18	18	18	18	17	17	17	17	17	17	0	0
12	20	20	20	20	19	19	19	19	19	18	18	18	0	0
13	22	22	21	21	21	21	21	20	20	20	20	19	0	0
14	24	23	23	23	23	22	22	22	22	21	21	21	0	0
15	25	25	25	24	24	24	24	23	23	23	23	23	0	0
16	27	27	26	26	26	26	25	25	25	25	24	24	1	0
17	29	28	28	28	27	27	27	27	26	26	26	25	1	0
18	30	30	30	29	29	29	28	28	28	28	27	27	1	0
19	32	32	31	31	31	30	30	30	29	29	29	29	1	0
20	34	33	33	33	32	32	32	31	31	31	30	30	1	0
21	35	35	35	34	34	34	33	33	33	32	32	31	1	0
22	37	37	36	36	36	35	35	34	34	34	33	33	1	0
23	39	38	38	38	37	37	36	36	36	35	35	35	1	0
24	40	40	40	39	39	38	38	38	37	37	36	36	1	0
25	42	42	41	41	40	40	40	39	39	38	38	37	1	0
26	44	43	43	42	42	42	41	41	40	40	39	39	1	0
27	45	45	45	44	44	43	43	42	42	41	41	41	1	0
28	47	47	46	46	45	45	44	44	43	43	42	42	1	0
29	49	48	48	47	47	46	46	45	45	44	44	43	1	0
30	50	50	50	49	48	48	48	47	46	46	46	45	1	0
31	52	52	51	51	50	50	49	49	48	48	47	47	1	1
32	54	53	53	52	52	51	51	50	50	49	49	48	1	1
33	56	55	54	54	53	53	52	52	51	51	50	49	1	1
34	57	57	56	56	55	54	54	53	53	52	52	51	1	1
35	59	58	58	57	57	56	55	55	54	54	53	53	1	1
36	61	60	59	59	58	58	57	56	56	55	55	54	1	1
37	62	62	61	60	60	59	59	58	57	57	56	55	1	1
38	64	63	63	62	61	61	60	60	59	58	58	57	1	1
39	66	65	64	64	63	62	62	61	60	60	59	59	1	1
40	67	67	66	65	65	64	63	63	62	61	61	60	1	1
41	69	68	68	67	66	66	65	64	64	63	62	61	1	1
42	71	70	69	69	68	67	66	66	65	64	64	63	1	1
43	72	72	71	70	70	69	68	67	67	66	65	65	1	1
44	74	73	73	72	71	70	70	69	68	67	67	66	1	1
45	76	75	74	73	73	72	71	71	70	69	68	67	2	1
46	77	77	76	75	74	74	73	72	71	71	70	69	2	1
47	79	78	78	77	76	75	74	74	73	72	71	71	2	1
48	81	80	79	78	78	77	76	75	74	74	73	72	2	1
49	82	82	81	80	79	78	78	77	76	75	74	73	2	1
50	84	83	82	82	81	80	79	78	78	77	76	75	2	1
51	86	85	84	83	82	81	80	79	78	77	77	76	2	1
52	88	87	86	85	84	83	82	81	81	80	79	78	2	1
53	89	88	87	87	86	85	84	83	82	81	80	79	2	1
54	91	90	89	88	87	86	86	85	84	83	82	81	2	1
55	93	92	91	90	89	88	87	86	85	84	83	83	2	1
56	94	93	92	91	91	90	89	88	87	86	85	84	2	1
57	96	95	94	93	92	91	90	89	88	87	86	85	2	1
58	98	97	96	95	94	93	92	91	90	89	88	87	2	1
59	99	98	97	96	95	94	93	92	91	90	89	89	2	1
60	101	100	99	98	97	96	95	94	93	92	91	90	2	1
"	101	100	99	98	97	96	95	94	93	92	91	90	2	1

Proportional Parts

8°

TABLE II

171°

"	l sin 9.	d 1'	l csc 10.	l tan 9.	d 1'	l cot 10.	l sec 10.	d 1'	l cos 9.	"
0	14356	89	85644	14780	92	85220	00425	1	99575	60
1	445	89	555	872	91	128	426	1	574	59
2	535	89	465	963	91	037	428	2	572	58
3	624	89	376	15054	91	84946	430	3	570	57
4	714	89	286	145	91	855	432	4	568	56
5	803	88	197	236	91	764	434	5	566	55
6	891	88	109	327	91	673	435	6	565	54
7	980	89	020	417	90	583	437	7	563	53
8	15069	88	84931	508	90	492	438	8	561	52
9	157	88	843	598	90	402	441	9	559	51
10	245	88	755	688	89	312	443	10	557	50
11	333	88	667	777	89	223	444	11	556	49
12	421	87	579	867	89	133	446	12	554	48
13	508	87	492	956	89	044	448	13	552	47
14	596	88	404	10046	90	83954	450	14	550	46
15	683	87	317	135	89	865	452	15	548	45
16	770	87	230	224	89	776	454	16	546	44
17	857	87	143	312	88	688	455	17	545	43
18	944	87	056	401	89	599	457	18	543	42
19	16030	86	83970	489	88	511	459	19	541	41
20	116	87	884	577	88	423	461	20	539	40
21	203	86	797	665	88	335	463	21	537	39
22	289	86	711	753	88	247	465	22	535	38
23	374	86	626	841	87	159	467	23	533	37
24	460	85	540	928	87	072	468	24	532	36
25	545	86	455	17016	87	82984	470	25	530	35
26	631	86	369	103	87	897	472	26	528	34
27	716	85	284	190	87	810	474	27	526	33
28	801	85	199	277	87	723	476	28	524	32
29	886	85	114	363	86	637	478	29	522	31
30	16970	84	83030	17450	87	82550	00480	30	99520	30
31	17055	84	82945	536	86	464	482	31	518	29
32	139	84	861	622	86	378	483	32	517	28
33	223	84	777	708	86	292	485	33	515	27
34	307	84	693	794	86	206	487	34	513	26
35	391	83	609	880	85	120	489	35	511	25
36	474	84	526	965	85	035	491	36	509	24
37	558	83	442	18051	86	81949	493	37	507	23
38	641	83	359	136	85	864	495	38	505	22
39	724	83	276	221	85	779	497	39	503	21
40	807	83	193	306	85	694	499	40	501	20
41	890	83	110	391	84	609	501	41	499	19
42	973	82	027	475	84	525	503	42	497	18
43	18055	82	81945	560	84	440	505	43	495	17
44	137	82	863	644	84	356	506	44	494	16
45	220	82	780	728	84	272	508	45	492	15
46	302	81	698	812	84	188	510	46	490	14
47	383	82	617	896	84	104	512	47	488	13
48	465	82	535	979	83	021	514	48	486	12
49	547	81	453	19063	83	80937	516	49	484	11
50	628	81	372	146	83	854	518	50	482	10
51	709	81	291	229	83	771	520	51	480	9
52	790	81	210	312	83	688	522	52	478	8
53	871	81	129	395	83	605	524	53	476	7
54	952	81	048	478	83	522	526	54	474	6
55	19033	80	80967	561	82	439	528	55	472	5
56	113	80	887	643	82	357	530	56	470	4
57	193	80	807	725	82	275	532	57	468	3
58	273	80	727	807	82	193	534	58	466	2
59	353	80	647	889	82	111	536	59	464	1
60	19433	80	80567	19971	82	80029	00538	60	99462	0
"	9	d	10	9.	d	10.	10.	d	9.	"
	l cos	1	l sec	l cot	1'	l tan	l csc	1'	l sin	

"	Proportional Parts		
"	92	91	90
0	0	0	0
1	2	2	1
2	3	3	3
3	5	5	5
4	6	6	6
5	8	8	7
6	9	9	9
7	11	11	11
8	12	12	12
9	14	14	13
10	15	15	15
11	17	17	17
12	18	18	18
13	20	20	19
14	21	21	21
15	23	23	23
16	25	24	24
17	26	26	25
18	28	27	27
19	29	29	29
20	31	30	30
21	32	32	31
22	34	33	33
23	35	35	35
24	37	36	36
25	38	38	37
26	40	39	39
27	41	41	41
28	43	42	42
29	44	44	43
30	46	46	45
31	48	47	47
32	49	49	48
33	51	50	49
34	52	52	51
35	54	53	53
36	55	55	54
37	57	56	55
38	58	58	57
39	60	59	59
40	61	61	60
41	63	62	61
42	64	64	63
43	66	65	65
44	67	67	66
45	69	68	67
46	71	70	69
47	72	71	71
48	74	73	72
49	75	74	73
50	77	76	75
51	78	77	77
52	80	79	78
53	81	80	79
54	83	82	81
55	84	83	83
56	86	85	84
57	87	86	85
58	89	88	87
59	90	89	89
60	92	91	90
"	92	91	90
	Proportional Parts		

98°

81°

TABLE II

"	Proportional Parts											
	89	88	87	86	85	84	83	82	81	80	2	1
0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	0	0
2	3	3	3	3	3	3	3	3	3	3	0	0
3	4	4	4	4	4	4	4	4	4	4	0	0
4	6	6	6	6	6	6	6	5	5	5	0	0
5	7	7	7	7	7	7	7	7	7	7	0	0
6	9	9	9	9	8	8	8	8	8	8	0	0
7	10	10	10	10	10	10	10	10	9	9	0	0
8	12	12	12	11	11	11	11	11	11	11	0	0
9	13	13	13	13	13	13	12	12	12	12	0	0
10	15	15	14	14	14	14	14	14	14	13	0	0
11	16	16	16	16	16	15	15	15	15	15	0	0
12	18	18	17	17	17	17	17	16	16	16	0	0
13	19	19	19	19	18	18	18	18	18	17	0	0
14	21	21	20	20	20	20	19	19	19	19	0	0
15	22	22	22	21	21	21	21	21	20	20	0	0
16	24	23	23	23	23	22	22	22	22	21	1	0
17	25	25	25	24	24	24	24	23	23	23	1	0
18	27	26	26	26	26	25	25	25	24	24	1	0
19	28	28	28	27	27	27	26	26	26	25	1	0
20	30	29	29	29	28	28	28	27	27	27	1	0
21	31	31	30	30	30	29	29	29	28	28	1	0
22	33	32	32	32	31	31	30	30	30	29	1	0
23	34	34	33	33	33	32	32	31	31	31	1	0
24	36	35	35	34	34	34	33	33	32	32	1	0
25	37	37	36	36	35	35	35	34	34	33	1	0
26	39	38	38	37	37	36	36	36	35	35	1	0
27	40	40	39	39	38	38	37	37	36	36	1	0
28	42	41	41	40	40	39	39	38	38	37	1	0
29	43	43	42	42	41	41	40	40	39	39	1	0
30	44	44	44	43	42	42	42	41	40	40	1	0
31	46	45	45	44	44	43	43	42	42	41	1	1
32	47	47	46	46	45	45	44	44	43	43	1	1
33	49	48	48	47	47	46	46	45	45	44	1	1
34	50	50	49	49	48	48	47	46	46	45	1	1
35	52	51	51	50	50	49	48	48	47	47	1	1
36	53	53	52	52	51	50	50	49	49	48	1	1
37	55	54	54	53	52	51	51	50	50	49	1	1
38	56	56	55	54	54	53	53	52	51	51	1	1
39	58	57	57	56	55	55	54	53	53	52	1	1
40	59	59	58	57	57	56	55	55	54	53	1	1
41	61	60	59	59	58	57	57	56	55	55	1	1
42	62	62	61	60	60	59	58	57	57	56	1	1
43	64	63	62	62	61	60	59	59	58	57	1	1
44	65	65	64	63	62	62	61	60	59	59	1	1
45	67	66	65	65	64	63	62	61	61	60	2	1
46	68	67	67	66	65	64	64	63	62	61	2	1
47	70	69	68	67	67	66	65	64	63	63	2	1
48	71	70	70	69	68	67	66	66	65	64	2	1
49	73	72	71	70	69	69	68	67	66	65	2	1
50	74	73	72	72	71	70	69	68	68	67	2	1
51	76	75	74	73	72	71	71	70	69	68	2	1
52	77	76	75	75	74	73	72	71	70	69	2	1
53	79	78	77	76	75	74	73	72	72	71	2	1
54	80	79	78	77	76	76	75	74	73	72	2	1
55	82	81	80	79	78	77	76	75	74	73	2	1
56	83	82	81	80	79	78	77	77	76	75	2	1
57	85	84	83	82	81	80	79	78	77	76	2	1
58	86	85	84	83	82	81	80	79	78	77	2	1
59	88	87	86	85	84	83	82	81	80	79	2	1
60	89	88	87	86	85	84	83	82	81	80	2	1
"	89	88	87	86	85	84	83	82	81	80	2	1

Proportional Parts

9°

TABLE II

170°

°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°	Proportional Parts																
											82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66
0	19433		80567	19971		80029	00538		99462	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	513	80	487	20053	82	79947	540	2	460	59	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	592	80	408	134	82	866	542	2	458	58	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
3	672	80	328	216	81	784	544	2	456	57	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
4	751	79	249	297	81	703	546	2	454	56	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
5	830	79	170	378	81	622	548	2	452	55	5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
6	909	79	091	459	81	541	550	2	450	54	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	
7	988	79	012	540	81	460	552	2	448	53	7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	
8	20067	79	79933	621	81	379	554	2	446	52	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	
9	145	78	855	701	81	299	556	2	444	51	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
10	223	78	777	782	80	218	558	2	442	50	10	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	
11	302	78	698	862	80	138	560	2	440	49	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	
12	380	78	620	942	80	058	562	2	438	48	12	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	
13	458	78	542	21022	80	78978	564	2	436	47	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	
14	535	77	465	102	80	898	566	2	434	46	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
15	613	78	387	182	79	818	568	3	432	45	15	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	
16	691	78	309	261	79	739	571	3	429	44	16	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	
17	768	77	232	341	80	659	573	2	427	43	17	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	
18	845	77	155	420	79	580	575	2	425	42	18	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	
19	922	77	078	499	79	501	577	2	423	41	19	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	
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21	21076	77	78924	657	79	343	581	2	419	39	21	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	
22	153	76	847	736	78	264	583	2	417	38	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	
23	229	76	771	814	78	186	585	2	415	37	23	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	
24	306	76	694	893	78	107	587	2	413	36	24	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
25	382	76	618	971	78	029	589	2	411	35	25	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	
26	458	76	542	22049	78	77951	591	2	409	34	26	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
27	534	76	466	127	78	873	593	3	407	33	27	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
28	610	76	390	205	78	795	596	2	404	32	28	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	
29	685	76	315	283	78	717	598	2	402	31	29	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	
30	21761	76	78239	22361	77	77639	00600	2	99400	30	30	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	
31	836	75	164	438	78	562	602	2	398	29	31	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	
32	912	75	088	516	78	484	604	2	396	28	32	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	
33	987	75	013	593	77	407	606	2	394	27	33	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	
34	22062	75	77938	670	77	330	608	2	392	26	34	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	
35	137	74	863	747	77	253	610	2	390	25	35	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
36	211	74	789	824	77	176	612	2	388	24	36	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	
37	286	74	714	901	77	099	615	2	385	23	37	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	
38	361	74	639	977	77	023	617	2	383	22	38	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	
39	435	74	565	23054	76	76946	619	2	381	21	39	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	
40	509	74	491	130	76	870	621	2	379	20	40	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	
41	583	74	417	206	76	794	623	2	377	19	41	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	
42	657	74	343	283	76	717	625	2	375	18	42	43	43	43	43	43	43	43	43	43	43	43	43	43	43	43	
43	731	74	269	359	76	641	628	3	372	17	43	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	
44	805	73	195	435	75	565	630	2	370	16	44	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	
45	878	73	122	510	76	490	632	2	368	15	45	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	
46	952	73	048	586	76	414	634	2	366	14	46	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	
47	23025	73	76975	661	76	339	636	2	364	13	47	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48	
48	098	73	902	737	75	263	638	3	362	12	48	49	49	49	49	49	49	49	49	49	49	49	49	49	49	49	
49	171	73	829	812	75	188	641	2	359	11	49	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	
50	244	73	756	887	75	113	643	2	357	10	50	51	51	51	51	51	51	51	51	51	51	51	51	51	51	51	
51	317	73	683	962	75	038	645	2	355	9	51	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	
52	390	73	610	24037	75	75963	647	2	353	8	52	53	53	53	53	53	53	53	53	53	53	53	53	53	53	53	
53	462	72	538	112	74	888	649	3	351	7	53	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54	
54	538	72	465	186	74	814	652	2	348	6	54	55	55	55	55	55	55	55	55	55	55	55	55	55	55	55	
55	607	72	593	261	74	739	654	2	346	5	55	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56	
56	679	72	521	335	75	665	656	2	344	4	56	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	
57	752	71	448	410																							

$\angle$	$\angle$ sin	$\angle$ cos	$\angle$ tan	$\angle$ cot	$\angle$ sec	$\angle$ csc
9.	10.	10.	9.	10.	10.	9.
0	23967	76033	24632	75368	00665	99335
1	24039	75961	706 74	294 667	2	333 59
2	110	890	779 73	221 669	2	331 58
3	181	819	853 74	147 672	3	328 57
4	253	747	926 73	074 674	2	326 56
5	324	676	25000	000 676	2	324 55
6	395	605	073 73	74927	2	322 54
7	466	534	146 73	854 681	3	319 53
8	536	464	219 73	781 683	2	317 52
9	607	393	292 73	708 685	2	315 51
10	677	323	365 73	635 687	2	313 50
11	748	252	437 72	563 690	3	310 49
12	818	182	510 73	490 692	2	308 48
13	888	112	582 72	418 694	2	306 47
14	958	042	655 73	345 696	2	304 46
15	25028	74972	727 72	273 699	3	301 45
16	098	902	799 72	201 701	2	299 44
17	168	832	871 72	129 703	3	297 43
18	237	763	943 72	057 706	3	294 42
19	307	693	26015 72	73985	2	292 41
20	376	624	086 71	914 710	2	290 40
21	445	555	158 72	842 712	2	288 39
22	514	486	229 71	771 715	3	286 38
23	583	417	301 72	699 717	2	283 37
24	652	348	372 71	628 719	2	281 36
25	721	279	443 71	557 722	3	278 35
26	790	210	514 71	486 724	2	276 34
27	858	142	585 71	415 726	2	274 33
28	927	073	655 70	345 729	3	271 32
29	995	005	726 71	274 731	2	269 31
30	26063	73937	26797 71	73203	00733	99267
31	131	869	867 70	133 736	3	264 29
32	199	801	937 70	063 738	2	262 28
33	267	733	27008 71	72992	740 2	260 27
34	335	665	078 70	922 743	3	257 26
35	403	597	148 70	852 745	2	255 25
36	470	530	218 70	782 748	3	252 24
37	538	462	288 70	712 750	2	250 23
38	605	395	357 69	643 752	2	248 22
39	672	328	427 70	573 755	3	245 21
40	739	261	496 69	504 757	2	243 20
41	806	194	566 70	434 759	2	241 19
42	873	127	635 69	365 762	3	238 18
43	940	060	704 69	296 764	2	236 17
44	27007	72993	773 69	227 767	3	233 16
45	073	927	842 69	158 769	2	231 15
46	140	860	911 69	089 771	2	229 14
47	206	794	980 69	020 774	3	226 13
48	273	727	28049 69	71951	776 2	224 12
49	339	661	117 68	883 779	3	221 11
50	405	595	186 69	814 781	2	219 10
51	471	529	254 68	746 783	2	217 9
52	537	463	323 69	677 786	3	214 8
53	602	398	391 68	609 788	2	212 7
54	668	332	459 68	541 791	2	209 6
55	734	266	527 68	473 793	3	207 5
56	799	201	595 68	405 796	2	204 4
57	864	136	662 67	338 798	2	202 3
58	930	070	730 68	270 800	2	200 2
59	995	005	798 68	202 803	3	197 1
60	28060	71940	28865 67	71135	00805	99195
9.	10.	9.	10.	10.	9.	0
$\angle$ cos	$\angle$ sec	$\angle$ cot	$\angle$ tan	$\angle$ csc	$\angle$ sin	

Proportional Parts													
74	73	72	71	70	69	68	67	66	65	3	2		
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2	2	2	2	2	2	2	2	2	2	2	2	0	0
3	4	4	4	4	4	4	4	4	4	4	4	0	0
4	5	5	5	5	5	5	5	5	5	5	5	0	0
5	6	6	6	6	6	6	6	6	6	6	6	5	0
6	7	7	7	7	7	7	7	7	7	7	7	6	0
7	9	9	8	8	8	8	8	8	8	8	8	8	0
8	10	10	10	9	9	9	9	9	9	9	9	9	0
9	11	11	11	11	11	11	11	11	11	11	11	10	0
10	12	12	12	12	12	12	12	12	12	12	12	11	0
11	14	13	13	13	13	13	13	13	13	13	13	12	1
12	15	15	14	14	14	14	14	14	14	14	14	13	1
13	16	16	16	15	15	15	15	15	15	15	15	14	1
14	17	17	17	17	16	16	16	16	16	16	16	15	1
15	19	18	18	18	17	17	17	17	17	17	17	16	1
16	20	19	19	19	19	18	18	18	18	18	18	17	1
17	21	21	20	20	20	20	20	20	20	20	20	19	1
18	22	22	22	21	21	21	21	21	21	21	21	20	1
19	23	23	23	22	22	22	22	22	22	22	22	21	1
20	25	24	24	24	23	23	23	23	23	23	23	22	1
21	26	26	25	25	25	24	24	24	24	24	24	23	1
22	27	27	26	26	26	25	25	25	25	25	25	24	1
23	28	28	28	27	27	27	26	26	26	26	26	25	1
24	30	29	29	28	28	28	27	27	27	27	27	26	1
25	31	30	30	30	29	29	28	28	28	28	28	27	1
26	32	32	31	31	30	30	29	29	29	29	29	28	1
27	33	33	32	32	31	31	31	30	30	30	30	29	1
28	35	34	34	33	33	33	32	32	32	32	32	31	1
29	36	35	35	34	34	34	33	33	33	33	33	32	1
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31	38	37	37	37	36	36	35	35	35	35	35	34	2
32	39	38	38	38	37	37	36	36	36	36	36	35	2
33	41	40	40	39	39	39	38	38	38	38	38	37	2
34	42	41	41	40	40	40	39	39	39	39	39	38	2
35	43	43	42	41	41	41	40	40	40	40	40	39	2
36	44	44	43	43	42	42	41	41	41	41	41	40	2
37	46	45	44	44	43	43	42	42	42	42	42	41	2
38	47	46	46	45	44	44	43	43	43	43	43	42	2
39	48	47	47	46	45	45	44	44	44	44	44	43	2
40	49	48	48	47	47	47	46	46	46	46	46	45	2
41	51	50	49	49	48	48	47	47	47	47	47	46	2
42	52	51	50	50	49	49	48	48	48	48	48	47	2
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45	55	55	54	53	53	52	51	50	50	50	50	49	2
46	57	56	55	54	54	53	52	51	51	51	51	50	2
47	58	57	56	56	55	54	53	53	52	52	52	51	2
48	59	58	58	57	56	55	54	54	53	53	53	52	2
49	60	60	59	58	57	56	56	55	54	54	54	53	2
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54	67	66	65	64	63	62	61	60	59	59	59	58	3
55	68	67	66	65	64	63	62	61	61	61	61	60	3
56	69	68	67	66	65	64	63	63	62	62	62	61	3
57	70	69	68	67	66	65	64	63	63	63	63	62	3
58	72	71	70	69	68	67	66	65	64	64	64	63	3
59	73	72	71	70	69	68	67	66	65	65	65	64	3
60	74	73	72	71	70	69	68	67	66	66	66	65	3
74	73	72	71	70	69	68	67	66	65	3	2		
Proportional Parts													

$\circ$	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	1'	10.	9.	1'	10.	10.	1'	9.	1'	10.
0	28060	71940	28865	71135	00805	99195	60	0	0	0
1	125	875	933	067	808	192	59	1	1	1
2	190	810	29000	000	810	190	58	2	2	2
3	254	746	067	70933	813	187	57	3	3	3
4	319	681	134	866	815	185	56	4	4	4
5	384	616	201	799	818	182	55	5	5	5
6	448	552	268	732	820	180	54	6	6	6
7	512	488	335	665	823	177	53	7	7	7
8	577	423	402	598	825	175	52	8	8	8
9	641	359	468	532	828	172	51	9	9	9
10	705	295	535	465	830	170	50	10	10	10
11	769	231	601	399	833	167	49	11	11	11
12	833	167	668	332	835	165	48	12	12	12
13	896	104	734	266	838	162	47	13	13	13
14	960	040	800	200	840	160	46	14	14	14
15	29024	70976	866	134	843	157	45	15	15	15
16	087	913	932	068	845	155	44	16	16	16
17	150	850	998	002	848	152	43	17	17	17
18	214	786	30064	69936	850	150	42	18	18	18
19	277	723	130	870	853	147	41	19	19	19
20	340	660	195	805	855	145	40	20	20	20
21	403	597	261	739	858	142	39	21	21	21
22	466	534	326	674	860	140	38	22	22	22
23	529	471	391	609	863	137	37	23	23	23
24	591	409	457	543	865	135	36	24	24	24
25	654	346	522	478	868	132	35	25	25	25
26	716	284	587	413	870	130	34	26	26	26
27	779	221	652	348	873	127	33	27	27	27
28	841	159	717	283	876	124	32	28	28	28
29	903	097	782	218	878	122	31	29	29	29
30	29966	70034	30846	69154	00881	99119	30	30	30	30
31	30028	69972	911	089	883	117	29	31	31	31
32	090	910	975	040	886	114	28	32	32	32
33	151	849	31040	68960	888	112	27	33	33	33
34	213	787	104	896	891	109	26	34	34	34
35	275	725	168	832	894	106	25	35	35	35
36	336	664	233	767	896	104	24	36	36	36
37	398	602	297	703	899	101	23	37	37	37
38	459	541	361	639	901	099	22	38	38	38
39	521	479	425	575	904	096	21	39	39	39
40	582	418	489	511	907	093	20	40	40	40
41	643	357	552	448	909	091	19	41	41	41
42	704	296	616	384	912	088	18	42	42	42
43	765	235	679	321	914	086	17	43	43	43
44	826	174	743	257	917	083	16	44	44	44
45	887	113	806	194	920	080	15	45	45	45
46	947	053	870	130	922	078	14	46	46	46
47	31008	68992	933	067	925	075	13	47	47	47
48	068	932	996	004	928	072	12	48	48	48
49	129	871	32059	67941	930	070	11	49	49	49
50	189	811	122	878	933	067	10	50	50	50
51	250	750	185	815	936	064	9	51	51	51
52	310	690	248	752	938	062	8	52	52	52
53	370	630	311	689	941	059	7	53	53	53
54	430	570	373	627	944	056	6	54	54	54
55	490	510	436	564	946	054	5	55	55	55
56	549	450	498	502	949	051	4	56	56	56
57	609	391	561	439	952	048	3	57	57	57
58	669	331	623	377	954	046	2	58	58	58
59	728	272	685	315	957	043	1	59	59	59
60	31788	68212	32747	67253	00960	99040	0	60	60	60
$\circ$	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	1'	10.	9.	1'	10.	10.	1'	9.	1'	10.

Proportional Parts										
68	67	66	65	64	63	62	61	60	59	3
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	0
2	2	2	2	2	2	2	2	2	2	0
3	3	3	3	3	3	3	3	3	3	0
4	4	4	4	4	4	4	4	4	4	0
5	5	5	5	5	5	5	5	5	5	0
6	6	6	6	6	6	6	6	6	6	0
7	7	7	7	7	7	7	7	7	7	0
8	8	8	8	8	8	8	8	8	8	0
9	9	9	9	9	9	9	9	9	9	0
10	10	10	10	10	10	10	10	10	10	0
11	11	11	11	11	11	11	11	11	11	0
12	12	12	12	12	12	12	12	12	12	0
13	13	13	13	13	13	13	13	13	13	0
14	14	14	14	14	14	14	14	14	14	0
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17	17	17	17	17	17	17	17	17	17	0
18	18	18	18	18	18	18	18	18	18	0
19	19	19	19	19	19	19	19	19	19	0
20	20	20	20	20	20	20	20	20	20	0
21	21	21	21	21	21	21	21	21	21	0
22	22	22	22	22	22	22	22	22	22	0
23	23	23	23	23	23	23	23	23	23	0
24	24	24	24	24	24	24	24	24	24	0
25	25	25	25	25	25	25	25	25	25	0
26	26	26	26	26	26	26	26	26	26	0
27	27	27	27	27	27	27	27	27	27	0
28	28	28	28	28	28	28	28	28	28	0
29	29	29	29	29	29	29	29	29	29	0
30	30	30	30	30	30	30	30	30	30	0
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62	62	62	62	62	62	62	62	62	62	0
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65	65	65	65	65	65	65	65	65	65	0
66	66	66	66	66	66	66	66	66	66	0
67	67	67	67	67	67	67	67	67	67	0
68	68	68	68	68	68	68	68	68	68	0

Proportional Parts



	$\angle$ sin	d	$\angle$ csc	$\angle$ tan	d	$\angle$ cot	$\angle$ sec	d	$\angle$ cos	
	9.	1'	10.	9.	1'	10.	10.	1'	9.	
0	31788		68212	32747		67253	00960		99040	60
1	847	59	153	810	62	190	962	2	038	59
2	907	60	093	872	62	128	965	3	035	58
3	966	59	034	933	61	067	968	3	032	57
4	32025	59	67975	995	62	005	970	2	030	56
5	084	59	916	33057	62	66943	973	3	027	55
6	143	59	857	119	62	881	976	3	024	54
7	202	59	798	180	61	820	978	3	022	53
8	261	58	739	242	62	758	981	3	019	52
9	319	59	681	303	62	697	984	3	016	51
10	378	59	622	365	61	635	987	2	013	50
11	437	58	563	426	61	574	989	3	011	49
12	495	58	505	487	61	513	992	3	008	48
13	553	58	447	548	61	452	995	3	005	47
14	612	58	388	609	61	391	998	2	002	46
15	670	58	330	670	61	330	01000	3	000	45
16	728	58	272	731	61	269	003	3	989	44
17	786	58	214	792	61	208	006	3	994	43
18	844	58	156	853	60	147	009	3	991	42
19	902	58	098	913	61	087	011	3	989	41
20	960	58	040	974	60	026	014	3	986	40
21	33018	57	66982	34034	61	65960	017	3	983	39
22	075	57	925	095	60	905	020	3	980	38
23	133	57	867	155	60	845	022	3	978	37
24	190	57	810	215	61	785	025	3	975	36
25	248	57	752	276	60	724	028	3	972	35
26	305	57	695	336	60	664	031	3	969	34
27	362	57	638	396	60	604	033	3	967	33
28	420	58	580	456	60	544	036	3	964	32
29	477	57	523	516	60	484	039	3	961	31
30	33534	57	66466	34576	59	65424	01042	3	98958	30
31	591	56	409	635	59	365	045	3	955	29
32	647	56	353	695	59	305	047	3	953	28
33	704	57	296	755	59	245	050	3	950	27
34	761	57	239	814	59	186	053	3	947	26
35	818	57	182	874	59	126	056	3	944	25
36	874	57	126	933	59	067	059	3	941	24
37	931	57	069	992	59	008	062	3	938	23
38	987	56	013	35051	59	64049	064	3	936	22
39	34043	56	65957	111	59	889	067	3	933	21
40	100	56	900	170	59	830	070	3	930	20
41	156	56	844	229	59	771	073	3	927	19
42	212	56	788	288	59	712	076	3	924	18
43	268	56	732	347	58	653	079	2	921	17
44	324	56	676	405	59	595	081	3	919	16
45	380	56	620	464	59	536	084	3	916	15
46	436	55	564	523	58	477	087	3	913	14
47	491	55	509	581	49	419	090	3	910	13
48	547	55	453	640	58	360	093	3	907	12
49	602	56	398	698	59	302	096	3	904	11
50	658	55	342	757	58	243	099	3	901	10
51	713	56	287	815	58	185	102	2	898	9
52	769	55	231	873	58	127	104	3	896	8
53	824	55	176	931	58	069	107	3	893	7
54	879	55	121	989	58	011	110	3	890	6
55	934	55	066	36047	58	63953	113	3	887	5
56	989	55	011	105	58	895	116	3	884	4
57	35044	55	64956	163	58	837	119	3	881	3
58	099	55	901	221	58	779	122	3	878	2
59	154	55	846	279	57	721	125	3	875	1
60	35209	55	64791	36336	57	63664	01128	3	98872	0
	$\angle$ sin	d	$\angle$ sec	$\angle$ cot	d	$\angle$ tan	$\angle$ csc	d	$\angle$ cos	
	9.	1'	10.	9.	1'	10.	10.	1'	9.	

Proportional Parts												
"	63	62	61	60	59	58	57	56	55	3	2	
0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	0
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3	3	3	3	3	3	3	3	3	3	3	3	0
4	4	4	4	4	4	4	4	4	4	4	4	0
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19	19	19	19	19	19	19	19	19	19	19	19	0
20	20	20	20	20	20	20	20	20	20	20	20	0
21	21	21	21	21	21	21	21	21	21	21	21	0
22	22	22	22	22	22	22	22	22	22	22	22	0
23	23	23	23	23	23	23	23	23	23	23	23	0
24	24	24	24	24	24	24	24	24	24	24	24	0
25	25	25	25	25	25	25	25	25	25	25	25	0
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57	57	57	57	57	57	57	57	57	57	57	57	0
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59	59	59	59	59	59	59	59	59	59	59	59	0
60	60	60	60	60	60	60	60	60	60	60	60	0
	$\angle$ sin	d	$\angle$ sec	$\angle$ cot	d	$\angle$ tan	$\angle$ csc	d	$\angle$ cos			
	9.	1'	10.	9.	1'	10.	10.	1'	9.			

Proportional Parts

13°

TABLE II

166°

	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$\cos$	
9.	1'	10.	10.	9.	1'	10.	10.	9.	1'
0	35209	64791	36336	58	63664	01128	98872	60	
1	263	737	394	58	606	131	869	59	
2	318	682	452	57	548	133	867	58	
3	373	627	509	57	491	136	864	57	
4	427	573	566	57	434	139	861	56	
5	481	519	624	57	376	142	858	55	
6	536	464	681	57	319	145	855	54	
7	590	410	738	57	262	148	852	53	
8	644	356	795	57	205	151	849	52	
9	698	302	852	57	148	154	846	51	
10	752	248	909	57	091	157	843	50	
11	806	194	966	57	034	160	840	49	
12	860	140	37023	57	62977	163	837	48	
13	914	086	080	57	920	166	834	47	
14	968	032	137	56	863	169	831	46	
15	36022	63978	193	57	807	172	828	45	
16	075	925	250	56	750	175	825	44	
17	129	871	306	56	694	178	822	43	
18	182	818	363	56	637	181	819	42	
19	236	764	419	57	581	184	816	41	
20	289	711	476	56	524	187	813	40	
21	342	658	532	56	468	190	810	39	
22	395	605	588	56	412	193	807	38	
23	449	551	644	56	356	196	804	37	
24	502	498	700	56	300	199	801	36	
25	555	445	756	56	244	202	798	35	
26	608	392	812	56	188	205	795	34	
27	660	340	868	56	132	208	792	33	
28	713	287	924	56	076	211	789	32	
29	766	234	980	55	020	214	786	31	
30	36819	63181	38035	56	61965	01217	98783	30	
31	871	129	091	56	909	220	780	29	
32	924	076	147	56	853	223	777	28	
33	976	024	202	55	798	226	774	27	
34	37028	62972	257	56	743	229	771	26	
35	081	919	313	55	687	232	768	25	
36	133	867	368	55	632	235	765	24	
37	185	815	423	55	577	238	762	23	
38	237	763	479	55	521	241	759	22	
39	289	711	534	55	466	244	756	21	
40	341	659	589	55	411	247	753	20	
41	393	607	644	55	356	250	750	19	
42	445	555	699	55	301	254	746	18	
43	497	503	754	55	246	257	743	17	
44	549	451	808	55	192	260	740	16	
45	600	400	863	55	137	263	737	15	
46	652	348	918	55	082	266	734	14	
47	703	297	972	55	028	269	731	13	
48	755	245	39027	55	60973	272	728	12	
49	806	194	082	54	918	275	725	11	
50	858	142	136	54	864	278	722	10	
51	909	091	190	55	810	281	719	9	
52	960	040	245	54	755	285	715	8	
53	38011	61989	299	54	701	288	712	7	
54	062	938	353	54	647	291	709	6	
55	113	887	407	54	593	294	706	5	
56	164	836	461	54	539	297	703	4	
57	215	785	515	54	485	300	700	3	
58	266	734	569	54	431	303	697	2	
59	317	683	623	54	377	306	694	1	
60	38368	61632	39677	54	60323	01310	98690	0	
	$\sin$	$d$	$\sec$	$\cot$	$\tan$	$\csc$	$\cos$		

Proportional Parts												
58	57	56	55	54	53	52	51	4	3	2		
0	0	0	0	0	0	0	0	0	0	0		
1	1	1	1	1	1	1	1	1	1	1		
2	2	2	2	2	2	2	2	2	2	2		
3	3	3	3	3	3	3	3	3	3	3		
4	4	4	4	4	4	4	4	4	4	4		
5	5	5	5	5	5	5	5	5	5	5		
6	6	6	6	6	6	6	6	6	6	6		
7	7	7	7	7	7	7	7	7	7	7		
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19	19	19	19	19	19	19	19	19	19	19		
20	20	20	20	20	20	20	20	20	20	20		
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23	23	23	23	23	23	23	23	23	23	23		
24	24	24	24	24	24	24	24	24	24	24		
25	25	25	25	25	25	25	25	25	25	25		
26	26	26	26	26	26	26	26	26	26	26		
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28	28	28	28	28	28	28	28	28	28	28		
29	29	29	29	29	29	29	29	29	29	29		
30	30	30	30	30	30	30	30	30	30	30		
31	31	31	31	31	31	31	31	31	31	31		
32	32	32	32	32	32	32	32	32	32	32		
33	33	33	33	33	33	33	33	33	33	33		
34	34	34	34	34	34	34	34	34	34	34		
35	35	35	35	35	35	35	35	35	35	35		
36	36	36	36	36	36	36	36	36	36	36		
37	37	37	37	37	37	37	37	37	37	37		
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39	39	39	39	39	39	39	39	39	39	39		
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44	44	44	44	44	44	44	44	44	44	44		
45	45	45	45	45	45	45	45	45	45	45		
46	46	46	46	46	46	46	46	46	46	46		
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57	57	57	57	57	57	57	57	57	57	57		
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59	59	59	59	59	59	59	59	59	59	59		
60	60	60	60	60	60	60	60	60	60	60		
	$\sin$	$d$	$\sec$	$\cot$	$\tan$	$\csc$	$\cos$					

103°

76°

14°

TABLE II

165°

$\circ$	$l \sin$	$d$	$l \csc$	$l \tan$	$d$	$l \cot$	$l \sec$	$d$	$l \cos$	$\circ$
9.	1'	10.	9.	1'	10.	10.	1'	9.	1'	9.
0	38368		61632	39677		60323	01310		98690	60
1	418	50	582	731	54	269	313	3	687	59
2	469	51	531	785	54	215	316	3	684	58
3	519	50	481	838	53	162	319	3	681	57
4	570	51	430	892	54	108	322	3	678	56
5	620	50	380	945	53	055	325	3	675	55
6	670	50	330	999	54	001	329	4	671	54
7	721	51	279	40052	53	5948	332	3	668	53
8	771	50	229	106	54	894	335	3	665	52
9	821	50	179	159	53	841	338	3	662	51
10	871	50	129	212	53	788	341	3	659	50
11	921	50	079	266	54	734	344	3	656	49
12	971	50	029	319	53	681	348	4	652	48
13	39021	50	60979	372	53	628	351	3	649	47
14	071	50	929	425	53	575	354	3	646	46
15	121	50	879	478	53	522	357	3	643	45
16	170	49	830	531	53	469	360	3	640	44
17	220	50	780	584	53	416	364	4	636	43
18	270	50	730	636	52	364	367	3	633	42
19	319	49	681	689	53	311	370	3	630	41
20	369	50	631	742	53	258	373	3	627	40
21	418	49	582	795	53	205	377	4	623	39
22	467	49	533	847	52	153	380	3	620	38
23	517	50	483	900	53	100	383	3	617	37
24	566	49	434	952	52	048	386	3	614	36
25	615	49	385	41005	52	58995	390	4	610	35
26	664	49	336	057	52	943	393	3	607	34
27	713	49	287	109	52	891	396	3	604	33
28	762	49	238	161	52	839	399	3	601	32
29	811	49	189	214	53	786	403	4	597	31
30	39860	49	60140	41266	52	58734	01406	3	98594	30
31	909	49	091	318	52	682	409	3	591	29
32	958	49	042	370	52	630	412	3	588	28
33	40006	49	59994	422	52	578	416	4	584	27
34	055	49	945	474	52	526	419	3	581	26
35	103	48	897	526	52	474	422	3	578	25
36	152	49	848	578	52	422	426	4	574	24
37	200	48	800	629	51	371	429	3	571	23
38	249	49	751	681	52	319	432	3	568	22
39	297	48	703	733	52	267	435	3	565	21
40	346	49	654	784	51	216	439	4	561	20
41	394	48	606	836	52	164	442	3	558	19
42	442	48	558	887	51	113	445	3	555	18
43	490	48	510	939	52	061	449	4	551	17
44	538	48	462	990	51	010	452	3	548	16
45	586	48	414	42041	52	57959	455	3	545	15
46	634	48	366	093	52	907	459	4	541	14
47	682	48	318	144	51	856	462	3	538	13
48	730	48	270	195	51	805	465	3	535	12
49	778	48	222	246	51	754	469	4	531	11
50	825	47	175	297	51	703	472	3	528	10
51	873	48	127	348	51	652	475	3	525	9
52	921	48	079	399	51	601	479	3	521	8
53	968	47	032	450	51	550	482	3	518	7
54	41016	48	58984	501	51	499	485	3	515	6
55	063	47	937	552	51	448	489	3	511	5
56	111	48	889	603	51	397	492	3	508	4
57	158	47	842	653	50	347	495	3	505	3
58	205	47	795	704	51	296	499	4	501	2
59	252	47	748	755	51	245	502	3	498	1
60	41300	48	58700	42805	50	57195	01506	3	98494	0
9.	d	10.	9.	d	10.	10.	d	9.	d	9.
$l \cos$	1'	$l \sec$	$l \cot$	1'	$l \tan$	$l \csc$	1'	$l \sin$		

Proportional Parts										
"	54	53	52	51	50	49	48	47	4	3
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	0
2	2	2	2	2	2	2	2	2	0	0
3	3	3	3	3	3	3	3	3	0	0
4	4	4	4	4	4	4	4	4	0	0
5	5	4	4	4	4	4	4	4	0	0
6	6	5	5	5	5	5	5	5	0	0
7	7	6	6	6	6	6	6	6	0	0
8	8	7	7	7	7	7	7	7	0	0
9	8	8	8	8	8	8	7	7	1	0
10	9	9	9	8	8	8	8	8	1	0
11	10	10	10	9	9	9	9	9	1	1
12	11	11	11	10	10	10	10	10	1	1
13	12	11	11	11	11	11	11	11	1	1
14	13	12	12	12	12	12	11	11	1	1
15	14	13	13	13	12	12	12	12	1	1
16	14	14	14	14	13	13	13	13	1	1
17	15	15	15	14	14	14	14	14	1	1
18	16	16	16	15	15	15	14	14	1	1
19	17	17	17	16	16	16	15	15	1	1
20	18	18	17	17	17	16	16	16	1	1
21	19	19	18	18	18	17	17	17	1	1
22	20	19	19	19	18	18	18	18	1	1
23	21	20	20	20	19	19	19	18	2	1
24	22	21	21	20	20	20	19	19	2	1
25	22	22	22	21	21	20	20	20	2	1
26	23	23	23	22	22	21	21	21	2	1
27	24	24	23	23	22	22	22	21	2	1
28	25	25	24	24	23	23	22	22	2	1
29	26	26	25	25	24	24	23	23	2	1
30	27	26	26	26	25	24	24	24	2	2
31	28	27	27	26	26	25	25	24	2	2
32	29	28	28	27	27	26	26	25	2	2
33	30	29	29	28	28	27	26	26	2	2
34	31	30	29	28	28	27	27	27	2	2
35	32	31	30	30	29	29	28	27	2	2
36	32	32	31	31	30	29	29	28	2	2
37	33	33	32	31	31	30	30	29	2	2
38	34	34	33	32	32	31	30	30	3	2
39	35	34	34	33	33	32	31	31	3	2
40	36	35	35	34	33	33	32	31	3	2
41	37	36	36	35	34	33	33	32	3	2
42	38	37	36	36	35	34	34	33	3	2
43	39	38	37	37	36	35	34	34	3	2
44	40	39	38	37	37	36	35	34	3	2
45	40	40	39	38	38	37	36	35	3	2
46	41	41	40	39	38	38	37	36	3	2
47	42	42	41	40	39	38	38	37	3	2
48	43	42	42	41	40	39	38	38	3	2
49	44	43	42	42	41	40	39	38	3	2
50	45	44	43	42	42	41	40	39	3	2
51	46	45	44	43	42	42	41	40	3	3
52	47	46	45	44	43	42	42	41	3	3
53	48	47	46	45	44	43	42	42	4	3
54	49	48	47	46	45	44	43	42	4	3
55	50	49	48	47	46	45	44	43	4	3
56	50	49	48	47	46	45	44	43	4	3
57	51	50	49	48	47	46	45	44	4	3
58	52	51	50	49	48	47	46	45	4	3
59	53	52	51	50	49	48	47	46	4	3
60	54	53	52	51	50	49	48	47	4	3
Proportional Parts										
"	54	53	52	51	50	49	48	47	4	3

104°

75°

15°

TABLE II

164°

	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\sec$
	9.	1'	10.	9.	1'	10.	10.	1'	9.	1'
0	1300		58700	42805		57195	01506		98494	60
1	347	47	653	856	51	144	509	3	491	59
2	394	47	606	906	50	094	512	3	488	58
3	441	47	559	957	51	043	516	4	484	57
4	488	47	512	43007	50	56993	519	3	481	56
5	535	47	465	057	51	943	523	3	477	55
6	582	47	418	108	50	892	526	4	474	54
7	628	46	372	158	50	842	529	3	471	53
8	675	47	325	208	50	792	533	4	467	52
9	722	47	278	258	50	742	536	3	464	51
10	768	47	232	308	50	692	540	3	460	50
11	815	47	185	358	50	642	543	4	457	49
12	861	46	139	408	50	592	547	3	453	48
13	908	47	092	458	50	542	550	3	450	47
14	954	46	046	508	50	492	553	4	447	46
15	42001	47	57999	558	49	442	557	3	443	45
16	047	46	953	607	49	393	560	4	440	44
17	093	47	907	657	50	343	564	3	436	43
18	140	47	860	707	50	293	567	4	433	42
19	186	46	814	756	49	244	571	3	429	41
20	232	46	768	806	49	194	574	4	426	40
21	278	46	722	855	50	145	578	3	422	39
22	324	46	676	905	49	095	581	4	419	38
23	370	46	630	954	50	046	585	3	415	37
24	416	45	584	44004	49	55996	588	4	412	36
25	461	46	539	053	49	947	591	3	409	35
26	507	46	493	102	49	898	595	4	405	34
27	553	46	447	151	50	849	598	3	402	33
28	599	45	401	201	49	799	602	4	398	32
29	644	46	356	250	49	750	605	3	395	31
30	42690	47	57310	44299	49	55701	01609	3	98391	30
31	735	46	265	348	48	652	612	4	388	29
32	781	46	219	397	49	603	616	3	384	28
33	826	46	174	446	49	554	619	4	381	27
34	872	45	128	495	49	505	623	3	377	26
35	917	45	083	544	48	456	627	4	373	25
36	962	45	038	592	48	408	630	3	370	24
37	43008	46	56992	641	49	359	634	4	366	23
38	053	45	947	690	48	310	637	3	363	22
39	098	45	902	738	49	262	641	4	359	21
40	143	45	857	787	49	213	644	3	356	20
41	188	45	812	836	49	164	648	4	352	19
42	233	45	767	884	49	116	651	3	349	18
43	278	45	722	933	48	067	655	4	345	17
44	323	44	677	981	48	019	658	3	342	16
45	367	45	633	45029	49	54971	662	4	338	15
46	412	45	588	078	48	922	666	3	334	14
47	457	45	543	126	48	874	669	4	331	13
48	502	44	498	174	48	826	673	3	327	12
49	546	44	454	222	48	778	676	4	324	11
50	591	44	409	271	48	729	680	3	320	10
51	635	45	365	319	48	681	683	4	317	9
52	680	44	320	367	48	633	687	3	313	8
53	724	44	276	415	45	585	691	4	309	7
54	769	44	231	463	48	537	694	3	306	6
55	813	44	187	511	48	489	698	4	302	5
56	857	44	143	559	47	441	701	3	299	4
57	901	45	099	606	47	394	705	4	295	3
58	946	44	054	654	48	346	709	3	291	2
59	990	44	010	702	48	298	712	4	288	1
60	44034	47	55966	48750	49	54250	01716	3	98284	0
	$\sin$	$d$	$\sec$	$\cot$	$d$	$\tan$	$\csc$	$d$	$\cos$	$\sec$
	9.	1'	10.	9.	1'	10.	10.	1'	9.	1'

Proportional Parts										
	51	50	49	48	47	46	45	44	4	3
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	0
2	2	2	2	2	2	2	2	2	0	0
3	3	3	3	3	3	3	3	3	0	0
4	4	4	4	4	4	4	4	4	0	0
5	5	5	5	5	5	5	5	5	0	0
6	6	6	6	6	6	6	6	6	0	0
7	7	7	7	7	7	7	7	7	0	0
8	8	8	8	8	8	8	8	8	0	0
9	9	9	9	9	9	9	9	9	0	0
10	10	10	10	10	10	10	10	10	1	0
11	11	11	11	11	11	11	11	11	1	1
12	12	12	12	12	12	12	12	12	1	1
13	13	13	13	13	13	13	13	13	1	1
14	14	14	14	14	14	14	14	14	1	1
15	15	15	15	15	15	15	15	15	1	1
16	16	16	16	16	16	16	16	16	1	1
17	17	17	17	17	17	17	17	17	1	1
18	18	18	18	18	18	18	18	18	1	1
19	19	19	19	19	19	19	19	19	1	1
20	20	20	20	20	20	20	20	20	1	1
21	21	21	21	21	21	21	21	21	1	1
22	22	22	22	22	22	22	22	22	1	1
23	23	23	23	23	23	23	23	23	1	1
24	24	24	24	24	24	24	24	24	1	1
25	25	25	25	25	25	25	25	25	1	1
26	26	26	26	26	26	26	26	26	1	1
27	27	27	27	27	27	27	27	27	1	1
28	28	28	28	28	28	28	28	28	1	1
29	29	29	29	29	29	29	29	29	1	1
30	30	30	30	30	30	30	30	30	1	1
31	31	31	31	31	31	31	31	31	1	1
32	32	32	32	32	32	32	32	32	1	1
33	33	33	33	33	33	33	33	33	1	1
34	34	34	34	34	34	34	34	34	1	1
35	35	35	35	35	35	35	35	35	1	1
36	36	36	36	36	36	36	36	36	1	1
37	37	37	37	37	37	37	37	37	1	1
38	38	38	38	38	38	38	38	38	1	1
39	39	39	39	39	39	39	39	39	1	1
40	40	40	40	40	40	40	40	40	1	1
41	41	41	41	41	41	41	41	41	1	1
42	42	42	42	42	42	42	42	42	1	1
43	43	43	43	43	43	43	43	43	1	1
44	44	44	44	44	44	44	44	44	1	1
45	45	45	45	45	45	45	45	45	1	1
46	46	46	46	46	46	46	46	46	1	1
47	47	47	47	47	47	47	47	47	1	1
48	48	48	48	48	48	48	48	48	1	1
49	49	49	49	49	49	49	49	49	1	1
50	50	50	50	50	50	50	50	50	1	1
51	51	51	51	51	51	51	51	51	1	1
52	52	52	52	52	52	52	52	52	1	1
53	53	53	53	53	53	53	53	53	1	1
54	54	54	54	54	54	54	54	54	1	1
55	55	55	55	55	55	55	55	55	1	1
56	56	56	56	56	56	56	56	56	1	1
57	57	57	57	57	57	57	57	57	1	1
58	58	58	58	58	58	58	58	58	1	1
59	59	59	59	59	59	59	59	59	1	1
60	60	60	60	60	60	60	60	60	1	1
	$\sin$	$d$	$\sec$	$\cot$	$d$	$\tan$	$\csc$	$d$	$\cos$	$\sec$
	51	50	49	48	47	46	45	44	4	3

Proportional Parts

105°

74°

16°

TABLE II

163°

°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°
9.	10.	9.	10.	10.	1'	10.	10.	1'	9.	
0	44034	44	55966	45750	47	54250	01716	47	98284	60
1	078	44	922	797	47	203	719	47	281	59
2	122	44	878	845	48	155	723	48	277	58
3	166	44	834	892	47	108	727	47	273	57
4	210	44	790	940	48	060	730	48	270	56
5	253	44	747	987	47	013	734	47	266	55
6	297	44	703	1035	48	53965	738	48	262	54
7	341	44	659	1082	47	918	741	47	259	53
8	385	44	615	1130	48	870	745	48	255	52
9	428	44	572	1177	47	823	749	47	251	51
10	472	44	528	1224	47	776	752	47	248	50
11	516	44	484	1271	47	729	756	47	244	49
12	559	43	441	1319	48	681	760	48	240	48
13	602	43	398	1366	47	634	763	47	237	47
14	646	43	354	1413	47	587	767	47	233	46
15	689	43	311	1460	47	540	771	47	229	45
16	733	43	267	1507	47	493	774	47	226	44
17	776	43	224	1554	47	446	778	47	222	43
18	819	43	181	1601	47	399	782	47	218	42
19	862	43	138	1648	46	352	785	46	215	41
20	905	43	095	1694	47	306	789	47	211	40
21	948	43	052	1741	47	259	793	47	207	39
22	992	43	008	1788	47	212	796	47	204	38
23	45035	43	54965	1835	46	165	800	46	200	37
24	077	43	923	1881	47	119	804	47	196	36
25	120	43	880	1928	47	072	808	47	192	35
26	163	43	837	1975	47	025	811	47	189	34
27	206	43	794	2021	47	52979	815	47	185	33
28	249	43	751	2068	47	932	819	47	181	32
29	292	43	708	2114	46	886	823	46	177	31
30	45334	43	54666	2160	47	52840	01826	47	98174	30
31	377	42	623	207	46	793	830	46	170	29
32	419	42	581	253	46	747	834	46	166	28
33	462	42	538	299	46	701	838	46	162	27
34	504	42	496	346	46	654	841	46	159	26
35	547	42	453	392	46	608	845	46	155	25
36	589	42	411	438	46	562	849	46	151	24
37	632	42	368	484	46	516	853	46	147	23
38	674	42	326	530	46	470	856	46	144	22
39	716	42	284	576	46	424	860	46	140	21
40	758	42	242	622	46	378	864	46	136	20
41	801	42	199	668	46	332	868	46	132	19
42	843	42	157	714	46	286	871	46	129	18
43	885	42	115	760	46	240	875	46	125	17
44	927	42	073	806	46	194	879	46	121	16
45	969	42	031	852	45	148	883	45	117	15
46	46011	42	53989	897	45	103	887	45	113	14
47	053	42	947	943	46	057	890	46	110	13
48	095	41	905	989	46	011	894	46	106	12
49	136	42	864	1035	45	51965	898	45	102	11
50	178	42	822	080	46	920	902	46	098	10
51	220	42	780	126	45	874	906	45	094	9
52	262	42	738	171	45	829	910	45	090	8
53	303	41	697	217	45	783	913	45	087	7
54	345	41	655	262	45	738	917	45	083	6
55	386	42	614	307	45	693	921	45	079	5
56	428	42	572	353	45	647	925	45	075	4
57	469	42	531	398	45	602	929	45	071	3
58	511	42	489	443	45	557	933	45	067	2
59	552	41	448	489	45	511	937	45	063	1
60	46594	41	53406	48534	45	51466	01940	45	98060	0
9.	d	10.	9.	d	10.	9.	d	10.	9.	d
l cos	1'	l sec	l cot	1'	l tan	l csc	1'	l sin	1'	

Proportional Parts										
''	48	47	46	45	44	43	42	41	4	3
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	0
2	2	2	2	2	2	2	2	2	0	0
3	2	2	2	2	2	2	2	2	0	0
4	3	3	3	3	3	3	3	3	0	0
5	4	4	4	4	4	4	4	4	0	0
6	5	5	5	5	5	5	5	5	0	0
7	6	5	5	5	5	5	5	5	0	0
8	6	6	6	6	6	6	6	6	1	0
9	7	7	7	7	7	7	7	7	1	0
10	8	8	8	8	7	7	7	7	1	0
11	9	9	8	8	8	8	8	8	1	1
12	10	9	9	9	9	9	9	8	1	1
13	10	10	10	10	10	9	9	9	1	1
14	11	11	11	11	10	10	10	10	1	1
15	12	12	12	11	11	11	10	10	1	1
16	13	13	12	12	12	11	11	11	1	1
17	14	13	13	13	12	12	12	12	1	1
18	14	14	14	14	13	13	13	12	1	1
19	15	15	15	14	14	14	13	13	1	1
20	16	16	15	15	15	14	14	14	1	1
21	17	16	16	16	15	15	15	14	1	1
22	18	17	17	16	16	16	15	15	1	1
23	18	18	18	17	17	17	16	16	2	1
24	19	19	18	18	18	17	17	17	2	1
25	20	20	19	19	18	18	18	17	2	1
26	21	20	20	20	19	19	18	18	2	1
27	22	21	21	20	20	19	19	18	2	1
28	22	22	21	21	21	20	20	19	2	1
29	23	23	22	22	22	21	20	20	2	1
30	24	24	23	22	22	22	21	20	2	2
31	25	24	24	23	23	22	22	21	2	2
32	26	25	25	24	23	23	22	22	2	2
33	26	26	25	25	24	24	23	23	2	2
34	27	27	26	26	25	24	24	23	2	2
35	28	27	27	26	26	25	24	24	2	2
36	29	28	28	27	26	26	25	25	2	2
37	30	29	28	28	27	27	26	25	2	2
38	30	30	29	28	28	27	27	26	3	2
39	31	31	30	29	29	28	27	27	3	2
40	32	31	31	30	29	29	28	27	3	2
41	33	32	31	31	30	29	29	28	3	2
42	34	33	32	32	31	30	29	29	3	2
43	34	34	33	32	32	31	30	29	3	2
44	35	34	34	33	32	32	31	30	3	2
45	36	35	34	34	33	32	32	31	3	2
46	37	36	35	34	34	33	32	31	3	2
47	38	37	36	35	34	34	33	32	3	2
48	38	38	37	36	35	34	34	33	3	2
49	39	38	38	37	36	35	34	33	3	2
50	40	39	38	38	37	36	35	34	3	2
51	41	40	39	38	37	37	36	35	3	3
52	42	41	40	39	38	37	36	36	3	3
53	42	42	41	40	39	38	37	36	4	3
54	43	42	41	40	40	39	38	37	4	3
55	44	43	42	41	40	39	38	38	4	3
56	45	44	43	42	41	40	39	38	4	3
57	46	45	44	43	42	41	40	39	4	3
58	46	45	44	43	42	41	40	40	4	3
59	47	46	45	44	43	42	41	40	4	3
60	48	47	46	45	44	43	42	41	4	3
Proportional Parts										
''	48	47	46	45	44	43	42	41	4	3

106°

73°

17°

TABLE II

162°

	$\angle$	$\sin$	$d$	$\angle$	$\sec$	$\angle$	$\tan$	$d$	$\angle$	$\cot$	$\angle$	$\sec$	$d$	$\angle$	$\cos$	$\angle$
		9.	1'		10.	9.	1'		10.	10.	1'		10.	9.		
0	46	594		53	406	48	534		51	466	01	940		98	060	60
1		635	41		365	570	45		421	944	4		056	59		59
2		676	41		324	624	45		376	948	4		052	58		58
3		717	41		283	669	45		331	952	4		048	57		57
4		758	42		242	714	45		286	956	4		044	56		56
5		800	41		200	759	45		241	960	4		040	55		55
6		841	41		159	804	45		196	964	4		036	54		54
7		882	41		118	849	45		151	968	3		032	53		53
8		923	41		077	894	45		106	971	3		029	52		52
9		964	41		036	939	45		061	975	4		025	51		51
10	47	005	52	29	95	984	45		016	979	4		021	50		50
11		045	41		955	49	029	50	50	971	4		017	49		49
12		086	41		914	073	45		927	987	4		013	48		48
13		127	41		873	118	45		882	991	4		009	47		47
14		168	41		832	163	45		837	995	4		005	46		46
15		209	40		791	207	45		793	999	4		001	45		45
16		249	40		751	252	45		748	020	03		97	997	44	
17		290	40		710	296	45		704	007	4		993	43		43
18		330	40		670	341	45		659	011	3		989	42		42
19		371	40		629	385	45		615	014	4		986	41		41
20		411	40		589	430	44		570	018	4		982	40		40
21		452	40		548	474	44		526	022	4		978	39		39
22		492	40		508	519	44		481	026	4		974	38		38
23		533	40		467	563	44		437	030	4		970	37		37
24		573	40		427	607	44		393	034	4		966	36		36
25		613	41		387	652	44		348	038	4		962	35		35
26		654	40		346	696	44		304	042	4		958	34		34
27		694	40		306	740	44		260	046	4		954	33		33
28		734	40		266	784	44		216	050	4		950	32		32
29		774	40		226	828	44		172	054	4		946	31		31
30	47	814	40	52	186	872	44	50	128	020	58	97	942	30		30
31		854	40		146	916	44		084	062	4		938	29		29
32		894	40		106	960	44		040	066	4		934	28		28
33		934	40		066	500	04	44	49	996	070	4	930	27		27
34		974	40		026	048	44		952	074	4		926	26		26
35	48	014	39	51	986	092	44	90	88	078	4	92	22	25		25
36		054	39		946	136	44		864	082	4		918	24		24
37		094	39		906	180	44		820	086	4		914	23		23
38		133	39		867	223	43		777	090	4		910	22		22
39		173	40		827	267	44		733	094	4		906	21		21
40		213	39		787	311	44		689	098	4		902	20		20
41		252	39		748	355	44		645	102	4		898	19		19
42		292	40		708	398	44		602	106	4		894	18		18
43		332	40		668	442	43		558	110	4		890	17		17
44		371	40		629	485	44		515	114	4		886	16		16
45		411	39		589	529	43		471	118	4		882	15		15
46		450	39		550	572	44		428	122	4		878	14		14
47		490	39		510	616	44		384	126	4		874	13		13
48		529	39		471	659	44		341	130	4		870	12		12
49		568	39		432	703	43		297	134	5		866	11		11
50		607	40		393	746	44		254	139	4		861	10		10
51		647	39		353	789	44		211	143	4		857	9		9
52		686	39		314	833	43		167	147	4		853	8		8
53		725	39		275	876	43		124	151	4		849	7		7
54		764	39		236	919	43		081	155	4		845	6		6
55		803	39		197	962	43		038	159	4		841	5		5
56		842	39		158	510	05	43	48	995	163	4	837	4		4
57		881	39		119	048	44		952	167	4		833	3		3
58		920	39		080	092	43		908	171	4		829	2		2
59		959	39		041	135	43		865	175	4		825	1		1
60	48	998	39	51	002	51	78	48	822	021	79	97	821	0		0
		9.	d		10.	9.	d		10.	10.	d		9.	d		
		$\angle$	$\cos$		$\angle$	$\sec$	$\angle$		$\angle$	$\cot$	$\angle$		$\angle$	$\sin$		

Proportional Parts											
"	45	44	43	42	41	40	39	5	4	3	
0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9
10	10	10	10	10	10	10	10	10	10	10	10
11	11	11	11	11	11	11	11	11	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12
13	13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14	14
15	15	15	15	15	15	15	15	15	15	15	15
16	16	16	16	16	16	16	16	16	16	16	16
17	17	17	17	17	17	17	17	17	17	17	17
18	18	18	18	18	18	18	18	18	18	18	18
19	19	19	19	19	19	19	19	19	19	19	19
20	20	20	20	20	20	20	20	20	20	20	20
21	21	21	21	21	21	21	21	21	21	21	21
22	22	22	22	22	22	22	22	22	22	22	22
23	23	23	23	23	23	23	23	23	23	23	23
24	24	24	24	24	24	24	24	24	24	24	24
25	25	25	25	25	25	25	25	25	25	25	25
26	26	26	26	26	26	26	26	26	26	26	26
27	27	27	27	27	27	27	27	27	27	27	27
28	28	28	28	28	28	28	28	28	28	28	28
29	29	29	29	29	29	29	29	29	29	29	29
30	30	30	30	30	30	30	30	30	30	30	30
31	31	31	31	31	31	31	31	31	31	31	31
32	32	32	32	32	32	32	32	32	32	32	32
33	33	33	33	33	33	33	33	33	33	33	33
34	34	34	34	34	34	34	34	34	34	34	34
35	35	35	35	35	35	35	35	35	35	35	35
36	36	36	36	36	36	36	36	36	36	36	36
37	37	37	37	37	37	37	37	37	37	37	37
38	38	38	38	38	38	38	38	38	38	38	38
39	39	39	39	39	39	39	39	39	39	39	39
40	40	40	40	40	40	40	40	40	40	40	40
41	41	41	41	41	41	41	41	41	41	41	41
42	42	42	42	42	42	42	42	42	42	42	42
43	43	43	43	43	43	43	43	43	43	43	43
44	44	44	44	44	44	44	44	44	44	44	44
45	45	45	45	45	45	45	45	45	45	45	45
46	46	46	46	46	46	46	46	46	46	46	46
47	47	47	47	47	47	47	47	47	47	47	47
48	48	48	48	48	48	48	48	48	48	48	48
49	49	49	49	49	49	49	49	49	49	49	49
50	50	50	50	50	50	50	50	50	50	50	50
51	51	51	51	51	51	51	51	51	51	51	51
52	52	52	52	52	52	52	52	52	52	52	52
53	53	53	53	53	53	53	53	53	53	53	53
54	54	54	54	54	54	54	54	54	54	54	54
55	55	55	55	55	55	55	55	55	55	55	55
56	56	56	56	56	56	56	56	56	56	56	56
57	57	57	57	57	57	57	57	57	57	57	57
58	58	58	58	58	58	58	58	58	58	58	58
59	59	59	59	59	59	59	59	59	59	59	59
60	60	60	60	60	60	60	60	60	60	60	60
"	45	44	43	42	41	40	39	5	4	3	

18°

TABLE II

161°

"	$\angle$ sin	d	$\angle$ csc	$\angle$ tan	d	$\angle$ cot	$\angle$ sec	d	$\angle$ cos	"
9.	10.	9.	10.	9.	10.	9.	10.	9.	10.	9.
0	48998		51002	51178		48822	02179		97821	60
1	49037	39	50963	221	43	779	183	4	817	59
2	076	39	924	264	43	736	188	4	812	58
3	115	39	885	306	42	694	192	4	808	57
4	153	38	847	349	43	651	196	4	804	56
5	192	38	808	392	43	608	200	4	800	55
6	231	38	769	435	43	565	204	4	796	54
7	269	38	731	478	42	522	208	4	792	53
8	308	39	692	520	42	480	212	4	788	52
9	347	38	653	563	43	437	216	4	784	51
10	385	39	615	606	42	394	221	4	779	50
11	424	38	576	648	43	352	225	4	775	49
12	462	38	538	691	43	309	229	4	771	48
13	500	39	500	734	42	266	233	4	767	47
14	539	38	461	776	43	224	237	4	763	46
15	577	38	423	819	42	181	241	5	759	45
16	615	39	385	861	42	139	246	4	754	44
17	654	38	346	903	43	097	250	4	750	43
18	692	38	308	946	42	054	254	4	746	42
19	730	38	270	988	43	012	258	4	742	41
20	768	38	232	52031	42	47969	262	4	738	40
21	806	38	194	073	42	927	266	4	734	39
22	844	38	156	115	42	885	271	5	729	38
23	882	38	118	157	43	843	275	4	725	37
24	920	38	080	200	43	800	279	4	721	36
25	958	38	042	242	42	758	283	4	717	35
26	996	39	004	284	42	716	287	4	713	34
27	50034	38	49966	326	42	674	292	5	708	33
28	072	38	928	368	42	632	296	4	704	32
29	110	38	890	410	43	590	300	4	700	31
30	50148	37	49852	52452	42	47548	02304	5	97696	30
31	185	38	815	494	42	506	309	4	691	29
32	223	38	777	536	42	464	313	4	687	28
33	261	37	739	578	42	422	317	4	683	27
34	298	37	702	620	41	380	321	5	679	26
35	336	36	664	661	42	339	326	4	674	25
36	374	37	626	703	42	297	330	4	670	24
37	411	37	589	745	42	255	334	4	666	23
38	449	38	551	787	42	213	338	4	662	22
39	486	37	514	829	41	171	343	5	657	21
40	523	37	477	870	41	130	347	4	653	20
41	561	38	439	912	42	088	351	4	649	19
42	598	37	402	953	42	047	355	4	645	18
43	635	38	365	995	42	005	360	5	640	17
44	673	37	327	53037	42	46963	364	4	636	16
45	710	37	290	078	42	922	368	4	632	15
46	747	37	253	120	41	880	372	4	628	14
47	784	37	216	161	41	839	377	5	623	13
48	821	37	179	202	42	798	381	4	619	12
49	858	38	142	244	41	756	385	5	615	11
50	896	38	104	285	41	715	390	4	610	10
51	933	37	067	327	42	673	394	4	606	9
52	970	37	030	368	41	632	398	4	602	8
53	51007	36	48993	409	41	591	403	5	597	7
54	043	36	957	450	41	550	407	4	593	6
55	080	37	920	492	41	508	411	5	589	5
56	117	37	883	533	41	467	416	4	584	4
57	154	37	846	574	41	426	420	4	580	3
58	191	37	809	615	41	385	424	5	576	2
59	227	36	773	656	41	344	429	4	571	1
60	51264	37	48736	53697	42	46303	02433	4	97567	0
9.	d	9.	d	9.	d	9.	d	9.	d	9.
$\angle$ cos	$\angle$ sec	$\angle$ cot	$\angle$ tan	$\angle$ csc	$\angle$ sin	$\angle$ cos	$\angle$ sec	$\angle$ cot	$\angle$ tan	$\angle$ csc

"	43	42	41	39	38	37	36	5	4
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2
4	3	3	3	3	3	3	3	3	3
5	4	4	4	4	4	4	4	4	4
6	4	4	4	4	4	4	4	4	4
7	5	5	5	5	5	5	5	5	5
8	6	6	6	6	6	6	6	6	6
9	6	6	6	6	6	6	6	6	6
10	7	7	7	7	7	7	7	7	7
11	8	8	8	8	8	8	8	8	8
12	9	9	9	9	9	9	9	9	9
13	9	9	9	9	9	9	9	9	9
14	10	10	10	10	10	10	10	10	10
15	11	11	11	11	11	11	11	11	11
16	11	11	11	11	11	11	11	11	11
17	12	12	12	12	12	12	12	12	12
18	13	13	13	13	13	13	13	13	13
19	14	14	14	14	14	14	14	14	14
20	14	14	14	14	14	14	14	14	14
21	15	15	15	15	15	15	15	15	15
22	16	16	16	16	16	16	16	16	16
23	16	16	16	16	16	16	16	16	16
24	17	17	17	17	17	17	17	17	17
25	18	18	18	18	18	18	18	18	18
26	19	19	19	19	19	19	19	19	19
27	19	19	19	19	19	19	19	19	19
28	20	20	20	20	20	20	20	20	20
29	21	21	21	21	21	21	21	21	21
30	22	22	22	22	22	22	22	22	22
31	22	22	22	22	22	22	22	22	22
32	23	23	23	23	23	23	23	23	23
33	24	24	24	24	24	24	24	24	24
34	24	24	24	24	24	24	24	24	24
35	25	25	25	25	25	25	25	25	25
36	26	26	26	26	26	26	26	26	26
37	27	27	27	27	27	27	27	27	27
38	27	27	27	27	27	27	27	27	27
39	28	28	28	28	28	28	28	28	28
40	29	29	29	29	29	29	29	29	29
41	29	29	29	29	29	29	29	29	29
42	30	30	30	30	30	30	30	30	30
43	31	31	31	31	31	31	31	31	31
44	32	32	32	32	32	32	32	32	32
45	32	32	32	32	32	32	32	32	32
46	33	33	33	33	33	33	33	33	33
47	34	34	34	34	34	34	34	34	34
48	34	34	34	34	34	34	34	34	34
49	35	35	35	35	35	35	35	35	35
50	36	36	36	36	36	36	36	36	36
51	37	37	37	37	37	37	37	37	37
52	37	37	37	37	37	37	37	37	37
53	38	38	38	38	38	38	38	38	38
54	39	39	39	39	39	39	39	39	39
55	39	39	39	39	39	39	39	39	39
56	40	40	40	40	40	40	40	40	40
57	41	41	41	41	41	41	41	41	41
58	42	42	42	42	42	42	42	42	42
59	42	42	42	42	42	42	42	42	42
60	43	43	43	43	43	43	43	43	43
"	43	42	41	39	38	37	36	5	4
	Proportional Parts								

108°

71°

19°

TABLE II

160°

	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	
	9.	1'	10.	9.	1'	10.	10.	1'	9.	
0	51264		48736	53697		46303	02433		97567	60
1	301	37	699	738	41	262	437	4	563	59
2	338	36	662	779	41	221	442	4	558	58
3	374	36	626	820	41	180	446	4	554	57
4	411	36	589	861	41	139	450	4	550	56
5	447	37	553	902	41	098	455	4	545	55
6	484	37	516	943	41	057	459	4	541	54
7	520	36	480	984	41	016	464	4	536	53
8	557	36	443	54025	41	45975	468	4	532	52
9	593	36	407	065	41	935	472	4	528	51
10	629	37	371	106	41	894	477	4	523	50
11	666	37	334	147	41	853	481	4	519	49
12	702	36	298	187	41	813	485	4	515	48
13	738	36	262	228	41	772	490	4	510	47
14	774	37	226	269	41	731	494	4	506	46
15	811	36	189	309	41	691	499	4	501	45
16	847	36	153	350	41	650	503	4	497	44
17	883	36	117	390	41	610	508	4	492	43
18	919	36	081	431	41	569	512	4	488	42
19	955	36	045	471	41	529	516	4	484	41
20	991	37	009	512	41	488	521	4	479	40
21	52027	37	47973	553	41	448	525	4	475	39
22	063	36	937	593	41	407	530	4	470	38
23	099	36	901	633	41	367	534	4	466	37
24	135	36	865	673	41	327	539	4	461	36
25	171	36	829	714	41	286	543	4	457	35
26	207	36	793	754	41	246	547	4	453	34
27	243	35	758	794	41	206	552	4	448	33
28	278	36	722	835	41	165	556	4	444	32
29	314	36	686	875	41	125	561	4	439	31
30	52350	36	47650	54915	41	45085	02565	4	97435	30
31	385	36	615	955	41	045	570	4	430	29
32	421	35	579	995	41	005	574	4	426	28
33	456	35	544	55038	41	44965	579	4	421	27
34	492	35	508	075	41	925	583	4	417	26
35	527	35	473	115	41	885	588	4	412	25
36	563	35	437	155	41	845	592	4	408	24
37	598	35	402	195	41	805	597	4	403	23
38	634	35	366	235	41	765	601	4	399	22
39	669	35	331	275	41	725	606	4	394	21
40	705	35	295	315	41	685	610	4	390	20
41	740	35	260	355	41	645	615	4	385	19
42	775	35	225	395	39	605	619	4	381	18
43	811	35	189	434	41	566	624	4	376	17
44	846	35	154	474	41	526	628	4	372	16
45	881	35	119	514	41	486	633	4	367	15
46	916	35	084	554	39	446	637	4	363	14
47	951	35	049	593	39	407	642	4	358	13
48	986	35	014	633	39	367	647	4	353	12
49	53021	35	46979	673	39	327	651	4	349	11
50	056	36	944	712	40	288	656	4	344	10
51	092	34	908	752	39	248	660	4	340	9
52	126	35	874	791	39	209	665	4	335	8
53	161	35	839	831	39	169	669	4	331	7
54	196	35	804	870	39	130	674	4	326	6
55	231	35	769	910	39	090	678	4	322	5
56	266	35	734	949	39	051	683	4	317	4
57	301	35	699	989	39	011	688	4	312	3
58	336	35	664	56028	39	43972	692	4	308	2
59	370	34	630	067	39	933	697	4	303	1
60	53405	35	46595	56107	39	43893	02701	4	97299	0
	9.	d	10.	9.	d	10.	10.	d	9.	
	l cos	1'	l sec	l cot	1'	l tan	l csc	1'	l sin	

Proportional Parts										
	41	40	39	37	36	35	34	5	4	
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2	2
4	3	3	3	3	3	3	3	3	3	3
5	3	3	3	3	3	3	3	3	3	3
6	4	4	4	4	4	4	4	4	4	4
7	5	5	5	5	5	5	5	5	5	5
8	5	5	5	5	5	5	5	5	5	5
9	6	6	6	6	6	6	6	6	6	6
10	7	7	7	7	7	7	7	7	7	7
11	8	8	8	8	8	8	8	8	8	8
12	8	8	8	8	8	8	8	8	8	8
13	9	9	9	9	9	9	9	9	9	9
14	10	10	10	10	10	10	10	10	10	10
15	10	10	10	10	10	10	10	10	10	10
16	11	11	11	11	11	11	11	11	11	11
17	12	12	12	12	12	12	12	12	12	12
18	12	12	12	12	12	12	12	12	12	12
19	13	13	13	13	13	13	13	13	13	13
20	14	14	14	14	14	14	14	14	14	14
21	14	14	14	14	14	14	14	14	14	14
22	15	15	15	15	15	15	15	15	15	15
23	15	15	15	15	15	15	15	15	15	15
24	16	16	16	16	16	16	16	16	16	16
25	17	17	17	17	17	17	17	17	17	17
26	17	17	17	17	17	17	17	17	17	17
27	18	18	18	18	18	18	18	18	18	18
28	19	19	19	19	19	19	19	19	19	19
29	20	20	20	20	20	20	20	20	20	20
30	20	20	20	20	20	20	20	20	20	20
31	21	21	21	21	21	21	21	21	21	21
32	22	22	22	22	22	22	22	22	22	22
33	23	23	23	23	23	23	23	23	23	23
34	23	23	23	23	23	23	23	23	23	23
35	24	24	24	24	24	24	24	24	24	24
36	25	25	25	25	25	25	25	25	25	25
37	25	25	25	25	25	25	25	25	25	25
38	26	26	26	26	26	26	26	26	26	26
39	27	27	27	27	27	27	27	27	27	27
40	27	27	27	27	27	27	27	27	27	27
41	28	28	28	28	28	28	28	28	28	28
42	29	29	29	29	29	29	29	29	29	29
43	29	29	29	29	29	29	29	29	29	29
44	30	30	30	30	30	30	30	30	30	30
45	31	31	31	31	31	31	31	31	31	31
46	31	31	31	31	31	31	31	31	31	31
47	32	32	32	32	32	32	32	32	32	32
48	33	33	33	33	33	33	33	33	33	33
49	33	33	33	33	33	33	33	33	33	33
50	34	34	34	34	34	34	34	34	34	34
51	35	35	35	35	35	35	35	35	35	35
52	36	36	36	36	36	36	36	36	36	36
53	36	36	36	36	36	36	36	36	36	36
54	37	37	37	37	37	37	37	37	37	37
55	38	38	38	38	38	38	38	38	38	38
56	38	38	38	38	38	38	38	38	38	38
57	39	39	39	39	39	39	39	39	39	39
58	40	40	40	40	40	40	40	40	40	40
59	40	40	40	40	40	40	40	40	40	40
60	41	41	41	41	41	41	41	41	41	41
Proportional Parts										
	41	40	39	37	36	35	34	5	4	

109°

70°



20°

TABLE II

159°

$\circ$	$l \sin$	$d$	$l \csc$	$l \tan$	$d$	$l \cot$	$l \sec$	$d$	$l \cos$	$l$
9.	10.	10.	9.	10.	10.	10.	10.	10.	9.	10.
0	53405	46595	56107	43893	02701	97299	60			
1	440	560	146	854	706	294	59			
2	475	525	185	815	711	289	58			
3	509	491	224	776	715	285	57			
4	544	456	264	736	720	280	56			
5	578	422	303	697	724	276	55			
6	613	387	342	658	729	271	54			
7	647	353	381	619	734	266	53			
8	682	318	420	580	738	262	52			
9	716	284	459	541	743	257	51			
10	751	249	498	502	748	252	50			
11	785	215	537	463	752	248	49			
12	819	181	576	424	757	243	48			
13	854	146	615	385	762	238	47			
14	888	112	654	346	766	234	46			
15	922	078	693	307	771	229	45			
16	957	043	732	268	776	224	44			
17	991	009	771	229	780	220	43			
18	54025	45975	810	190	785	215	42			
19	059	941	849	151	790	210	41			
20	093	907	887	113	794	206	40			
21	127	873	926	074	799	201	39			
22	161	839	965	035	804	196	38			
23	195	805	57004	42996	808	192	37			
24	229	771	042	395	813	187	36			
25	263	737	081	356	818	182	35			
26	297	703	120	317	822	178	34			
27	331	669	158	278	827	173	33			
28	365	635	197	239	832	168	32			
29	399	601	235	200	837	163	31			
30	54433	45567	57274	42726	02841	97159	30			
31	466	534	312	688	846	154	29			
32	500	500	351	649	851	149	28			
33	534	466	389	611	855	145	27			
34	567	433	428	572	860	140	26			
35	601	399	466	534	865	135	25			
36	635	365	504	496	870	130	24			
37	668	332	543	457	874	126	23			
38	702	298	581	419	879	121	22			
39	735	265	619	381	884	116	21			
40	769	231	658	342	889	111	20			
41	802	198	696	304	893	107	19			
42	836	164	734	266	898	102	18			
43	869	131	772	228	903	97	17			
44	903	97	810	190	908	92	16			
45	936	064	849	151	913	87	15			
46	969	031	887	113	917	83	14			
47	55003	44997	925	075	922	078	13			
48	036	964	963	037	927	073	12			
49	069	931	58001	41999	932	068	11			
50	102	898	039	961	937	063	10			
51	136	864	077	923	941	059	9			
52	169	831	115	885	946	054	8			
53	202	798	153	847	951	049	7			
54	235	765	191	809	956	044	6			
55	268	732	229	771	961	039	5			
56	301	699	267	733	965	035	4			
57	334	666	304	696	970	030	3			
58	367	633	342	658	975	025	2			
59	400	600	380	620	980	020	1			
60	55433	44567	58418	41582	02985	97015	0			
9.	10.	9.	10.	10.	9.	10.	9.			
$l \cos$	$l \sec$	$l \cot$	$l \tan$	$l \csc$	$l \sin$					

Proportional Parts										
"	40	39	38	37	35	34	33	5	4	
0	0	0	0	0	0	0	0	0	0	
1	1	1	1	1	1	1	1	0	0	
2	1	1	1	1	1	1	1	0	0	
3	2	2	2	2	2	2	2	0	0	
4	3	3	3	3	2	2	2	0	0	
5	3	3	3	3	3	3	3	0	0	
6	4	4	4	4	4	4	4	0	0	
7	5	5	5	5	4	4	4	1	0	
8	5	5	5	5	5	5	5	1	1	
9	6	6	6	6	5	5	5	1	1	
10	7	6	6	6	6	6	6	1	1	
11	7	7	7	7	6	6	6	1	1	
12	8	8	8	8	7	7	7	1	1	
13	9	8	8	8	8	8	8	1	1	
14	9	9	9	9	8	8	8	1	1	
15	10	10	10	10	9	9	9	1	1	
16	11	10	10	10	9	9	9	1	1	
17	11	11	11	10	10	10	10	1	1	
18	12	12	11	11	10	10	10	2	1	
19	13	12	12	12	11	11	11	2	1	
20	13	13	13	12	12	11	11	2	1	
21	14	14	13	13	12	12	12	2	1	
22	15	14	14	13	12	12	12	2	1	
23	15	15	15	14	13	13	13	2	2	
24	16	16	15	15	14	14	13	2	2	
25	17	16	16	15	15	14	14	2	2	
26	17	17	16	16	15	15	14	2	2	
27	18	17	17	17	16	15	15	2	2	
28	19	18	18	17	16	16	15	2	2	
29	19	18	18	18	17	16	16	2	2	
30	20	20	19	18	18	17	16	2	2	
31	21	20	20	19	18	17	16	3	2	
32	21	21	20	20	19	18	17	3	2	
33	22	21	21	20	19	18	17	3	2	
34	23	22	22	21	20	19	18	3	2	
35	23	23	22	22	21	20	19	3	2	
36	24	23	23	22	21	20	19	3	2	
37	25	24	23	23	22	21	20	3	2	
38	25	25	24	23	22	21	20	3	3	
39	26	25	25	24	23	22	21	3	3	
40	27	26	25	25	24	23	22	3	3	
41	27	27	26	25	24	23	22	3	3	
42	28	27	27	26	24	24	23	4	3	
43	29	28	27	27	25	24	24	4	3	
44	29	29	28	27	26	25	24	4	3	
45	30	29	28	28	26	26	25	4	3	
46	31	30	29	28	27	26	25	4	3	
47	31	31	30	29	27	27	26	4	3	
48	32	31	30	30	28	27	26	4	3	
49	33	32	31	30	29	28	27	4	3	
50	33	32	32	31	29	28	28	4	3	
51	34	33	32	31	30	29	28	4	3	
52	35	34	33	32	30	29	29	4	3	
53	35	34	34	33	31	30	29	4	4	
54	36	35	34	33	32	31	30	4	4	
55	37	36	35	34	32	31	30	5	4	
56	37	36	35	35	33	32	31	5	4	
57	38	37	36	35	33	32	31	5	4	
58	39	38	37	36	34	33	32	5	4	
59	39	38	37	36	34	33	32	5	4	
60	40	39	38	37	35	34	33	5	4	
"	40	39	38	37	35	34	33	5	4	
Proportional Parts										

110°

69°

										Proportional Parts									
°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°	38	37	36	35	34	33	32	31	30
0	55433		44567	58418		41582	02985		97015	60	0	0	0	0	0	0	0	0	0
1	466	33	534	455	37	545	990	5	010	59	1	1	1	1	1	1	1	1	1
2	499	33	501	493	38	507	995	5	005	58	2	1	1	1	1	1	1	1	1
3	532	33	468	531	38	469	999	5	001	57	3	2	2	2	2	2	2	2	2
4	564	33	436	569	37	431	03004	5	96996	56	4	3	2	2	2	2	2	2	2
5	597	33	403	606	38	394	009	5	991	55	5	3	3	3	3	3	3	3	3
6	630	33	370	644	37	356	014	5	986	54	6	4	4	4	4	4	4	4	4
7	663	33	337	681	37	319	019	5	981	53	7	4	4	4	4	4	4	4	4
8	695	33	305	719	38	281	024	5	976	52	8	5	5	5	5	5	5	5	5
9	728	33	272	757	37	243	029	5	971	51	9	6	6	6	6	6	6	6	6
10	761	33	239	794	38	206	034	4	966	50	10	6	6	6	6	6	6	6	6
11	793	33	207	832	38	168	038	4	962	49	11	7	7	7	7	7	7	7	7
12	826	33	174	869	37	131	043	5	957	48	12	8	7	7	7	7	7	7	7
13	858	33	142	907	38	093	048	5	952	47	13	8	8	8	8	8	8	8	8
14	891	33	109	944	37	056	053	5	947	46	14	9	9	9	9	9	9	9	9
15	923	33	077	981	38	019	058	5	942	45	15	10	9	9	9	9	9	9	9
16	956	33	044	59019	37	40981	063	5	937	44	16	10	10	10	10	10	10	10	10
17	988	33	012	056	38	944	068	5	932	43	17	11	10	10	9	9	9	9	9
18	56021	33	43979	094	37	906	073	5	927	42	18	11	11	11	10	10	9	9	9
19	053	33	947	131	37	869	078	5	922	41	19	12	12	11	10	10	10	10	10
20	085	33	915	168	37	832	083	5	917	40	20	13	12	12	11	11	10	10	10
21	118	33	882	205	38	795	088	5	912	39	21	13	13	13	12	11	11	11	11
22	150	33	850	243	37	757	093	5	907	38	22	14	14	13	12	12	11	11	11
23	182	33	818	280	37	720	097	4	903	37	23	15	14	14	13	12	12	12	12
24	215	33	785	317	37	683	102	5	898	36	24	15	15	14	13	13	12	12	12
25	247	33	753	354	37	646	107	5	893	35	25	16	15	15	14	13	13	12	12
26	279	33	721	391	38	609	112	5	888	34	26	16	16	16	14	14	13	13	12
27	311	33	689	429	37	571	117	5	883	33	27	17	17	16	15	14	14	13	12
28	343	33	657	466	37	534	122	5	878	32	28	18	17	17	15	15	14	13	12
29	375	33	625	503	37	497	127	5	873	31	29	18	18	17	16	15	15	14	13
30	56408	33	43592	59540	37	40460	03132	5	96368	30	30	19	18	18	16	16	16	16	16
31	440	33	560	577	37	423	137	5	863	29	31	20	19	19	17	17	16	16	16
32	472	33	528	614	37	386	142	5	858	28	32	20	20	19	18	17	17	17	17
33	504	33	496	651	37	349	147	5	853	27	33	21	20	20	18	18	17	17	17
34	536	33	464	688	37	312	152	5	848	26	34	22	21	20	19	18	18	18	18
35	568	33	432	725	37	275	157	5	843	25	35	22	22	21	19	19	18	18	18
36	599	33	401	762	37	238	162	5	838	24	36	23	22	22	20	19	19	19	19
37	631	33	369	799	37	201	167	5	833	23	37	23	23	22	20	20	19	19	19
38	663	33	337	835	36	165	172	5	828	22	38	24	23	23	21	20	20	19	19
39	695	33	305	872	37	128	177	5	823	21	39	25	24	23	21	21	20	19	19
40	727	33	273	909	37	091	182	5	818	20	40	25	25	24	22	21	21	21	21
41	759	33	241	946	37	054	187	5	813	19	41	26	25	25	23	22	21	21	21
42	790	33	210	983	37	017	192	5	808	18	42	27	26	25	23	22	22	21	21
43	822	33	178	60019	37	39981	197	5	803	17	43	27	27	26	24	23	22	22	22
44	854	33	146	056	37	944	202	5	798	16	44	28	27	26	24	23	23	23	23
45	886	33	114	093	37	907	207	5	793	15	45	28	28	27	25	24	23	23	23
46	917	33	083	130	36	870	212	5	788	14	46	29	28	28	25	25	24	24	24
47	949	33	051	166	37	834	217	5	783	13	47	30	29	28	26	26	25	25	25
48	980	33	020	203	37	797	222	5	778	12	48	30	30	29	26	26	25	25	25
49	57012	33	42988	240	36	760	228	0	772	11	49	31	30	29	27	26	25	25	25
50	044	33	956	276	37	724	233	5	767	10	50	32	31	30	28	27	26	26	26
51	075	33	925	313	36	687	238	5	762	9	51	32	31	31	28	27	26	26	26
52	107	33	893	349	37	651	243	5	757	8	52	33	32	31	29	28	27	27	27
53	138	33	862	386	36	614	248	5	752	7	53	34	33	32	29	28	27	27	27
54	169	33	831	422	37	578	253	5	747	6	54	34	33	32	30	29	28	28	28
55	201	33	799	459	37	541	258	5	742	5	55	35	34	33	30	29	28	28	28
56	232	33	768	495	36	505	263	5	737	4	56	35	35	34	31	30	29	29	29
57	264	33	736	532	36	468	268	5	732	3	57	36	35	34	31	30	29	29	29
58	295	33	705	568	36	432	273	5	727	2	58	37	36	35	32	31	30	30	30
59	326	33	674	605	36	395	278	5	722	1	59	37	36	35	32	31	30	30	30
60	57358	33	42642	60641	37	39359	03283	5	96717	0	60	38	37	36	33	32	31	31	31
9.	l cos	d	l sec	l cot	d	l tan	l csc	d	l sin	9.	38	37	36	35	34	33	32	31	30

"	<i>l</i> sin 9.	<i>d</i> 1'	<i>l</i> csc 10.	<i>l</i> tan 9.	<i>d</i> 1'	<i>l</i> cot 10.	<i>l</i> sec 10.	<i>d</i> 1'	<i>l</i> cos 9.	"
0	57358		42642	60641		39359	03283		96717	60
1	389	31	611	677	36	323	289	6	711	59
2	420	31	580	714	37	286	294	5	706	58
3	451	31	549	750	36	250	299	5	701	57
4	482	31	518	786	36	214	304	5	696	56
5	514	31	486	823	37	177	309	5	691	55
6	545	31	455	859	36	141	314	5	686	54
7	576	31	424	895	36	105	319	5	681	53
8	607	31	393	931	36	069	324	5	676	52
9	638	31	362	967	36	033	330	6	670	51
10	669	31	331	1004	37	38996	335	5	665	50
11	700	31	300	040	36	960	340	5	660	49
12	731	31	269	076	36	924	345	5	655	48
13	762	31	238	112	36	888	350	5	650	47
14	793	31	207	148	36	852	355	5	645	46
15	824	31	176	184	36	816	360	6	640	45
16	855	31	145	220	36	780	366	5	634	44
17	885	31	115	256	36	744	371	5	629	43
18	916	31	084	292	36	708	376	5	624	42
19	947	31	053	328	36	672	381	5	619	41
20	978	30	022	364	36	636	386	6	614	40
21	58008	31	41992	400	36	600	392	5	608	39
22	039	31	961	436	36	564	397	5	603	38
23	070	31	930	472	36	528	402	5	598	37
24	101	31	899	508	36	492	407	5	593	36
25	131	31	869	544	35	456	412	6	588	35
26	162	30	838	579	35	421	418	5	582	34
27	192	30	808	615	35	385	423	5	577	33
28	223	30	777	651	34	349	428	5	572	32
29	253	30	747	687	34	313	433	5	567	31
30	58284	31	41716	61722	36	38278	03438	6	96562	30
31	314	31	686	758	36	242	444	5	556	29
32	345	31	655	794	36	206	449	5	551	28
33	375	31	625	830	36	170	454	5	546	27
34	406	30	594	865	35	135	459	6	541	26
35	436	31	564	901	35	099	465	5	535	25
36	467	31	533	936	35	064	470	5	530	24
37	497	30	503	972	35	028	475	5	525	23
38	527	30	473	62008	35	37992	480	6	520	22
39	557	31	443	043	36	957	486	5	514	21
40	588	30	412	079	35	921	491	5	509	20
41	618	30	382	114	35	886	496	5	504	19
42	648	30	352	150	35	850	502	6	498	18
43	678	31	322	185	35	815	507	5	493	17
44	709	31	291	221	35	779	512	5	488	16
45	739	30	261	256	36	744	517	6	483	15
46	769	30	231	292	35	708	523	6	477	14
47	799	30	201	327	35	673	528	6	472	13
48	829	30	171	362	35	638	533	6	467	12
49	859	30	141	398	35	602	539	6	461	11
50	889	30	111	433	35	567	544	5	456	10
51	919	30	081	468	36	532	549	5	451	9
52	949	30	051	504	35	496	555	5	445	8
53	979	30	021	539	35	461	560	5	440	7
54	59009	31	40991	574	35	426	565	6	435	6
55	039	31	961	609	36	391	571	5	429	5
56	069	30	931	644	35	355	576	5	424	4
57	098	30	902	680	35	320	581	6	419	3
58	128	30	872	715	35	285	587	5	413	2
59	158	30	842	750	35	250	592	5	408	1
60	59188	31	40812	62785	36	37215	03597	6	96403	0
"	9.	"	10.	9.	"	10.	10.	"	9.	"
"	<i>l</i> cos	"	<i>l</i> sec	<i>l</i> cot	"	<i>l</i> tan	<i>l</i> csc	"	<i>l</i> sin	"

Proportional Parts										
"	37	36	35	32	31	30	29	6	5	
0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2	2
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7	4	4	4	4	4	4	4	4	4	4
8	5	5	5	5	5	5	5	5	5	5
9	6	5	5	5	5	5	5	5	5	5
10	6	6	6	6	5	5	5	5	5	5
11	7	7	6	6	6	6	6	6	6	6
12	7	7	7	6	6	6	6	6	6	6
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18	11	11	10	10	10	10	10	10	10	10
19	12	11	11	10	10	10	10	10	10	10
20	12	12	12	11	11	11	11	11	11	11
21	13	13	12	12	12	12	12	12	12	12
22	14	13	13	12	12	12	12	12	12	12
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24	15	14	14	13	12	12	12	12	12	12
25	15	15	15	14	13	13	13	13	13	13
26	16	15	15	14	14	14	14	14	14	14
27	16	16	16	15	14	14	14	14	14	14
28	17	17	16	15	15	15	15	15	15	15
29	18	17	17	16	15	15	15	15	15	15
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31	19	18	18	17	17	17	17	17	17	17
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35	22	21	21	20	19	19	19	19	19	19
36	22	22	22	21	20	20	20	20	20	20
37	23	22	22	22	21	21	21	21	21	21
38	23	23	22	22	22	22	22	22	22	22
39	24	23	23	22	22	22	22	22	22	22
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51	31	31	30	27	26	26	26	26	26	26
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53	33	32	31	28	27	27	27	27	27	27
54	33	32	32	29	28	28	28	28	28	28
55	34	33	32	29	28	28	28	28	28	28
56	35	34	33	30	29	29	29	29	29	29
57	35	34	33	30	29	29	29	29	29	29
58	36	35	34	31	30	30	30	30	30	30
59	36	35	34	31	30	30	30	30	30	30
60	37	36	35	32	31	31	31	31	31	31
"	37	36	35	32	31	31	31	31	31	31
Proportional Parts										

$\circ$	$\prime$	$\sin$	$d$	$\prime$	$\sec$	$\tan$	$d$	$\prime$	$\cot$	$\sec$	$d$	$\prime$	$\cos$	$\prime$
0	59	188	1	40812	62785	37	17	10	10	10	1	9	9	60
1	218	782	820	35	180	603	6	5	5	5	5	5	397	59
2	247	753	855	35	145	608	392	58					392	58
3	277	723	890	35	110	613	387	57					387	57
4	307	693	926	35	74	619	381	56					381	56
5	336	664	961	35	39	624	376	55					376	55
6	366	634	996	35	004	630	370	54					370	54
7	396	604	63031	35	36969	635	365	53					365	53
8	425	575	066	35	934	640	360	52					360	52
9	455	545	101	34	899	646	354	51					354	51
10	484	516	135	35	865	651	349	50					349	50
11	514	486	170	35	830	657	343	49					343	49
12	543	457	205	35	795	662	338	48					338	48
13	573	427	240	35	760	667	333	47					333	47
14	602	398	275	35	725	673	327	46					327	46
15	632	368	310	35	690	678	322	45					322	45
16	661	339	345	34	655	684	316	44					316	44
17	690	310	379	34	621	689	311	43					311	43
18	720	280	414	34	586	695	305	42					305	42
19	749	251	449	35	551	700	300	41					300	41
20	778	222	484	35	516	706	294	40					294	40
21	808	192	519	34	481	711	289	39					289	39
22	837	163	553	34	447	716	284	38					284	38
23	866	134	588	34	412	722	278	37					278	37
24	895	105	623	34	377	727	273	36					273	36
25	924	076	657	35	343	733	267	35					267	35
26	954	046	692	35	308	738	262	34					262	34
27	983	017	726	34	274	744	256	33					256	33
28	60012	39988	761	35	239	749	251	32					251	32
29	041	959	796	34	204	755	245	31					245	31
30	60070	39930	63830	35	36170	03760	96240	30					96240	30
31	099	901	865	35	135	766	234	29					234	29
32	128	872	899	35	101	771	229	28					229	28
33	157	843	934	35	066	777	223	27					223	27
34	186	814	968	35	032	782	218	26					218	26
35	215	785	64003	35	35997	788	212	25					212	25
36	244	756	037	35	963	793	207	24					207	24
37	273	727	072	35	928	799	201	23					201	23
38	302	698	106	35	894	804	196	22					196	22
39	331	669	140	35	860	810	190	21					190	21
40	359	641	175	34	825	815	185	20					185	20
41	388	612	209	34	791	821	179	19					179	19
42	417	583	243	35	757	826	174	18					174	18
43	446	554	278	34	722	832	168	17					168	17
44	474	526	312	34	688	838	162	16					162	16
45	503	497	346	35	654	843	157	15					157	15
46	532	468	381	35	619	849	151	14					151	14
47	561	439	415	34	585	854	146	13					146	13
48	589	411	449	34	551	860	140	12					140	12
49	618	382	483	34	517	865	135	11					135	11
50	646	354	517	35	483	871	129	10					129	10
51	675	325	552	34	448	877	123	9					123	9
52	704	296	586	34	414	882	118	8					118	8
53	732	268	620	34	380	888	112	7					112	7
54	761	239	654	34	346	893	107	6					107	6
55	789	211	688	34	312	899	101	5					101	5
56	818	182	722	34	278	905	095	4					095	4
57	846	154	756	34	244	910	090	3					090	3
58	875	125	790	34	210	916	084	2					084	2
59	903	097	824	34	176	921	079	1					079	1
60	60931	39069	64858	35	35142	03927	96073	0					96073	0
$\circ$	$\prime$	$\sin$	$d$	$\prime$	$\sec$	$\tan$	$d$	$\prime$	$\cot$	$\sec$	$d$	$\prime$	$\cos$	$\prime$
9	10	10	1	9	10	10	1	9	10	10	1	9	9	9

"	Proportional Parts							
	36	35	34	30	29	28	6	5
0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0
2	1	1	1	1	1	1	0	0
3	2	2	2	2	2	2	0	0
4	2	2	2	2	2	2	0	0
5	3	3	3	2	2	2	0	0
6	3	3	3	3	3	3	1	0
7	4	4	4	4	4	4	1	1
8	5	5	5	4	4	4	1	1
9	5	5	5	4	4	4	1	1
10	6	6	6	5	5	5	1	1
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12	7	7	7	6	6	6	1	1
13	8	8	8	7	7	7	1	1
14	8	8	8	7	7	7	1	1
15	9	9	9	8	8	8	2	1
16	10	10	10	8	8	8	2	1
17	10	10	10	9	9	9	2	2
18	11	11	11	10	10	10	2	2
19	11	11	11	10	10	10	2	2
20	12	12	12	10	10	10	2	2
21	13	13	13	11	11	11	2	2
22	13	13	13	11	11	11	2	2
23	14	14	14	12	12	12	2	2
24	14	14	14	12	12	12	2	2
25	15	15	15	12	12	12	2	2
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27	16	16	16	14	14	14	3	2
28	17	17	17	14	14	14	3	2
29	17	17	17	14	14	14	3	2
30	18	18	18	15	15	15	3	3
31	19	19	19	16	16	16	3	3
32	19	19	19	16	16	16	3	3
33	20	20	20	17	17	17	4	3
34	20	20	20	17	17	17	4	3
35	21	21	21	18	18	18	4	3
36	22	22	22	18	18	18	4	3
37	22	22	22	18	18	18	4	3
38	23	23	23	19	19	19	4	3
39	23	23	23	20	20	20	4	3
40	24	24	24	20	20	20	4	3
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45	27	27	27	23	23	23	5	4
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57	34	34	34	28	28	28	6	5
58	35	35	35	30	30	30	6	5
59	35	35	35	30	30	30	6	5
60	36	36	36	30	30	30	6	5
"	36	35	34	30	29	28	6	5

24°

TABLE II

155°

	$\ell$	$\sin$	$d$	$\ell$	$\tan$	$d$	$\ell$	$\cot$	$d$	$\ell$	$\cos$	$d$	$\ell$	$\sin$
	9.	1'	10.	9.	1'	10.	10.	1'	10.	1'	9.	1'	10.	1'
0	60931		39069	64858		35142	03927		96073	60				
1	960	28	040	892	34	108	933	6	067	59				
2	988	28	012	926	34	074	938	5	062	58				
3	61016	28	38084	960	34	040	944	6	056	57				
4	045	20	955	994	34	006	950	6	050	56				
5	073	28	927	65028	34	34972	955	6	045	55				
6	101	28	899	062	34	938	961	6	039	54				
7	129	28	871	096	34	904	966	5	034	53				
8	158	20	842	130	34	870	972	6	028	52				
9	186	28	814	164	33	836	978	5	022	51				
10	214	28	786	197	34	803	983	6	017	50				
11	242	28	758	231	34	769	989	6	011	49				
12	270	28	730	265	34	735	995	5	005	48				
13	298	28	702	299	34	701	04000	6	000	47				
14	326	28	674	333	33	667	006	6	95994	46				
15	354	28	646	366	34	634	012	6	988	45				
16	382	28	618	400	34	600	018	6	982	44				
17	411	20	589	434	34	566	023	5	977	43				
18	438	28	562	467	33	533	029	6	971	42				
19	466	28	534	501	34	499	035	5	965	41				
20	494	28	506	535	33	465	040	6	960	40				
21	522	28	478	568	34	432	046	6	954	39				
22	550	28	450	602	34	398	052	6	948	38				
23	578	28	422	636	34	364	058	5	942	37				
24	606	28	394	669	34	331	063	6	937	36				
25	634	28	366	703	33	297	069	6	931	35				
26	662	27	338	736	34	264	075	5	925	34				
27	689	27	311	770	33	230	080	6	920	33				
28	717	27	283	803	33	197	086	6	914	32				
29	745	27	255	837	34	163	092	6	908	31				
30	61773	27	38227	65870	34	34130	04098	5	95902	30				
31	800	28	200	904	34	096	103	5	897	29				
32	828	28	172	937	34	063	109	6	891	28				
33	856	28	144	971	34	029	115	6	885	27				
34	883	27	117	66004	34	33996	121	6	879	26				
35	911	28	089	038	34	962	127	6	873	25				
36	939	27	061	071	33	929	132	5	868	24				
37	966	27	034	104	33	896	138	6	862	23				
38	994	28	006	138	34	862	144	6	856	22				
39	62021	27	37979	171	33	829	150	6	850	21				
40	049	27	951	204	34	796	156	5	844	20				
41	076	27	924	238	34	762	161	6	839	19				
42	104	27	896	271	33	729	167	6	833	18				
43	131	27	869	304	33	696	173	6	827	17				
44	159	27	841	337	34	663	179	6	821	16				
45	186	28	814	371	33	629	185	5	815	15				
46	214	28	786	404	33	596	190	6	810	14				
47	241	27	759	437	33	563	196	6	804	13				
48	268	28	732	470	33	530	202	6	798	12				
49	296	27	704	503	34	497	208	6	792	11				
50	323	27	677	537	33	463	214	6	786	10				
51	350	27	650	570	33	430	220	5	780	9				
52	377	28	623	603	33	397	225	6	775	8				
53	405	27	595	636	33	364	231	6	769	7				
54	432	27	568	669	33	331	237	6	763	6				
55	459	27	541	702	33	298	243	5	757	5				
56	486	27	514	735	33	265	249	6	751	4				
57	513	28	487	768	33	232	255	6	745	3				
58	541	27	459	801	33	199	261	6	739	2				
59	568	27	432	834	33	166	267	5	733	1				
60	62595	27	37405	66867	33	33133	04272	5	95728	0				
	9.	$d$	10.	9.	$d$	10.	10.	$d$	9.	$d$	10.	$d$	9.	$d$
	$\ell$	$\cos$	$\ell$	$\sec$	$\ell$	$\cot$	$\ell$	$\tan$	$\ell$	$\csc$	$\ell$	$\sin$	$\ell$	$\cos$

"	Proportional Parts						
	34	33	29	28	27	6	5
0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0
2	1	1	1	1	1	0	0
3	2	2	1	1	1	0	0
4	2	2	2	2	2	0	0
5	3	3	2	2	2	0	0
6	3	3	3	3	3	1	0
7	4	4	3	3	3	1	1
8	5	4	4	4	4	1	1
9	5	5	4	4	4	1	1
10	6	6	5	5	4	1	1
11	6	6	5	5	5	1	1
12	7	7	6	6	5	1	1
13	7	7	6	6	6	1	1
14	8	8	7	7	6	1	1
15	8	8	7	7	7	2	1
16	9	9	8	7	7	2	1
17	10	9	8	8	8	2	1
18	10	10	9	8	8	2	2
19	11	10	9	9	9	2	2
20	11	11	10	9	9	2	2
21	12	12	10	10	9	2	2
22	12	12	11	10	10	2	2
23	13	13	11	11	10	2	2
24	14	13	12	11	11	2	2
25	14	14	12	12	11	2	2
26	15	14	13	12	12	3	2
27	15	15	13	13	12	3	2
28	16	15	14	13	13	3	2
29	16	16	14	14	13	3	2
30	17	16	14	14	14	3	2
31	18	17	15	14	14	3	3
32	18	18	15	15	14	3	3
33	19	18	16	15	15	3	3
34	19	19	16	16	15	3	3
35	20	19	17	16	16	4	3
36	20	20	17	17	16	4	3
37	21	20	18	17	17	4	3
38	22	21	18	18	17	4	3
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50	28	28	24	23	22	5	4
51	29	28	25	24	23	5	4
52	29	29	25	24	23	5	4
53	30	29	26	25	24	5	4
54	31	30	26	25	24	5	4
55	31	30	27	26	25	6	5
56	32	31	27	26	25	6	5
57	32	31	28	27	26	6	5
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59	33	32	29	28	27	6	5
60	34	33	29	28	27	6	5
"	34	33	29	28	27	6	5
Proportional Parts							

Proportional Parts

114°

65°

25°

TABLE II

154°

	$l \sin$	$d$	$l \csc$	$l \tan$	$d$	$l \cot$	$l \sec$	$d$	$l \cos$	$l$
	9.	1'	10.	9.	1'	10.	10.	1'	9.	
0	62595		37405	66867		33133	04272		95728	60
1	622	27	378	900	33	100	278	6	722	59
2	649	27	351	933	33	067	284	6	716	58
3	676	27	324	966	33	034	290	6	710	57
4	703	27	297	999	33	001	296	6	704	56
5	730	27	270	67032	33	32968	302	6	698	55
6	757	27	243	065	33	935	308	6	692	54
7	784	27	216	098	33	902	314	6	686	53
8	811	27	189	131	33	869	320	6	680	52
9	838	27	162	163	33	837	326	6	674	51
10	865	27	135	196	33	804	332	6	668	50
11	892	27	108	229	33	771	337	6	663	49
12	918	26	082	262	33	738	343	6	657	48
13	945	27	055	295	32	705	349	6	651	47
14	972	27	028	327	32	673	355	6	645	46
15	999	27	001	360	33	640	361	6	639	45
16	63026	26	36974	393	33	607	367	6	633	44
17	052	27	948	426	33	574	373	6	627	43
18	079	27	921	458	32	542	379	6	621	42
19	106	27	894	491	33	509	385	6	615	41
20	133	26	867	524	32	476	391	6	609	40
21	159	27	841	556	33	444	397	6	603	39
22	186	27	814	589	33	411	403	6	597	38
23	213	27	787	622	32	378	409	6	591	37
24	239	26	761	654	32	346	415	6	585	36
25	266	26	734	687	32	313	421	6	579	35
26	292	27	708	719	32	281	427	6	573	34
27	319	26	681	752	32	248	433	6	567	33
28	345	26	655	785	31	215	439	6	561	32
29	372	27	628	817	32	183	445	6	555	31
30	63398	27	36602	67850	33	32150	04451	6	95549	30
31	425	26	575	832	33	118	457	6	543	29
32	451	26	549	865	33	085	463	6	537	28
33	478	26	522	897	32	053	469	6	531	27
34	504	27	496	930	32	020	475	6	525	26
35	531	27	469	68012	33	31988	481	6	519	25
36	557	27	443	044	32	956	487	6	513	24
37	583	27	417	077	32	923	493	6	507	23
38	610	27	390	109	32	891	500	7	500	22
39	636	26	364	142	32	858	506	6	494	21
40	662	26	338	174	32	826	512	6	488	20
41	689	27	311	206	32	794	518	6	482	19
42	715	26	285	239	33	761	524	6	476	18
43	741	26	259	271	32	729	530	6	470	17
44	767	26	233	303	33	697	536	6	464	16
45	794	27	206	336	32	664	542	6	458	15
46	820	26	180	368	32	632	548	6	452	14
47	846	26	154	400	32	600	554	6	446	13
48	872	26	128	432	32	568	560	6	440	12
49	898	26	102	465	32	535	566	7	434	11
50	924	26	076	497	32	503	573	6	427	10
51	950	26	050	529	32	471	579	6	421	9
52	976	26	024	561	32	439	585	6	415	8
53	64002	26	35998	593	33	407	591	6	409	7
54	028	26	972	626	32	374	597	6	403	6
55	054	26	946	658	32	342	603	6	397	5
56	080	26	920	690	32	310	609	7	391	4
57	106	26	894	722	32	278	616	6	384	3
58	132	26	868	754	32	246	622	6	378	2
59	158	26	842	786	32	214	628	6	372	1
60	64184	26	35816	68818	33	31182	04634	6	95366	0
	9.	$d$	10.	9.	$d$	10.	10.	$d$	9.	
	$l \cos$	1'	$l \sec$	$l \cot$	1'	$l \tan$	$l \csc$	1'	$l \sin$	

Proportional Parts							
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2	1	1	1	1	0	0	0
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4	2	2	2	2	0	0	0
5	3	3	2	2	1	0	0
6	3	3	3	3	1	1	0
7	4	4	3	3	1	1	1
8	4	4	4	3	1	1	1
9	5	5	4	4	1	1	1
10	6	5	4	4	1	1	1
11	6	6	5	5	1	1	1
12	7	6	5	5	1	1	1
13	7	7	6	6	2	1	1
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59	32	31	27	26	7	6	5
60	33	32	27	26	7	6	5
	33	32	27	26	7	6	5
Proportional Parts							

115°

64°

	$\sin$	$\csc$	$\tan$	$\cot$	$\sec$	$\cos$
9.	10.	9.	10.	10.	9.	10.
0	64184	35816	68818	31182	04634	95366
1	210	790	850	150	640	360 59
2	236	764	882	118	646	354 58
3	262	738	914	086	652	348 57
4	288	712	946	054	659	341 56
5	313	687	978	022	665	335 55
6	339	661	990	000	671	329 54
7	365	635	042	958	677	323 53
8	391	609	074	926	683	317 52
9	417	583	106	894	690	310 51
10	442	558	138	862	696	304 50
11	468	532	170	830	702	298 49
12	494	506	202	798	708	292 48
13	519	481	234	766	714	286 47
14	545	455	266	734	721	279 46
15	571	429	298	702	727	273 45
16	596	404	329	671	733	267 44
17	622	378	361	639	739	261 43
18	647	353	393	607	746	254 42
19	673	327	425	575	752	248 41
20	698	302	457	543	758	242 40
21	724	276	488	512	764	236 39
22	749	251	520	480	771	229 38
23	775	225	552	448	777	223 37
24	800	200	584	416	783	217 36
25	826	174	615	385	789	211 35
26	851	149	647	353	796	204 34
27	877	123	679	321	802	198 33
28	902	098	710	290	808	192 32
29	927	073	742	258	815	185 31
30	64953	35047	69774	30226	04821	95179 30
31	978	022	805	195	827	173 29
32	65003	34997	837	163	833	167 28
33	029	971	868	132	840	160 27
34	054	946	900	100	846	154 26
35	079	921	932	068	852	148 25
36	104	896	963	037	859	141 24
37	130	870	995	005	865	135 23
38	155	845	70026	29974	871	129 22
39	180	820	058	942	878	122 21
40	205	795	089	911	884	116 20
41	230	770	121	879	890	110 19
42	255	745	152	848	897	103 18
43	281	719	184	816	903	097 17
44	306	694	215	785	910	090 16
45	331	669	247	753	916	084 15
46	356	644	278	722	922	078 14
47	381	619	309	691	929	071 13
48	406	594	341	659	935	065 12
49	431	569	372	628	941	059 11
50	456	544	404	596	948	052 10
51	481	519	435	565	954	046 9
52	506	494	466	534	961	039 8
53	531	469	498	502	967	033 7
54	556	444	529	471	973	027 6
55	580	420	560	440	980	020 5
56	605	395	592	408	986	014 4
57	630	370	623	377	993	007 3
58	655	345	654	346	999	001 2
59	680	320	685	315	05005	94995 1
60	65705	34295	70717	29283	05012	94988 0
9.	10.	9.	10.	10.	9.	10.
$\cos$	$\sec$	$\cot$	$\tan$	$\csc$	$\sin$	

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4	2	2	2	2	2	0	0
5	3	3	2	2	2	1	0
6	3	3	3	2	2	1	1
7	4	4	3	3	3	1	1
8	4	4	3	3	3	1	1
9	5	5	4	4	4	1	1
10	5	5	4	4	4	1	1
11	6	6	5	5	5	1	1
12	6	6	5	5	5	1	1
13	7	7	6	6	6	2	1
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17	9	9	7	7	7	2	2
18	10	9	8	8	8	2	2
19	10	10	8	8	8	2	2
20	11	10	9	8	8	2	2
21	11	11	9	9	9	3	2
22	12	11	10	9	9	3	2
23	12	12	10	10	10	3	2
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40	21	21	17	17	16	5	4
41	22	21	18	17	16	5	4
42	22	22	18	18	17	5	4
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44	23	23	19	18	18	5	4
45	24	23	20	19	18	5	4
46	25	24	20	19	18	5	5
47	25	24	20	20	19	5	5
48	26	25	21	20	19	6	5
49	26	25	21	20	20	6	5
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55	29	28	24	23	22	6	6
56	30	29	24	23	22	7	6
57	30	29	25	24	23	7	6
58	31	30	25	24	23	7	6
59	31	30	26	25	24	7	6
60	32	31	26	25	24	7	6
''	32	31	26	25	24	7	6

Proportional Parts

$\circ$	$\iota$	$\sin$	$d$	$\iota$	$\sec$	$\iota$	$\tan$	$d$	$\iota$	$\cot$	$\iota$	$\sec$	$d$	$\iota$	$\cos$	$\circ$
		9.	1'		10.	9.	1'		10.	10.		10.	1'		9.	
0	65	705			34295	70717			29283	05012			6	7	94988	60
1		729			271	748			252	018			6	7	982	59
2		754			246	779			221	025			6	7	975	58
3		779			221	810			190	031			6	7	969	57
4		804			196	841			159	038			6	7	962	56
5		828			172	873			127	044			6	7	956	55
6		853			147	904			096	051			6	7	949	54
7		878			122	935			065	057			6	7	943	53
8		902			098	966			034	064			6	7	936	52
9		927			073	997			003	070			6	7	930	51
10		952			048	71028			28972	077			6	7	923	50
11		976			024	059			941	083			6	6	917	49
12	66	001			33999	090			910	089			6	7	911	48
13		025			975	121			879	096			6	7	904	47
14		050			950	153			847	102			6	7	898	46
15		075			925	184			816	109			6	6	891	45
16		099			901	215			785	115			6	6	885	44
17		124			876	246			754	122			6	7	878	43
18		148			852	277			723	129			6	7	871	42
19		173			827	308			692	135			6	7	865	41
20		197			803	339			661	142			6	7	858	40
21		221			779	370			630	148			6	7	852	39
22		246			754	401			599	155			6	7	845	38
23		270			730	431			569	161			6	7	839	37
24		295			705	462			538	168			6	7	832	36
25		319			681	493			507	174			6	7	826	35
26		343			657	524			476	181			6	7	819	34
27		368			632	555			445	187			6	7	813	33
28		392			608	586			414	194			6	7	806	32
29		416			584	617			383	201			6	7	799	31
30	66	441			33559	71648			28352	05207			6	7	94793	30
31		465			535	679			321	214			6	7	786	29
32		489			511	709			291	220			6	7	780	28
33		513			487	740			260	227			6	7	773	27
34		537			463	771			229	233			6	7	767	26
35		562			438	802			198	240			6	7	760	25
36		586			414	833			167	247			6	7	753	24
37		610			390	863			137	253			6	7	747	23
38		634			366	894			106	260			6	7	740	22
39		658			342	925			075	266			6	7	734	21
40		682			318	955			045	273			6	7	727	20
41		706			294	986			014	280			6	7	720	19
42		731			269	72017			27983	286			6	7	714	18
43		755			245	048			952	293			6	7	707	17
44		779			221	078			922	300			6	7	700	16
45		803			197	109			891	306			6	7	694	15
46		827			173	140			860	313			6	7	687	14
47		851			149	170			830	320			6	7	680	13
48		875			125	201			799	326			6	7	674	12
49		899			101	231			769	333			6	7	667	11
50		922			078	262			738	340			6	6	660	10
51		946			054	293			707	346			6	6	654	9
52		970			030	323			677	353			6	7	647	8
53		994			006	354			646	360			6	7	640	7
54	67	018			32982	384			616	366			6	7	634	6
55		042			958	415			585	373			6	7	627	5
56		066			934	445			555	380			6	7	620	4
57		090			910	476			524	386			6	7	614	3
58		113			887	506			494	393			6	7	607	2
59		137			863	537			463	400			6	7	600	1
60	67	161			33839	72567			27433	05407			6	7	94593	0
	$\iota$	$\cos$	$d$	$\iota$	$\sec$	$\iota$	$\cot$	$d$	$\iota$	$\tan$	$\iota$	$\sec$	$d$	$\iota$	$\sin$	$\circ$
		9.	1'		10.	9.	1'		10.	10.		10.	1'		9.	

"	Proportional Parts							
	32	31	30	25	24	23	7	6
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3	2	2	2	1	1	1	0	0
4	2	2	2	2	2	2	0	0
5	3	3	2	2	2	2	1	0
6	3	3	3	2	2	2	1	1
7	4	4	4	3	3	3	1	1
8	4	4	4	3	3	3	1	1
9	5	5	4	4	4	3	1	1
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15	8	8	8	6	6	6	2	2
16	9	8	8	7	6	6	2	2
17	9	9	8	7	7	7	2	2
18	10	9	9	8	7	7	2	2
19	10	10	10	8	8	7	2	2
20	11	10	10	8	8	8	2	2
21	11	11	10	9	8	8	2	2
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31	17	16	16	13	12	12	4	3
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39	21	20	20	16	16	15	5	4
40	21	21	20	17	16	15	5	4
41	22	21	20	17	16	16	5	4
42	22	22	21	18	17	16	5	4
43	23	22	22	18	17	16	5	4
44	23	23	22	18	18	17	5	4
45	24	23	22	19	18	17	5	4
46	25	24	23	19	18	18	5	5
47	25	24	24	20	19	18	5	5
48	26	25	24	20	19	18	6	5
49	26	25	24	20	20	19	6	5
50	27	26	25	21	20	19	6	5
51	27	26	26	21	20	20	6	5
52	28	27	26	22	21	20	6	5
53	28	27	26	22	21	20	6	5
54	29	28	27	22	22	21	6	5
55	29	28	28	23	22	21	6	6
56	30	29	28	23	22	21	7	6
57	30	29	28	24	23	22	7	6
58	31	30	29	24	23	22	7	6
59	31	30	30	25	24	23	7	6
60	32	31	30	25	24	23	7	6
"	32	31	30	25	24	23	7	6



°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°
9.	10.	9.	10.	10.	1.	10.	10.	1.	9.	9.
0	67161	24	32839	72567	31	27433	05407	6	94593	60
1	185	24	815	598	31	402	413	6	587	59
2	208	23	792	628	30	372	420	7	580	58
3	232	23	768	659	31	341	427	7	573	57
4	256	24	744	689	30	311	433	6	567	56
5	280	23	720	720	28	280	440	7	560	55
6	303	23	697	750	29	250	447	7	553	54
7	327	24	673	780	30	220	454	6	546	53
8	350	23	650	811	31	189	460	7	540	52
9	374	24	626	841	30	159	467	7	533	51
10	398	23	602	872	31	128	474	7	526	50
11	421	24	579	902	30	998	481	6	519	49
12	445	23	555	932	31	968	487	7	513	48
13	468	24	532	963	30	937	494	7	506	47
14	492	23	508	993	31	907	501	7	499	46
15	515	24	485	73023	28	877	508	7	492	45
16	539	23	461	054	31	846	515	6	485	44
17	562	24	438	084	30	816	521	7	479	43
18	586	23	414	114	31	786	528	7	472	42
19	609	24	391	144	30	756	535	7	465	41
20	633	23	367	175	31	725	542	7	458	40
21	656	24	344	205	30	695	549	6	451	39
22	680	23	320	235	31	665	555	7	445	38
23	703	24	297	265	30	635	562	7	438	37
24	726	23	274	295	31	605	569	7	431	36
25	750	24	250	326	30	574	576	7	424	35
26	773	23	227	356	31	544	583	7	417	34
27	796	24	204	386	30	514	590	6	410	33
28	820	23	180	416	31	484	596	7	404	32
29	843	24	157	446	30	454	603	7	397	31
30	866	23	134	476	31	423	610	7	390	30
31	890	24	110	507	30	393	617	7	383	29
32	913	23	87	537	31	363	624	7	376	28
33	936	24	64	567	30	333	631	7	369	27
34	959	23	41	597	31	303	638	7	362	26
35	982	24	18	627	30	273	645	6	355	25
36	68006	23	31994	657	31	243	651	7	349	24
37	029	24	971	687	30	213	658	7	342	23
38	052	23	948	717	31	183	665	7	335	22
39	075	24	925	747	30	153	672	7	328	21
40	098	23	902	777	31	123	679	7	321	20
41	121	24	879	807	30	93	686	7	314	19
42	144	23	856	837	31	63	693	7	307	18
43	167	24	833	867	30	33	700	7	300	17
44	190	23	810	897	31	3	707	7	293	16
45	213	24	787	927	30	273	714	7	286	15
46	237	23	763	957	31	243	721	7	279	14
47	260	24	740	987	30	213	727	7	273	13
48	283	23	717	74017	31	183	734	7	266	12
49	305	24	695	047	30	153	741	7	259	11
50	328	23	672	077	31	123	748	7	252	10
51	351	24	649	107	30	93	755	7	245	9
52	374	23	626	137	31	63	762	7	238	8
53	397	24	603	166	30	33	769	7	231	7
54	420	23	580	196	31	3	776	7	224	6
55	443	24	557	226	30	273	783	7	217	5
56	466	23	534	256	31	243	790	7	210	4
57	489	24	511	286	30	213	797	7	203	3
58	512	23	488	316	31	183	804	7	196	2
59	534	24	466	345	30	153	811	7	189	1
60	68557	23	31443	74375	28	25625	05818	6	94182	0
	l cos	d	l sec	l cot	d	l tan	l csc	d	l sin	°

Proportional Parts									
31	30	29	28	27	26	25	24	23	22
0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1
3	2	2	1	1	1	1	1	1	0
4	2	2	2	2	2	2	2	1	0
5	3	2	2	2	2	2	2	2	1
6	3	3	3	3	2	2	2	2	1
7	4	4	3	3	3	3	3	3	1
8	4	4	4	3	3	3	3	3	1
9	5	4	4	4	4	3	3	3	1
10	5	5	5	4	4	4	4	4	1
11	6	5	5	5	4	4	4	4	1
12	6	6	6	5	5	5	5	5	1
13	7	6	6	6	5	5	5	5	2
14	7	7	7	6	5	5	5	5	2
15	8	8	7	6	6	6	6	6	2
16	8	8	8	6	6	6	6	6	2
17	9	8	8	7	7	7	7	7	2
18	9	9	9	7	7	7	7	7	2
19	10	10	9	8	7	7	7	7	2
20	10	10	10	8	8	8	8	8	2
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23	12	12	11	9	9	9	9	9	2
24	12	12	12	10	9	9	9	9	3
25	13	12	12	10	10	10	10	10	3
26	13	13	13	10	10	10	10	10	3
27	14	14	13	11	10	10	10	10	3
28	14	14	14	11	11	11	11	11	3
29	15	14	14	12	11	11	11	11	3
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52	27	26	25	21	20	20	20	20	6
53	27	26	26	21	21	21	21	21	6
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55	28	28	27	22	22	22	22	22	6
56	29	28	27	22	22	22	22	22	7
57	29	28	28	23	22	22	22	22	7
58	30	29	28	23	23	23	23	23	7
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60	31	30	29	24	24	24	24	24	7
	31	30	29	24	23	22	22	22	7
	31	30	29	24	23	22	22	22	7
Proportional Parts									

29°

TABLE II

150°

	$\angle$	$\sin$	$d$	$\angle$	$\cos$	$d$	$\angle$	$\tan$	$d$	$\angle$	$\cot$	$d$	$\angle$	$\sec$	$d$	$\angle$	$\cos$	$d$
	$\angle$	9.	1'	10.	9.	1'	10.	10.	1'	10.	10.	1'	9.	1'	10.	10.	1'	9.
0	68	557		31443	74375		25625	05818		94182	60		0					
1	580		23	420	405	30	595	825	7	175	59		0					
2	603		22	397	435	30	565	832	7	168	58		2	1	1	1	0	0
3	625		23	375	465	29	535	839	7	161	57		3	2	1	1	0	0
4	648		23	352	494	29	506	846	7	154	56		4	2	2	2	1	0
5	671		23	329	524	30	476	853	7	147	55		5	2	2	2	1	1
6	694		23	306	554	30	446	860	7	140	54		6	3	3	2	2	1
7	716		22	284	583	29	417	867	7	133	53		7	4	3	3	3	1
8	739		22	261	613	30	387	874	7	126	52		8	4	4	3	3	1
9	762		22	238	643	30	357	881	7	119	51		9	4	4	3	3	1
10	784		23	216	673	30	327	888	7	112	50		10	5	5	4	4	1
11	807		23	193	702	29	298	895	7	105	49		11	6	5	4	4	1
12	829		22	171	732	30	268	902	8	98	48		12	6	6	5	4	2
13	852		23	148	762	30	238	910	8	90	47		13	6	6	5	5	2
14	875		22	125	791	29	209	917	7	83	46		14	7	7	5	5	2
15	897		23	103	821	30	179	924	7	76	45		15	8	7	6	6	2
16	920		23	080	851	30	149	931	7	69	44		16	8	8	6	6	2
17	942		22	058	880	30	120	938	7	62	43		17	8	8	7	6	2
18	965		23	035	910	29	090	945	7	55	42		18	9	9	7	7	2
19	987		22	013	939	29	061	952	7	48	41		19	10	9	7	7	3
20	690	10	22	30990	969	29	031	959	7	41	40		20	10	10	8	7	3
21	032		23	968	998	30	002	966	7	34	39		21	10	10	8	8	3
22	055		22	945	75028	29	24972	973	7	27	38		22	11	11	8	8	3
23	077		23	923	058	29	942	980	8	20	37		23	12	11	9	8	3
24	100		22	900	087	29	913	988	7	12	36		24	12	12	9	9	3
25	122		22	878	117	29	883	995	7	05	35		25	12	12	10	9	3
26	144		23	856	146	30	854	06002	7	93998	34		26	13	13	10	10	3
27	167		22	833	176	30	824	009	7	991	33		27	14	13	10	10	4
28	189		23	811	205	30	795	016	7	984	32		28	14	14	11	10	4
29	212		22	788	235	30	765	023	7	977	31		29	14	14	11	11	4
30	69234		22	30766	75264	29	24736	06030	7	93970	30		30	15	14	12	11	4
31	256		23	744	294	30	706	037	8	963	29		31	16	15	12	11	4
32	279		22	721	323	30	677	045	7	955	28		32	16	15	12	12	4
33	301		22	699	353	29	647	052	7	948	27		33	16	16	13	12	4
34	323		22	677	382	29	618	059	7	941	26		34	17	16	13	12	5
35	345		23	655	411	30	589	066	7	934	25		35	18	17	13	13	5
36	368		22	632	441	29	559	073	7	927	24		36	18	17	14	13	5
37	390		22	610	470	30	530	080	7	920	23		37	18	18	14	14	5
38	412		22	588	500	30	500	088	8	912	22		38	19	18	15	14	5
39	434		22	566	529	29	471	095	7	905	21		39	20	19	15	14	5
40	456		23	544	558	29	442	102	7	898	20		40	20	19	15	15	5
41	479		23	521	588	30	412	109	7	891	19		41	20	20	16	15	5
42	501		22	499	617	29	383	116	7	884	18		42	21	20	16	15	6
43	523		22	477	647	30	353	124	8	876	17		43	22	21	16	16	6
44	545		22	455	676	29	324	131	7	869	16		44	22	21	17	16	6
45	567		23	433	705	30	295	138	7	862	15		45	22	22	17	16	6
46	589		22	411	735	29	265	145	8	855	14		46	23	22	18	17	6
47	611		22	389	764	29	236	153	8	847	13		47	24	23	18	17	6
48	633		22	367	793	29	207	160	7	840	12		48	24	23	18	18	6
49	655		22	345	822	30	178	167	7	833	11		49	24	24	19	18	7
50	677		23	323	852	30	148	174	7	826	10		50	25	24	19	18	7
51	699		22	301	881	29	119	181	7	819	9		51	26	25	20	19	7
52	721		22	279	910	29	090	189	8	811	8		52	26	25	20	19	7
53	743		23	257	939	30	061	196	7	804	7		53	26	26	20	19	7
54	765		22	235	969	29	031	203	8	797	6		54	27	26	21	20	7
55	787		23	213	998	30	002	211	7	789	5		55	28	27	21	20	7
56	809		22	191	76027	29	23973	218	7	782	4		56	28	27	21	21	7
57	831		22	169	056	30	944	225	7	775	3		57	28	28	22	21	8
58	853		23	147	086	30	914	232	7	768	2		58	29	28	22	21	8
59	875		22	125	115	29	885	240	8	760	1		59	30	29	23	22	8
60	69897		22	30103	76144	29	23856	06247	7	93753	0		60	30	29	23	22	8
	9.	d		10.	9.	d	10.	10.	d	9.	d			30	29	23	22	8
	$\angle$	$\cos$	$1'$	$\angle$	$\sec$	$1'$	$\angle$	$\cot$	$1'$	$\angle$	$\tan$	$1'$	$\angle$	$\cos$	$1'$	$\angle$	$\sin$	$1'$

"	Proportional Parts						"
	30	29	23	22	8	7	
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3	2	1	1	1	0	0	0
4	2	2	2	1	1	0	0
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6	3	3	2	2	1	1	1
7	4	3	3	3	1	1	1
8	4	4	3	3	1	1	1
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59	30	29	23	22	8	7	7
60	30	29	23	22	8	7	7
"	30	29	23	22	8	7	7
Proportional Parts							

	$l \sin$	$d$	$l \csc$	$l \tan$	$d$	$l \cot$	$l \sec$	$d$	$l \cos$	$l$
	9.	1'	10.	9.	1'	10.	10.	1'	9.	
0	69897		30103	76144		23856	06247		93753	60
1	919	22	081	173	20	827	254	7	746	59
2	941	22	059	202	20	798	262	7	738	58
3	963	22	037	231	20	769	269	7	731	57
4	984	21	016	261	30	739	276	7	724	56
5	70006	22	29994	290	20	710	283	8	717	55
6	028	22	972	319	29	681	291	8	709	54
7	050	22	950	348	29	652	298	7	702	53
8	072	22	928	377	29	623	305	7	695	52
9	093	21	907	406	29	594	313	8	687	51
10	115	22	885	435	29	565	320	7	680	50
11	137	22	863	464	29	536	327	7	673	49
12	159	21	841	493	29	507	335	8	665	48
13	180	21	820	522	29	478	342	7	658	47
14	202	22	798	551	29	449	350	8	650	46
15	224	21	776	580	29	420	357	7	643	45
16	245	22	755	609	30	391	364	8	636	44
17	267	22	733	639	30	361	372	7	628	43
18	288	21	712	668	29	332	379	7	621	42
19	310	22	690	697	28	303	386	8	614	41
20	332	21	668	725	29	275	394	7	606	40
21	353	22	647	754	29	246	401	8	599	39
22	375	21	625	783	29	217	409	7	591	38
23	396	21	604	812	29	188	416	7	584	37
24	418	21	582	841	29	159	423	8	577	36
25	439	22	561	870	29	130	431	7	569	35
26	461	22	539	899	29	101	438	8	562	34
27	482	22	518	928	29	072	446	7	554	33
28	504	21	496	957	29	043	453	8	547	32
29	525	21	475	986	28	014	461	7	539	31
30	70547	22	29453	77015	29	22985	06468	7	93532	30
31	568	22	432	044	29	956	475	8	525	29
32	590	21	410	073	29	927	483	7	517	28
33	611	21	389	101	29	899	490	8	510	27
34	633	22	367	130	29	870	498	7	502	26
35	654	21	346	159	29	841	505	8	495	25
36	675	22	325	188	29	812	513	7	487	24
37	697	22	303	217	29	783	520	8	480	23
38	718	21	282	246	28	754	528	7	472	22
39	739	22	261	274	29	726	535	8	465	21
40	761	21	239	303	29	697	543	7	457	20
41	783	21	218	332	29	668	550	8	450	19
42	803	21	197	361	29	639	558	7	442	18
43	824	21	176	390	28	610	565	8	435	17
44	846	22	154	418	28	582	573	7	427	16
45	867	21	133	447	29	553	580	8	420	15
46	888	21	112	476	29	524	588	7	412	14
47	909	22	091	505	28	495	595	8	405	13
48	931	22	069	533	28	467	603	7	397	12
49	952	21	048	562	29	438	610	8	390	11
50	973	21	027	591	28	409	618	7	382	10
51	994	21	006	619	29	381	625	8	375	9
52	71015	22	28985	648	29	352	633	7	367	8
53	036	22	964	677	29	323	640	8	360	7
54	058	22	942	706	28	294	648	7	352	6
55	079	21	921	734	28	266	656	8	344	5
56	100	21	900	763	28	237	663	7	337	4
57	121	21	879	791	29	209	671	8	329	3
58	142	21	858	820	29	180	678	7	322	2
59	163	21	837	849	28	151	686	8	314	1
60	71184	22	28816	77877	29	22123	06693	7	93307	0
	9.	10.	9.	10.	9.	10.	10.	9.	10.	
	$l \cos$	$l$	$l \sec$	$l \cot$	$l$	$l \tan$	$l \csc$	$l$	$l \sin$	

Proportional Parts							
	30	29	28	27	26	25	24
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	1	1	1	1	1	1	0
3	2	1	1	1	1	1	0
4	2	2	2	1	1	1	0
5	2	2	2	2	2	1	1
6	3	3	3	2	2	1	1
7	4	3	3	3	2	1	1
8	4	4	4	3	3	1	1
9	4	4	4	3	3	1	1
10	5	5	5	4	4	1	1
11	6	5	5	4	4	1	1
12	6	6	6	4	4	2	1
13	6	6	6	5	5	2	2
14	7	7	7	5	5	2	2
15	8	7	7	6	5	2	2
16	8	8	7	6	6	2	2
17	8	8	8	6	6	2	2
18	9	9	8	7	6	2	2
19	10	9	9	7	7	3	2
20	10	10	9	7	7	3	2
21	10	10	10	8	7	3	2
22	11	11	10	8	8	3	3
23	12	11	11	8	8	3	3
24	12	12	11	9	8	3	3
25	12	12	12	9	9	3	3
26	13	13	12	10	9	3	3
27	14	13	13	10	9	4	3
28	14	14	13	10	10	4	3
29	14	14	14	11	10	4	3
30	15	14	14	11	10	4	4
31	16	15	14	11	11	4	4
32	16	15	15	12	11	4	4
33	16	16	15	12	12	4	4
34	17	16	16	12	12	5	4
35	18	17	16	13	12	5	4
36	18	17	17	13	13	5	4
37	18	18	17	14	13	5	4
38	19	18	18	14	13	5	4
39	20	19	18	14	14	5	5
40	20	19	19	15	14	5	5
41	20	20	19	15	14	5	5
42	21	20	20	15	15	6	5
43	22	21	20	16	15	6	5
44	22	21	21	16	15	6	5
45	22	22	21	16	16	6	5
46	23	22	21	17	16	6	5
47	24	23	22	17	16	6	5
48	24	23	22	18	17	6	6
49	24	24	23	18	17	7	6
50	25	24	23	18	18	7	6
51	26	25	24	19	18	7	6
52	26	25	24	19	18	7	6
53	26	26	25	19	19	7	6
54	27	26	25	20	19	7	6
55	28	27	26	20	19	7	6
56	28	27	26	21	20	7	7
57	28	28	27	21	20	8	7
58	29	28	27	21	20	8	7
59	30	29	28	22	21	8	7
60	30	29	28	22	21	8	7
	30	29	28	22	21	8	7
Proportional Parts							

31°

TABLE II

148°

	$l \sin$	$d$	$l \csc$	$l \tan$	$d$	$l \cot$	$l \sec$	$d$	$l \cos$	$l$
	9.	1'	10.	9.	1'	10.	10.	1'	9.	0
0	71184		28816	77877		22123	06693		93307	60
1	205	21	795	906	29	094	701	8	299	59
2	226	21	774	935	29	065	709	8	291	58
3	247	21	753	963	28	037	716	7	284	57
4	268	21	732	992	28	008	724	8	276	56
5	289	21	711	78020	29	21980	731	8	269	55
6	310	21	690	049	29	951	739	8	261	54
7	331	21	669	077	28	923	747	8	253	53
8	352	21	648	106	29	894	754	7	246	52
9	373	20	627	135	28	865	762	8	238	51
10	393	21	607	163	29	837	770	7	230	50
11	414	21	586	192	28	808	777	8	223	49
12	435	21	565	220	28	780	785	8	215	48
13	456	21	544	249	29	751	793	7	207	47
14	477	21	523	277	28	723	800	8	200	46
15	498	21	502	306	28	694	808	8	192	45
16	519	20	481	334	29	666	816	7	184	44
17	539	20	461	363	28	637	823	8	177	43
18	560	21	440	391	29	609	831	8	169	42
19	581	21	419	419	29	581	839	7	161	41
20	602	20	398	448	28	552	846	8	154	40
21	622	21	378	476	29	524	854	8	146	39
22	643	21	357	505	28	495	862	7	138	38
23	664	21	336	533	29	467	869	8	131	37
24	685	20	315	562	28	438	877	8	123	36
25	705	21	295	590	28	410	885	7	115	35
26	726	21	274	618	29	382	892	8	108	34
27	747	20	253	647	28	353	900	8	100	33
28	767	21	233	675	29	325	908	8	092	32
29	788	21	212	704	28	296	916	7	084	31
30	71809		28191	78732		21268	06923		93077	30
31	829	21	171	760	29	240	931	8	069	29
32	850	21	150	789	28	211	939	8	061	28
33	870	21	130	817	28	183	947	7	053	27
34	891	20	109	845	29	155	954	8	046	26
35	911	21	089	874	28	126	962	8	038	25
36	932	20	068	902	28	098	970	8	030	24
37	952	21	048	930	28	070	978	8	022	23
38	973	21	027	959	28	041	986	8	014	22
39	994	20	006	987	28	013	993	8	007	21
40	72014		27986	79015		20985	07001		92999	20
41	034	21	966	043	28	957	009	8	991	19
42	055	21	945	072	28	928	017	8	983	18
43	075	21	925	100	28	900	024	7	976	17
44	096	21	904	128	28	872	032	8	968	16
45	116	20	884	156	28	844	040	8	960	15
46	137	21	863	185	28	815	048	8	952	14
47	157	20	843	213	28	787	056	8	944	13
48	177	21	823	241	28	759	064	8	936	12
49	198	20	802	269	28	731	071	7	929	11
50	218	21	782	297	29	703	079	8	921	10
51	238	21	762	326	28	674	087	8	913	9
52	259	21	741	354	28	646	095	8	905	8
53	279	20	721	382	28	618	103	8	897	7
54	299	21	701	410	28	590	111	8	889	6
55	320	20	680	438	28	562	119	8	881	5
56	340	20	660	466	29	534	126	7	874	4
57	360	21	640	495	28	505	134	8	866	3
58	381	21	619	523	28	477	142	8	858	2
59	401	20	599	551	28	449	150	8	850	1
60	72421		27579	79579		20421	07158		92842	0
	9.	d	10.	9.	d	10.	10.	d	9.	d
	$l \cos$	1'	$l \sec$	$l \cot$	1'	$l \tan$	$l \csc$	1'	$l \sin$	

Proportional Parts						
"	29	28	21	20	8	7
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	1	1	0	0
3	1	1	1	1	0	0
4	2	2	1	1	1	0
5	2	2	2	2	1	1
6	3	3	2	2	1	1
7	3	3	2	2	1	1
8	4	4	3	3	1	1
9	4	4	3	3	1	1
10	5	5	4	3	1	1
11	5	5	4	4	1	1
12	6	6	4	4	2	2
13	6	6	5	4	2	2
14	7	7	5	5	2	2
15	7	7	5	5	2	2
16	8	7	6	5	2	2
17	8	8	6	6	2	2
18	9	8	6	6	2	2
19	9	9	7	6	3	2
20	10	9	7	7	3	2
21	10	10	7	7	3	2
22	11	10	8	7	3	3
23	11	11	8	8	3	3
24	12	11	8	8	3	3
25	12	12	9	8	3	3
26	13	12	9	9	3	3
27	13	13	9	9	4	3
28	14	13	10	9	4	3
29	14	14	10	10	4	3
30	14	14	10	10	4	4
31	15	14	11	10	4	4
32	15	15	11	11	4	4
33	16	15	12	11	4	4
34	16	16	12	11	5	4
35	17	16	12	12	5	4
36	17	17	13	12	5	4
37	18	17	13	12	5	4
38	18	18	13	13	5	4
39	19	18	14	13	5	5
40	19	19	14	13	5	5
41	20	19	14	14	5	5
42	20	20	15	14	6	5
43	21	20	15	14	6	5
44	21	21	15	15	6	5
45	22	21	16	15	6	5
46	22	21	16	15	6	5
47	23	22	16	16	6	5
48	23	22	17	16	6	6
49	24	23	17	16	7	6
50	24	23	18	17	7	6
51	25	24	18	17	7	6
52	25	24	18	17	7	6
53	26	25	19	18	7	6
54	26	25	19	18	7	6
55	27	26	19	18	7	6
56	27	26	20	19	7	7
57	28	27	20	19	8	7
58	28	27	20	19	8	7
59	29	28	21	20	8	7
60	29	28	21	20	8	7
"	29	28	21	20	8	7
Proportional Parts						

121°

58°

32°

TABLE II

147°

°	l sin	d	l csc	l tan	d	l cot	l sec	d	l cos	°
9.	1'	10.	9.	1'	10.	10.	1'	10.	9.	1'
0	72421	20	27579	79579	28	20421	07158	8	92842	66
1	441	20	559	607	28	393	166	8	834	56
2	461	20	539	635	28	365	174	8	826	58
3	482	21	518	663	28	337	182	8	818	57
4	502	20	498	691	28	309	190	8	810	56
5	522	20	478	719	28	281	197	8	803	56
6	542	20	458	747	28	253	205	8	795	54
7	562	20	438	776	29	224	213	8	787	53
8	582	20	418	804	28	196	221	8	779	52
9	602	20	398	832	28	168	229	8	771	51
10	622	21	378	860	28	140	237	8	763	50
11	643	20	357	888	28	112	245	8	755	49
12	663	20	337	916	28	84	253	8	747	48
13	683	20	317	944	28	56	261	8	739	47
14	703	20	297	972	28	28	269	8	731	46
15	723	20	277	80000	28	000	277	8	723	45
16	743	20	257	028	28	19072	285	8	715	44
17	763	20	237	056	28	944	293	8	707	43
18	783	20	217	084	28	916	301	8	699	42
19	803	20	197	112	28	888	309	8	691	41
20	823	20	177	140	28	860	317	8	683	40
21	843	20	157	168	28	832	325	8	675	39
22	863	20	137	195	28	805	333	8	667	38
23	883	19	117	223	28	777	341	8	659	37
24	902	19	098	251	28	749	349	8	651	36
25	922	20	078	279	28	721	357	8	643	35
26	942	20	058	307	28	693	365	8	635	34
27	962	20	038	335	28	665	373	8	627	33
28	982	20	018	363	28	637	381	8	619	32
29	73002	20	26998	391	28	609	389	8	611	31
30	73022	20	26978	80419	28	19581	07397	8	92603	30
31	041	20	959	447	27	553	405	8	595	29
32	061	20	939	474	27	526	413	8	587	28
33	081	20	919	502	28	498	421	8	579	27
34	101	20	899	530	28	470	429	8	571	26
35	121	20	879	558	28	442	437	8	563	25
36	140	20	860	586	28	414	445	8	555	24
37	160	20	840	614	28	386	454	8	546	23
38	180	20	820	642	27	358	462	8	538	22
39	200	19	800	669	27	331	470	8	530	21
40	219	19	781	697	28	303	478	8	522	20
41	239	20	761	725	28	275	486	8	514	19
42	259	20	741	753	28	247	494	8	506	18
43	278	19	722	781	27	219	502	8	498	17
44	298	20	702	808	27	192	510	8	490	16
45	318	19	682	836	28	164	518	8	482	15
46	337	19	663	864	28	136	527	8	473	14
47	357	20	643	892	27	108	535	8	465	13
48	377	19	623	919	27	81	543	8	457	12
49	396	20	604	947	28	53	551	8	449	11
50	416	19	584	975	27	025	559	8	441	10
51	435	19	565	81003	27	18997	567	8	433	9
52	455	20	545	030	28	970	575	8	425	8
53	474	20	526	058	28	942	584	8	416	7
54	494	19	506	086	27	914	592	8	408	6
55	513	19	487	113	28	887	600	8	400	5
56	533	20	467	141	28	859	608	8	392	4
57	552	20	448	169	27	831	616	8	384	3
58	572	20	428	196	28	804	624	8	376	2
59	591	19	409	224	28	776	633	8	367	1
60	73611	20	26389	81252	28	18748	07641	8	92359	0
°	l cos	1'	l sec	l cot	1'	l tan	l csc	1'	l sin	°

Proportional Parts										
°	29	28	27	26	25	24	23	22	21	20
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	2	2	2	2	2	2	2	2	2	2
5	2	2	2	2	2	2	2	2	2	2
6	3	3	3	3	3	3	3	3	3	3
7	3	3	3	3	3	3	3	3	3	3
8	4	4	4	4	4	4	4	4	4	4
9	4	4	4	4	4	4	4	4	4	4
10	5	5	5	5	5	5	5	5	5	5
11	5	5	5	5	5	5	5	5	5	5
12	6	6	6	6	6	6	6	6	6	6
13	6	6	6	6	6	6	6	6	6	6
14	7	7	7	7	7	7	7	7	7	7
15	7	7	7	7	7	7	7	7	7	7
16	8	8	8	8	8	8	8	8	8	8
17	8	8	8	8	8	8	8	8	8	8
18	9	9	9	9	9	9	9	9	9	9
19	9	9	9	9	9	9	9	9	9	9
20	10	10	10	10	10	10	10	10	10	10
21	10	10	10	10	10	10	10	10	10	10
22	11	11	11	11	11	11	11	11	11	11
23	11	11	11	11	11	11	11	11	11	11
24	12	12	12	12	12	12	12	12	12	12
25	12	12	12	12	12	12	12	12	12	12
26	13	13	13	13	13	13	13	13	13	13
27	13	13	13	13	13	13	13	13	13	13
28	14	14	14	14	14	14	14	14	14	14
29	14	14	14	14	14	14	14	14	14	14
30	14	14	14	14	14	14	14	14	14	14
31	15	15	15	15	15	15	15	15	15	15
32	15	15	15	15	15	15	15	15	15	15
33	16	16	16	16	16	16	16	16	16	16
34	16	16	16	16	16	16	16	16	16	16
35	17	17	17	17	17	17	17	17	17	17
36	17	17	17	17	17	17	17	17	17	17
37	18	18	18	18	18	18	18	18	18	18
38	18	18	18	18	18	18	18	18	18	18
39	19	19	19	19	19	19	19	19	19	19
40	19	19	19	19	19	19	19	19	19	19
41	20	20	20	20	20	20	20	20	20	20
42	20	20	20	20	20	20	20	20	20	20
43	21	21	21	21	21	21	21	21	21	21
44	21	21	21	21	21	21	21	21	21	21
45	22	22	22	22	22	22	22	22	22	22
46	22	22	22	22	22	22	22	22	22	22
47	23	23	23	23	23	23	23	23	23	23
48	23	23	23	23	23	23	23	23	23	23
49	24	24	24	24	24	24	24	24	24	24
50	24	24	24	24	24	24	24	24	24	24
51	25	25	25	25	25	25	25	25	25	25
52	25	25	25	25	25	25	25	25	25	25
53	26	26	26	26	26	26	26	26	26	26
54	26	26	26	26	26	26	26	26	26	26
55	27	27	27	27	27	27	27	27	27	27
56	27	27	27	27	27	27	27	27	27	27
57	28	28	28	28	28	28	28	28	28	28
58	28	28	28	28	28	28	28	28	28	28
59	29	29	29	29	29	29	29	29	29	29
60	29	29	29	29	29	29	29	29	29	29
°	29	28	27	26	25	24	23	22	21	20
Proportional Parts										

122°

57°

	$\sin$	$\cos$	$\tan$	$\cot$	$\sec$	$\csc$
9.	10.	9.	10.	10.	10.	9.
0	73611	26389	81252	18748	07641	92359
1	630	370	279	721	649	351
2	650	350	307	693	657	343
3	669	331	335	665	665	335
4	689	311	362	638	674	326
5	708	292	390	610	682	318
6	727	273	418	582	690	310
7	747	253	445	555	698	302
8	766	234	473	527	707	293
9	785	215	500	500	715	285
10	805	195	528	472	723	277
11	824	176	556	444	731	269
12	843	157	583	417	740	260
13	863	137	611	389	748	252
14	882	118	638	362	756	244
15	901	099	666	334	765	235
16	921	079	693	307	773	227
17	940	060	721	279	781	219
18	959	041	748	252	789	211
19	978	022	776	224	798	202
20	997	003	803	197	806	194
21	74017	25983	831	169	814	186
22	036	964	858	142	823	177
23	055	945	886	114	831	169
24	074	926	913	887	839	161
25	093	907	941	059	848	152
26	113	887	968	032	856	144
27	132	868	996	004	864	136
28	151	849	82023	17977	873	127
29	170	830	051	949	881	119
30	74189	25811	82078	17922	07889	92111
31	208	792	106	894	898	102
32	227	773	133	867	906	094
33	246	754	161	839	914	086
34	265	735	188	812	923	077
35	284	716	215	785	931	069
36	303	697	243	757	940	060
37	322	678	270	730	948	052
38	341	659	298	702	956	044
39	360	640	325	675	965	035
40	379	621	352	648	973	027
41	398	602	380	620	982	018
42	417	583	407	593	990	010
43	436	564	435	565	998	002
44	455	545	462	538	08007	91993
45	474	526	489	511	015	985
46	493	507	517	483	024	976
47	512	488	544	456	032	968
48	531	469	571	429	041	959
49	549	451	599	401	049	951
50	568	432	626	374	058	942
51	587	413	653	347	066	934
52	606	394	681	319	075	925
53	625	375	708	292	083	917
54	644	356	735	265	092	908
55	662	338	762	238	100	900
56	681	319	790	210	109	891
57	700	300	817	183	117	883
58	719	281	844	156	126	874
59	737	263	871	129	134	866
60	74756	25244	82899	17101	08143	91857
9.	10.	9.	10.	10.	10.	9.
$\sin$	$\cos$	$\tan$	$\cot$	$\sec$	$\csc$	$\sin$

Proportional Parts							
28	27	20	19	18	9	8	
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	1	1	1	1	1	0	0
3	1	1	1	1	1	0	0
4	2	2	1	1	1	1	1
5	2	2	2	2	2	1	1
6	3	3	2	2	2	1	1
7	3	3	2	2	2	1	1
8	4	4	3	3	2	1	1
9	4	4	3	3	3	1	1
10	5	4	3	3	3	2	1
11	5	5	4	3	3	2	1
12	6	5	4	4	4	2	2
13	6	6	4	4	4	2	2
14	7	6	5	4	4	2	2
15	7	7	5	5	5	2	2
16	7	7	5	5	5	2	2
17	8	8	6	5	5	3	2
18	8	8	6	6	5	3	2
19	9	9	6	6	6	3	3
20	9	9	7	6	6	3	3
21	10	9	7	7	6	3	3
22	10	10	7	7	7	3	3
23	11	10	8	7	7	3	3
24	11	11	8	8	7	4	3
25	12	11	8	8	8	4	3
26	12	12	9	8	8	4	3
27	13	12	9	9	8	4	4
28	13	13	9	9	8	4	4
29	14	13	10	9	9	4	4
30	14	14	10	10	9	4	4
31	14	14	10	10	9	5	4
32	15	14	11	10	10	5	4
33	15	15	11	10	10	5	4
34	16	15	11	11	10	5	5
35	16	16	12	11	10	5	5
36	17	16	12	11	11	5	5
37	17	17	12	12	11	6	5
38	18	17	13	12	11	6	5
39	18	18	13	12	12	6	5
40	19	18	13	13	12	6	5
41	19	18	14	13	12	6	5
42	20	19	14	13	13	6	6
43	20	19	14	14	13	6	6
44	21	20	15	14	13	7	6
45	21	20	15	14	14	7	6
46	21	21	15	15	14	7	6
47	22	21	16	15	14	7	6
48	22	22	16	15	14	7	6
49	23	22	16	16	15	7	7
50	23	22	17	16	15	8	7
51	24	23	17	16	15	8	7
52	24	23	17	16	16	8	7
53	25	24	18	17	16	8	7
54	25	24	18	17	16	8	7
55	26	25	18	17	16	8	7
56	26	25	19	18	17	8	7
57	27	26	19	18	17	9	8
58	27	26	19	18	17	9	8
59	28	27	20	19	18	9	8
60	28	27	20	19	18	9	8
Proportional Parts							
28	27	20	19	18	9	8	

34°

TABLE II

145°

$\angle$	$\angle \sin$	$d$	$\angle \csc$	$\angle \tan$	$d$	$\angle \cot$	$\angle \sec$	$d$	$\angle \cos$	$\angle$
9.	10.	9.	10.	9.	10.	9.	10.	9.	10.	9.
0	74756		25244	82899		17101	08143		91857	60
1	775		225	926		074	151		849	59
2	794	19	206	953	27	047	160	8	840	58
3	812	18	188	980	27	020	168	8	832	57
4	831	19	169	83008	28	16992	177	9	823	56
5	850	18	150	035	27	965	185	9	815	55
6	868	18	132	062	27	938	194	9	806	54
7	887	19	113	089	27	911	202	8	798	53
8	906	19	094	117	28	883	211	9	789	52
9	924	19	076	144	27	856	219	8	781	51
10	943	18	057	171	27	829	228	9	772	50
11	961	18	039	198	27	802	237	9	763	49
12	980	19	020	225	27	775	245	8	755	48
13	999	19	001	252	27	748	254	9	746	47
14	75017		24983	280	28	720	262	8	738	46
15	036	18	964	307	27	693	271	9	729	45
16	054	18	946	334	27	666	280	8	720	44
17	073	18	927	361	27	639	288	8	712	43
18	091	19	909	388	27	612	297	8	703	42
19	110	18	890	415	27	585	305	9	695	41
20	128	19	872	442	28	558	314	9	686	40
21	147	18	853	470	27	530	323	8	677	39
22	165	18	835	497	27	503	331	9	669	38
23	184	19	816	524	27	476	340	9	660	37
24	202	19	798	551	27	449	349	8	651	36
25	221	18	779	578	27	422	357	9	643	35
26	239	18	761	605	27	395	366	9	634	34
27	258	19	742	632	27	368	375	8	625	33
28	276	18	724	659	27	341	383	8	617	32
29	294	19	706	686	27	314	392	9	608	31
30	75313		24687	83713		16287	08401		91599	30
31	331	19	669	740	28	260	409	9	591	29
32	350	18	650	768	27	232	418	8	582	28
33	368	18	632	795	27	205	427	9	573	27
34	386	19	614	822	27	178	435	8	565	26
35	405	18	595	849	27	151	444	9	556	25
36	423	18	577	876	27	124	453	9	547	24
37	441	18	559	903	27	097	462	8	538	23
38	459	19	541	930	27	070	470	8	530	22
39	478	18	522	957	27	043	479	9	521	21
40	496	18	504	984	27	016	488	8	512	20
41	514	19	486	84011	28	15989	496	9	504	19
42	533	18	467	038	27	962	505	8	495	18
43	551	18	449	065	27	935	514	9	486	17
44	569	18	431	092	27	908	523	8	477	16
45	587	19	413	119	27	881	531	9	469	15
46	605	18	395	146	27	854	540	9	460	14
47	624	18	376	173	27	827	549	9	451	13
48	642	19	358	200	27	800	558	8	442	12
49	660	18	340	227	27	773	567	8	433	11
50	678	18	322	254	26	746	575	9	425	10
51	696	18	304	280	27	720	584	9	416	9
52	714	18	286	307	27	693	593	8	407	8
53	733	19	267	334	27	666	602	9	398	7
54	751	18	249	361	27	639	611	8	389	6
55	769	18	231	388	27	612	619	9	381	5
56	787	18	213	415	27	585	628	8	372	4
57	805	18	195	442	27	558	637	9	363	3
58	823	19	177	469	27	531	646	8	354	2
59	841	18	159	496	27	504	655	9	345	1
60	75859		24141	84523		15477	08664		91336	0
	$\angle \cos$	$d$	$\angle \sec$	$\angle \cot$	$d$	$\angle \tan$	$\angle \csc$	$d$	$\angle \sin$	$\angle$
	9.	10.	9.	10.	9.	10.	9.	10.	9.	10.

Proportional Parts									
"	28	27	26	19	18	9	8		
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	0	0	0
3	1	1	1	1	1	1	0	0	0
4	2	2	2	1	1	1	1	1	1
5	2	2	2	2	2	1	1	1	1
6	3	3	3	2	2	1	1	1	1
7	3	3	3	2	2	1	1	1	1
8	4	4	3	3	2	1	1	1	1
9	4	4	4	3	3	1	1	1	1
10	5	4	4	3	3	2	1	1	1
11	5	5	5	3	3	2	1	1	1
12	6	5	5	4	4	2	2	2	2
13	6	6	6	4	4	2	2	2	2
14	7	6	6	4	4	2	2	2	2
15	7	7	6	5	4	2	2	2	2
16	7	7	7	5	5	2	2	2	2
17	8	8	7	5	5	3	2	2	2
18	8	8	8	6	5	3	2	2	2
19	9	9	8	6	6	3	3	3	3
20	9	9	9	6	6	3	3	3	3
21	10	9	9	7	6	3	3	3	3
22	10	10	10	7	7	3	3	3	3
23	11	10	10	7	7	3	3	3	3
24	11	11	10	8	7	4	3	3	3
25	12	11	11	8	8	4	3	3	3
26	12	12	11	8	8	4	3	3	3
27	13	12	12	9	8	4	4	4	4
28	13	13	12	9	8	4	4	4	4
29	14	13	13	9	9	4	4	4	4
30	14	14	13	10	9	4	4	4	4
31	14	14	13	10	9	5	4	4	4
32	15	14	14	10	10	5	4	4	4
33	15	15	14	10	10	5	4	4	4
34	16	15	15	11	10	5	5	5	5
35	16	16	15	11	10	5	5	5	5
36	17	16	16	11	11	5	5	5	5
37	17	17	16	12	11	6	5	5	5
38	18	17	16	12	11	6	5	5	5
39	18	18	17	12	12	6	5	5	5
40	19	18	17	13	12	6	5	5	5
41	19	18	18	13	12	6	5	5	5
42	20	19	18	13	13	6	6	6	6
43	20	19	19	14	13	6	6	6	6
44	21	20	19	14	13	7	6	6	6
45	21	20	20	14	14	7	6	6	6
46	21	21	20	15	14	7	6	6	6
47	22	21	20	15	14	7	6	6	6
48	22	22	21	15	14	7	6	6	6
49	23	22	21	16	15	7	7	7	7
50	23	22	22	16	15	8	7	7	7
51	24	23	22	16	15	8	7	7	7
52	24	23	23	16	16	8	7	7	7
53	25	24	23	17	16	8	7	7	7
54	25	24	23	17	16	8	7	7	7
55	26	25	24	17	16	8	7	7	7
56	26	25	24	18	17	8	7	7	7
57	27	26	25	18	17	9	8	8	8
58	27	26	25	18	17	9	8	8	8
59	28	27	26	19	18	9	8	8	8
60	28	27	26	19	18	9	8	8	8
"	28	27	26	19	18	9	8	8	8
Proportional Parts									

124°

55°

35°

TABLE II

144°

	$\sin$	$\cos$	$\tan$	$\cot$	$\sec$	$\csc$
°	9.	10.	9.	10.	10.	9.
0	75859	24141	84523	15477	08664	91336
1	877	123	550	450	672	328
2	895	105	576	424	681	319
3	913	87	603	397	690	310
4	931	69	630	370	699	301
5	949	51	657	343	708	292
6	967	33	684	316	717	283
7	985	15	711	289	726	274
8	76003	23997	738	262	734	266
9	021	979	764	236	743	257
10	039	961	791	209	752	248
11	057	943	818	182	761	239
12	075	925	845	155	770	230
13	093	907	872	128	779	221
14	111	889	899	101	788	212
15	129	871	925	075	797	203
16	146	854	952	048	806	194
17	164	836	979	021	815	185
18	182	818	85006	14994	824	176
19	200	800	033	967	833	167
20	218	782	059	941	842	158
21	236	764	086	914	851	149
22	253	747	113	887	859	141
23	271	729	140	860	868	132
24	289	711	166	834	877	123
25	307	693	193	807	886	114
26	324	676	220	780	895	105
27	342	658	247	753	904	096
28	360	640	273	727	913	087
29	378	622	300	700	922	078
30	76395	23605	85327	14673	08931	91069
31	413	587	354	646	940	060
32	431	569	380	620	949	051
33	448	552	407	593	958	042
34	466	534	434	566	967	033
35	484	516	460	540	977	023
36	501	499	487	513	986	014
37	519	481	514	486	995	005
38	537	463	540	460	09004	00996
39	554	446	567	434	013	087
40	572	428	594	406	022	078
41	590	410	620	380	031	069
42	607	393	647	353	040	060
43	625	375	674	326	049	051
44	642	358	700	300	058	042
45	660	340	727	273	067	033
46	677	323	754	246	076	024
47	695	305	780	220	085	015
48	712	288	807	193	094	006
49	730	270	834	166	104	089
50	747	253	860	140	113	087
51	765	235	887	113	122	078
52	782	218	913	087	131	069
53	800	200	940	060	140	060
54	817	183	967	033	149	051
55	835	165	993	007	158	042
56	852	148	86020	13980	168	033
57	870	130	046	954	177	023
58	887	113	073	927	186	014
59	904	096	100	900	195	005
60	76922	23078	86126	13874	09204	90796
°	9.	10.	9.	10.	10.	9.
	$\cos$	$\sec$	$\cot$	$\tan$	$\csc$	$\sin$

	27	26	18	17	10	9	8
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0
3	1	1	1	1	0	0	0
4	2	2	1	1	1	1	1
5	2	2	2	1	1	1	1
6	3	3	2	2	1	1	1
7	3	3	2	2	1	1	1
8	4	3	2	2	1	1	1
9	4	4	3	3	2	1	1
10	4	4	3	3	2	2	1
11	5	5	3	3	2	2	1
12	5	5	4	3	2	2	2
13	6	6	4	4	2	2	2
14	6	6	4	4	2	2	2
15	7	6	4	4	2	2	2
16	7	7	5	5	3	2	2
17	8	7	5	5	3	3	2
18	8	8	5	5	3	3	2
19	9	8	6	5	3	3	3
20	9	9	6	6	3	3	3
21	9	9	6	6	4	3	3
22	10	10	7	6	4	3	3
23	10	10	7	7	4	3	3
24	11	10	7	7	4	4	3
25	11	11	8	7	4	4	3
26	12	11	8	7	4	4	3
27	12	12	8	8	4	4	4
28	13	12	8	8	5	4	4
29	13	13	9	8	5	4	4
30	14	13	9	8	5	4	4
31	14	13	9	9	5	4	4
32	14	14	10	9	5	5	4
33	15	14	10	9	6	5	4
34	15	15	10	10	6	5	5
35	16	15	10	10	6	5	5
36	16	16	11	10	6	5	5
37	17	16	11	10	6	6	5
38	17	16	11	11	6	6	5
39	18	17	12	11	6	6	5
40	18	17	12	11	7	6	5
41	18	18	12	12	7	6	5
42	19	18	13	12	7	6	6
43	19	19	13	12	7	6	6
44	20	19	13	12	7	7	6
45	20	20	14	13	8	7	6
46	21	20	14	13	8	7	6
47	21	20	14	13	8	7	6
48	22	21	14	14	8	7	6
49	22	21	15	14	8	7	7
50	22	22	15	14	8	8	7
51	23	22	15	14	8	8	7
52	23	23	16	15	9	8	7
53	24	23	16	15	9	8	7
54	24	23	16	15	9	8	7
55	25	24	16	16	9	8	7
56	25	24	17	16	9	8	7
57	26	25	17	16	10	9	8
58	26	25	17	16	10	9	8
59	27	26	18	17	10	9	8
60	27	26	18	17	10	9	8
°	27	26	18	17	10	9	8

Proportional Parts

125°

54°



$\circ$	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	1'	10.	9.	10.	10.	10.	10.	1'	9.	
0	76922	23078	86126	13874	09204	90796	60			
1	939	061	153	847	213	787	59			
2	957	043	179	821	223	777	58			
3	974	026	206	794	232	768	57			
4	991	009	232	768	241	759	56			
5	77009	22991	259	741	250	750	55			
6	026	974	285	715	259	741	54			
7	043	957	312	688	269	731	53			
8	061	939	338	662	278	722	52			
9	078	922	365	635	287	713	51			
10	095	905	392	608	296	704	50			
11	112	888	418	582	306	694	49			
12	130	870	445	555	315	685	48			
13	147	853	471	529	324	676	47			
14	164	836	498	502	333	667	46			
15	181	819	524	476	343	657	45			
16	199	801	551	449	352	648	44			
17	216	784	577	423	361	639	43			
18	233	767	603	397	370	630	42			
19	250	750	630	370	380	620	41			
20	268	732	656	344	389	611	40			
21	285	715	683	317	398	602	39			
22	302	698	709	291	408	592	38			
23	319	681	736	264	417	583	37			
24	336	664	762	238	426	574	36			
25	353	647	789	211	435	565	35			
26	370	630	815	185	445	555	34			
27	387	613	842	158	454	546	33			
28	405	595	868	132	463	537	32			
29	422	578	894	106	473	527	31			
30	77439	22561	86921	13079	09482	90518	30			
31	456	544	947	053	491	509	29			
32	473	527	974	026	501	499	28			
33	490	510	87000	000	510	490	27			
34	507	493	027	12973	520	480	26			
35	524	476	053	947	529	471	25			
36	541	459	079	921	538	462	24			
37	558	442	106	894	548	452	23			
38	575	425	132	868	557	443	22			
39	592	408	158	842	566	434	21			
40	609	391	185	815	576	424	20			
41	626	374	211	789	585	415	19			
42	643	357	238	762	595	405	18			
43	660	340	264	736	604	396	17			
44	677	323	290	710	614	386	16			
45	694	306	317	683	623	377	15			
46	711	289	343	657	632	368	14			
47	728	272	369	631	642	358	13			
48	744	256	396	604	651	349	12			
49	761	239	422	578	661	339	11			
50	778	222	448	552	670	330	10			
51	795	205	475	525	680	320	9			
52	812	188	501	499	689	311	8			
53	829	171	527	473	699	301	7			
54	846	154	554	446	708	292	6			
55	862	138	580	420	718	282	5			
56	879	121	606	394	727	273	4			
57	896	104	633	367	737	263	3			
58	913	087	659	341	746	254	2			
59	930	070	685	315	756	244	1			
60	77946	22054	87711	12289	09765	90235	0			
$\circ$	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	1'	10.	9.	10.	10.	10.	10.	1'	9.	

Proportional Parts								
"	27	26	18	17	16	10	9	
0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	
2	1	1	1	1	1	0	0	
3	1	1	1	1	1	0	0	
4	2	2	1	1	1	1	1	
5	2	2	2	1	1	1	1	
6	3	3	2	2	2	1	1	
7	3	3	2	2	2	1	1	
8	4	3	2	2	2	1	1	
9	4	4	3	3	2	2	1	
10	4	4	3	3	3	2	2	
11	5	5	3	3	3	2	2	
12	5	5	4	3	3	2	2	
13	6	6	4	4	3	2	2	
14	6	6	4	4	4	2	2	
15	7	6	4	4	4	2	2	
16	7	7	5	5	4	3	2	
17	8	7	5	5	5	3	3	
18	8	8	5	5	5	3	3	
19	9	8	6	5	5	3	3	
20	9	9	6	6	5	3	3	
21	9	9	6	6	6	4	3	
22	10	10	7	6	6	4	3	
23	10	10	7	7	6	4	3	
24	11	10	7	7	6	4	4	
25	11	11	8	7	7	4	4	
26	12	11	8	7	7	4	4	
27	12	12	8	8	7	4	4	
28	13	12	8	8	7	5	4	
29	13	13	8	8	8	5	4	
30	14	13	9	8	8	5	4	
31	14	13	9	9	8	5	5	
32	14	14	10	9	9	5	5	
33	15	14	10	9	9	6	5	
34	15	15	10	10	9	6	5	
35	16	15	10	10	9	6	5	
36	16	16	11	10	10	6	5	
37	17	16	11	10	10	6	5	
38	17	16	11	11	10	6	6	
39	18	17	12	11	10	6	6	
40	18	17	12	11	11	7	6	
41	18	18	12	12	11	7	6	
42	19	18	13	12	11	7	6	
43	19	19	13	12	11	7	6	
44	20	19	13	12	12	7	7	
45	20	20	14	13	12	8	7	
46	21	20	14	13	12	8	7	
47	21	20	14	13	13	8	7	
48	22	21	14	14	13	8	7	
49	22	21	15	14	13	8	7	
50	22	22	15	14	13	8	8	
51	23	22	15	14	14	8	8	
52	23	23	16	15	14	9	8	
53	24	23	16	15	14	9	8	
54	24	23	16	15	14	9	8	
55	25	24	16	16	15	9	8	
56	25	24	17	16	15	9	8	
57	26	25	17	16	15	10	9	
58	26	25	17	16	15	10	9	
59	27	26	18	17	16	10	9	
60	27	26	18	17	16	10	9	
"	27	26	18	17	16	10	9	
Proportional Parts								

37°

TABLE II

142°

	<i>l</i> sin	<i>d</i>	<i>l</i> csc	<i>l</i> tan	<i>d</i>	<i>l</i> cot	<i>l</i> sec	<i>d</i>	<i>l</i> cos	
	9.	1'	10.	9.	1'	10.	10.	1'	9.	
0	77946		22054	87711		12289	09765		90235	60
1	963		037	738	27	262	775		225	59
2	980	17	020	764	26	236	784	9	216	58
3	997	17	003	790	26	210	794	10	206	57
4	78013	16	21987	817	27	183	803	9	197	56
5	030	17	970	843	26	157	813	10	187	55
6	047	17	953	869	26	131	822	9	178	54
7	063	16	937	895	26	105	832	10	168	53
8	080	17	920	922	27	078	841	9	159	52
9	097	16	903	948	26	052	851	10	149	51
10	113	17	887	974	26	026	861	9	139	50
11	130	17	870	88000	27	000	870	10	130	49
12	147	17	853	027	27	11973	880	10	120	48
13	163	17	837	053	26	947	889	9	111	47
14	180	17	820	079	26	921	899	10	101	46
15	197	16	803	105	26	895	909	9	091	45
16	213	17	787	131	26	869	918	10	082	44
17	230	16	770	158	27	842	928	9	072	43
18	246	17	754	184	26	816	937	10	063	42
19	263	17	737	210	26	790	947	9	053	41
20	280	16	720	236	26	764	957	10	043	40
21	296	17	704	262	27	738	966	9	034	39
22	313	16	687	289	26	711	976	10	024	38
23	329	17	671	315	26	685	986	9	014	37
24	346	16	654	341	26	659	995	10	005	36
25	362	17	638	367	26	633	10005	9	89995	35
26	379	17	621	393	27	607	015	10	985	34
27	395	16	605	420	26	580	024	9	976	33
28	412	17	588	446	26	554	034	10	966	32
29	428	16	572	472	26	528	044	9	956	31
30	78445	17	21555	88498	26	11502	10053	10	89947	30
31	461	17	539	524	26	476	063	9	937	29
32	478	16	522	550	26	450	073	10	927	28
33	494	17	506	577	26	423	082	9	918	27
34	510	16	490	603	26	397	092	10	908	26
35	527	17	473	629	26	371	102	9	898	25
36	543	16	457	655	26	345	112	10	888	24
37	560	17	440	681	26	319	121	9	879	23
38	576	16	424	707	26	293	131	10	869	22
39	592	17	408	733	26	267	141	9	859	21
40	609	16	391	759	27	241	151	10	849	20
41	625	17	375	786	26	214	160	9	840	19
42	642	16	358	812	26	188	170	10	830	18
43	658	17	342	838	26	162	180	9	820	17
44	674	16	326	864	26	136	190	10	810	16
45	691	17	309	890	26	110	199	9	801	15
46	707	16	293	916	26	084	209	10	791	14
47	723	17	277	942	26	058	219	9	781	13
48	739	16	261	968	26	032	229	10	771	12
49	756	17	244	994	26	006	239	9	761	11
50	772	16	228	89020	27	10980	248	10	752	10
51	788	17	212	046	26	954	258	9	742	9
52	805	16	195	073	26	927	268	10	732	8
53	821	17	179	099	26	901	278	9	722	7
54	837	16	163	125	26	875	288	10	712	6
55	853	17	147	151	26	849	298	9	702	5
56	869	16	131	177	26	823	307	10	693	4
57	886	17	114	203	26	797	317	9	683	3
58	902	16	098	229	26	771	327	10	673	2
59	918	17	082	255	26	745	337	9	663	1
60	78934	16	21066	89281	27	10719	10347	10	89653	0
	<i>l</i> cos	<i>d</i>	<i>l</i> sec	<i>l</i> cot	<i>d</i>	<i>l</i> tan	<i>l</i> csc	<i>d</i>	<i>l</i> sin	
	9.	1'	10.	9.	1'	10.	10.	1'	9.	

	Proportional Parts					
	27	28	17	16	10	9
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	1	1	0	0
3	1	1	1	1	0	0
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	3	2	2	1	1
7	3	3	2	2	1	1
8	4	3	2	2	1	1
9	4	4	3	2	2	1
10	4	4	3	3	2	2
11	5	5	3	3	2	2
12	5	5	3	3	2	2
13	6	6	4	3	2	2
14	6	6	4	4	2	2
15	7	6	4	4	2	2
16	7	7	5	4	3	2
17	8	7	5	5	3	3
18	8	8	5	5	3	3
19	9	8	5	5	3	3
20	9	9	6	5	3	3
21	9	9	6	6	4	3
22	10	10	6	6	4	3
23	10	10	7	6	4	3
24	11	10	7	6	4	4
25	11	11	7	7	4	4
26	12	11	7	7	4	4
27	12	12	8	7	4	4
28	13	12	8	7	5	4
29	13	13	8	8	5	4
30	14	13	8	8	5	4
31	14	13	9	8	5	5
32	14	14	9	9	5	5
33	15	14	9	9	6	5
34	15	15	10	9	6	5
35	16	15	10	9	6	5
36	16	16	10	10	6	5
37	17	16	10	10	6	6
38	17	16	11	10	6	6
39	18	17	11	10	6	6
40	18	17	11	11	7	6
41	18	18	12	11	7	6
42	19	18	12	11	7	6
43	19	19	12	11	7	6
44	20	19	12	12	7	7
45	20	20	13	12	8	7
46	21	20	13	12	8	7
47	21	20	13	13	8	7
48	22	21	14	13	8	7
49	22	21	14	13	8	7
50	22	22	14	13	8	8
51	23	22	14	14	8	8
52	23	23	15	14	9	8
53	24	23	15	14	9	8
54	24	23	15	14	9	8
55	25	24	16	15	9	8
56	25	24	16	15	9	8
57	26	25	16	15	10	9
58	26	25	16	15	10	9
59	27	26	17	16	10	9
60	27	26	17	16	10	9
	Proportional Parts					
	27	26	17	16	10	9

127°

52°

"	$\sin$	d	$\csc$	$\tan$	d	$\cot$	$\sec$	d	$\cos$	"
9.	1'	10.	9.	1'	10.	10.	1'	10.	9.	"
0	78934	21066	89281	10719	10347	89653	60	89653	60	0
1	950	050	307	693	357	643	59	643	59	1
2	967	033	333	667	367	633	58	633	58	2
3	983	017	359	641	376	624	57	624	57	3
4	999	001	385	615	386	614	56	614	56	4
5	79015	20985	411	589	396	604	55	604	55	5
6	031	969	437	563	406	594	54	594	54	6
7	047	953	463	537	416	584	53	584	53	7
8	063	937	489	511	426	574	52	574	52	8
9	079	921	515	485	436	564	51	564	51	9
10	095	905	541	459	446	554	50	554	50	10
11	111	889	567	433	456	544	49	544	49	11
12	128	872	593	407	466	534	48	534	48	12
13	144	856	619	381	476	524	47	524	47	13
14	160	840	645	355	486	514	46	514	46	14
15	176	824	671	329	496	504	45	504	45	15
16	192	808	697	303	505	495	44	495	44	16
17	208	792	723	277	515	485	43	485	43	17
18	224	776	749	251	525	475	42	475	42	18
19	240	760	775	225	535	465	41	465	41	19
20	256	744	801	199	545	455	40	455	40	20
21	272	728	827	173	555	445	39	445	39	21
22	288	712	853	147	565	435	38	435	38	22
23	304	696	879	121	575	425	37	425	37	23
24	319	681	905	95	585	415	36	415	36	24
25	335	665	931	69	595	405	35	405	35	25
26	351	649	957	43	605	395	34	395	34	26
27	367	633	983	17	615	385	33	385	33	27
28	383	617	9009	9	625	375	32	375	32	28
29	399	601	935	26	636	364	31	364	31	29
30	79415	20585	90061	99939	10646	89354	30	89354	30	30
31	431	569	086	914	656	344	29	344	29	31
32	447	553	112	888	666	334	28	334	28	32
33	463	537	138	862	676	324	27	324	27	33
34	478	522	164	836	686	314	26	314	26	34
35	494	506	190	810	696	304	25	304	25	35
36	510	490	216	784	706	294	24	294	24	36
37	526	474	242	758	716	284	23	284	23	37
38	542	458	268	732	726	274	22	274	22	38
39	558	442	294	706	736	264	21	264	21	39
40	573	427	320	680	746	254	20	254	20	40
41	589	411	346	654	756	244	19	244	19	41
42	605	395	371	629	767	233	18	233	18	42
43	621	379	397	603	777	223	17	223	17	43
44	636	364	423	577	787	213	16	213	16	44
45	652	348	449	551	797	203	15	203	15	45
46	668	332	475	525	807	193	14	193	14	46
47	684	316	501	499	817	183	13	183	13	47
48	699	301	527	473	827	173	12	173	12	48
49	715	285	553	447	838	162	11	162	11	49
50	731	269	578	422	848	152	10	152	10	50
51	746	254	604	396	858	142	9	142	9	51
52	762	238	630	370	868	132	8	132	8	52
53	778	222	656	344	878	122	7	122	7	53
54	793	207	682	318	888	112	6	112	6	54
55	809	191	708	292	899	101	5	101	5	55
56	825	175	734	266	909	91	4	91	4	56
57	840	160	759	241	919	81	3	81	3	57
58	856	144	785	215	929	71	2	71	2	58
59	872	128	811	189	940	60	1	60	1	59
60	79887	20113	90837	99163	10950	89050	0	89050	0	60
9.	$\cos$	1'	$\sec$	$\cot$	1'	$\tan$	$\csc$	1'	$\sin$	"
9.	10.	9.	10.	9.	10.	10.	9.	10.	9.	"

"	26	25	17	16	15	11	10	9
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0
3	1	1	1	1	1	1	1	1
4	2	2	1	1	1	1	1	1
5	2	2	1	1	1	1	1	1
6	3	2	2	2	2	1	1	1
7	3	3	2	2	2	1	1	1
8	3	3	2	2	2	1	1	1
9	4	4	3	2	2	2	2	2
10	4	4	3	3	2	2	2	2
11	5	5	3	3	3	2	2	2
12	5	5	3	3	3	2	2	2
13	6	5	4	3	3	2	2	2
14	6	6	4	4	3	2	2	2
15	6	6	4	4	3	2	2	2
16	7	7	5	4	4	3	3	3
17	7	7	5	5	4	3	3	3
18	8	8	5	5	4	3	3	3
19	8	8	5	5	5	3	3	3
20	9	8	6	5	5	4	3	3
21	9	9	6	6	5	4	4	3
22	10	9	6	6	6	4	4	3
23	10	10	7	6	6	4	4	3
24	10	10	7	6	6	4	4	4
25	11	10	7	7	6	5	4	4
26	11	11	7	7	6	5	4	4
27	12	11	8	7	7	5	4	4
28	12	12	8	7	7	5	5	4
29	13	12	8	8	7	5	5	4
30	13	12	8	8	8	6	5	4
31	13	13	9	8	8	6	5	5
32	14	13	9	9	8	6	5	5
33	14	14	9	9	8	6	6	5
34	15	14	10	9	8	6	6	5
35	15	15	10	9	9	6	6	5
36	16	15	10	10	9	7	6	5
37	16	15	10	10	9	7	6	6
38	16	16	11	10	10	7	6	6
39	17	16	11	10	10	7	6	6
40	17	17	11	11	10	7	7	6
41	18	17	12	11	10	8	7	6
42	18	18	12	11	10	8	7	6
43	19	18	12	11	11	8	7	6
44	19	18	12	12	11	8	7	7
45	20	19	13	12	11	8	8	7
46	20	19	13	12	12	8	8	7
47	20	20	13	13	12	9	8	7
48	21	20	14	13	12	9	8	7
49	21	20	14	13	12	9	8	7
50	22	21	14	13	12	9	8	8
51	22	21	14	14	13	9	8	8
52	23	22	15	14	13	10	9	8
53	23	22	15	14	13	10	9	8
54	23	22	15	14	14	10	9	8
55	24	23	16	15	14	10	9	8
56	24	23	16	15	14	10	9	8
57	25	24	16	15	14	10	10	9
58	25	24	16	15	14	11	10	9
59	26	25	17	16	15	11	10	9
60	26	25	17	16	15	11	10	9
"	26	25	17	16	15	11	10	9

Proportional Parts

39°

TABLE II

140°

	$\angle$	$\sin$	$d$	$\angle$	$\csc$	$\tan$	$d$	$\angle$	$\cot$	$\sec$	$d$	$\angle$	$\cos$	$\angle$
	9.	1'	10.	9.	1'	10.	1'	10.	1'	10.	1'	9.	1'	
0	79887		20113	90837		09163	10950		89050	60				
1	903	16	097	863	26	137	960	10	040	59				
2	918	15	082	889	26	111	970	10	030	58				
3	934	16	066	914	25	086	980	10	020	57				
4	950	16	050	940	26	060	991	11	009	56				
5	965	15	035	966	26	034	11001	10	88999	55				
6	981	16	019	992	26	008	011	10	989	54				
7	996	16	004	91018	25	08982	022	11	978	53				
8	80012	16	19988	043	25	957	032	10	968	52				
9	027	16	973	069	26	931	042	10	958	51				
10	043	15	957	095	26	905	052	11	948	50				
11	058	15	942	121	26	879	063	11	937	49				
12	074	16	926	147	26	853	073	10	927	48				
13	089	15	911	172	25	828	083	10	917	47				
14	105	15	895	198	26	802	094	11	906	46				
15	120	16	880	224	26	776	104	10	890	45				
16	136	15	864	250	26	750	114	10	886	44				
17	151	15	849	276	26	724	125	11	875	43				
18	166	15	834	301	25	699	135	10	865	42				
19	182	15	818	327	26	673	145	10	855	41				
20	197	16	803	353	26	647	156	11	844	40				
21	213	15	787	379	25	621	166	10	834	39				
22	228	16	772	404	26	596	176	11	824	38				
23	244	15	756	430	26	570	187	10	813	37				
24	259	15	741	456	26	544	197	10	803	36				
25	274	16	726	482	25	518	207	11	793	35				
26	290	15	710	507	26	493	218	10	782	34				
27	305	15	695	533	26	467	228	11	772	33				
28	320	16	680	559	25	441	239	10	761	32				
29	336	15	664	585	26	415	249	10	751	31				
30	80351	15	19649	91610	25	08390	11259	10	88741	30				
31	366	16	634	636	26	364	270	10	730	29				
32	382	16	618	662	26	338	280	10	720	28				
33	397	15	603	688	25	312	291	10	709	27				
34	412	15	588	713	26	287	301	10	699	26				
35	428	16	572	739	26	261	312	11	688	25				
36	443	15	557	765	26	235	322	10	678	24				
37	458	15	542	791	26	209	332	10	668	23				
38	473	15	527	816	25	184	343	11	657	22				
39	489	16	511	842	26	158	353	11	647	21				
40	504	15	496	868	25	132	364	10	636	20				
41	519	15	481	893	26	107	374	10	626	19				
42	534	16	466	919	25	081	385	11	615	18				
43	550	15	450	945	26	055	395	10	605	17				
44	565	15	435	971	25	029	406	10	594	16				
45	580	15	420	996	26	004	416	11	584	15				
46	595	15	405	92022	26	07978	427	10	573	14				
47	610	15	390	048	25	952	437	11	563	13				
48	625	16	375	073	26	927	448	10	552	12				
49	641	15	359	099	26	901	458	11	542	11				
50	656	15	344	125	25	875	469	10	531	10				
51	671	15	329	150	26	850	479	11	521	9				
52	686	15	314	176	26	824	490	11	510	8				
53	701	15	299	202	26	798	501	11	499	7				
54	716	15	284	227	25	773	511	11	489	6				
55	731	15	269	253	26	747	522	10	478	5				
56	746	16	254	279	25	721	532	11	468	4				
57	762	15	238	304	26	696	543	10	457	3				
58	777	15	223	330	26	670	553	10	447	2				
59	792	15	208	356	25	644	564	11	436	1				
60	80807	15	19193	92381	25	07619	11575	10	88425	0				
	9.	d	10.	9.	d	10.	9.	d	10.	9.	d	10.	9.	d
	$\angle$	$\cos$	$1'$	$\angle$	$\sec$	$1'$	$\angle$	$\cot$	$1'$	$\angle$	$\tan$	$1'$	$\angle$	$\sin$

Proportional Parts						
	26	25	16	15	11	10
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	1	0	0	0
3	1	1	1	1	1	0
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	2	2	2	1	1
7	3	3	2	2	1	1
8	3	3	2	2	1	1
9	4	4	2	2	2	2
10	4	4	3	2	2	2
11	5	5	3	3	2	2
12	5	5	3	3	2	2
13	6	5	3	3	2	2
14	6	6	4	4	3	2
15	6	6	4	4	3	2
16	7	7	4	4	3	2
17	7	7	5	4	3	3
18	8	8	5	4	3	3
19	8	8	5	5	3	3
20	9	8	5	5	4	3
21	9	9	6	5	4	4
22	10	9	6	6	4	4
23	10	10	6	6	4	4
24	10	10	6	6	4	4
25	11	10	7	6	5	4
26	11	11	7	6	5	4
27	12	11	7	7	5	4
28	12	12	7	7	5	5
29	13	12	8	7	5	5
30	13	12	8	8	6	5
31	13	13	8	8	6	5
32	14	13	9	8	6	5
33	14	14	9	8	6	6
34	15	14	9	8	6	6
35	15	15	9	9	6	6
36	16	15	10	9	7	6
37	16	15	10	9	7	6
38	16	16	10	10	7	6
39	17	16	10	10	7	6
40	17	17	11	10	7	7
41	18	17	11	10	8	7
42	18	18	11	10	8	7
43	19	18	11	11	8	7
44	19	18	12	11	8	7
45	20	19	12	11	8	8
46	20	19	12	12	8	8
47	20	20	13	12	9	8
48	21	20	13	12	9	8
49	21	20	13	12	9	8
50	22	21	13	12	9	8
51	22	21	14	13	9	8
52	23	22	14	13	10	9
53	23	22	14	13	10	9
54	23	22	14	14	10	9
55	24	23	15	14	10	9
56	24	23	15	14	10	9
57	25	24	15	14	10	10
58	25	24	15	14	11	10
59	26	25	16	15	11	10
60	26	25	16	15	11	10
Proportional Parts						
	26	25	16	15	11	10

129°

50°

40°

TABLE II

139°

$\circ$	$\sin$	$d$	$\csc$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	10.	9.	10.	10.	10.	10.	10.	10.	9.	10.
0	80807	15	19193	92381	26	07619	11575	10	88425	60
1	822	15	178	407	26	593	585	10	415	59
2	837	15	163	433	26	567	596	10	404	58
3	852	15	148	458	26	542	606	10	394	57
4	867	15	133	484	26	516	617	10	383	56
5	882	15	118	510	26	490	628	10	372	55
6	897	15	103	535	26	465	638	10	362	54
7	912	15	088	561	26	439	649	10	351	53
8	927	15	073	587	26	413	660	10	340	52
9	942	15	058	612	26	388	670	10	330	51
10	957	15	043	638	26	362	681	10	319	50
11	972	15	028	663	26	337	692	10	308	49
12	987	15	013	689	26	311	702	10	298	48
13	81002	15	18908	715	26	285	713	10	287	47
14	017	15	983	740	26	260	724	10	276	46
15	032	15	968	766	26	234	734	10	266	45
16	047	15	953	792	26	208	745	10	255	44
17	061	15	939	817	26	183	756	10	244	43
18	076	15	924	843	26	157	766	10	234	42
19	091	15	909	868	26	132	777	10	223	41
20	106	15	894	894	26	106	788	10	212	40
21	121	15	879	920	26	080	799	10	201	39
22	136	15	864	945	26	055	809	10	191	38
23	151	15	849	971	26	029	820	10	180	37
24	166	15	834	996	26	004	831	10	169	36
25	180	15	820	93022	26	06978	842	10	158	35
26	195	15	805	048	26	952	852	10	148	34
27	210	15	790	073	26	927	863	10	137	33
28	225	15	775	099	26	901	874	10	126	32
29	240	15	760	124	26	876	885	10	115	31
30	81254	15	18746	93150	26	06850	11895	10	88105	30
31	269	15	731	175	26	825	906	10	094	29
32	284	15	716	201	26	799	917	10	083	28
33	299	15	701	227	26	773	928	10	072	27
34	314	15	686	252	26	748	939	10	061	26
35	328	15	672	278	26	722	949	10	051	25
36	343	15	657	303	26	697	960	10	040	24
37	358	15	642	329	26	671	971	10	029	23
38	372	15	628	354	26	646	982	10	018	22
39	387	15	613	380	26	620	993	10	007	21
40	402	15	598	406	26	594	12004	10	87996	20
41	417	15	583	431	26	569	015	10	985	19
42	431	15	569	457	26	543	025	10	975	18
43	446	15	554	482	26	518	036	10	964	17
44	461	15	539	508	26	492	047	10	953	16
45	475	15	525	533	26	467	058	10	942	15
46	490	15	510	559	26	441	069	10	931	14
47	505	15	495	584	26	416	080	10	920	13
48	519	15	481	610	26	390	091	10	909	12
49	534	15	466	636	26	364	102	10	898	11
50	549	15	451	661	26	339	113	10	887	10
51	563	15	437	687	26	313	123	10	877	9
52	578	15	422	712	26	288	134	10	866	8
53	592	15	408	738	26	262	145	10	855	7
54	607	15	393	763	26	237	156	10	844	6
55	622	15	378	789	26	211	167	10	833	5
56	636	15	364	814	26	186	178	10	822	4
57	651	15	349	840	26	160	189	10	811	3
58	665	15	335	865	26	135	200	10	800	2
59	680	15	320	891	26	109	211	10	789	1
60	81694	15	18306	93916	26	06084	12222	10	87778	0
9.	10.	9.	10.	10.	10.	10.	10.	10.	9.	10.
$\cos$	$\sec$	$\cot$	$\tan$	$\sec$	$\sin$					

Proportional Parts						
"	26	25	15	14	11	10
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	0	0	0	0
3	1	1	1	1	1	0
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	2	2	1	1	1
7	3	3	2	2	1	1
8	3	3	2	2	1	1
9	4	4	2	2	2	2
10	4	4	2	2	2	2
11	5	5	3	3	2	2
12	5	5	3	3	2	2
13	6	5	3	3	2	2
14	6	6	4	3	3	2
15	6	6	4	4	3	2
16	7	7	4	4	3	3
17	7	7	4	4	3	3
18	8	8	4	4	3	3
19	8	8	5	4	3	3
20	9	8	5	5	4	3
21	9	9	5	5	4	4
22	10	9	6	5	4	4
23	10	10	6	5	4	4
24	10	10	6	6	4	4
25	11	10	6	6	5	4
26	11	11	6	6	5	4
27	12	11	7	6	5	4
28	12	12	7	7	5	5
29	13	12	7	7	5	5
30	13	12	8	7	6	5
31	13	13	8	7	6	5
32	14	13	8	7	6	5
33	14	14	8	8	6	6
34	15	14	8	8	6	6
35	15	15	9	8	6	6
36	16	15	9	8	7	6
37	16	15	9	9	7	6
38	16	16	10	9	7	6
39	17	16	10	9	7	6
40	17	17	10	9	7	7
41	18	17	10	10	8	7
42	18	18	10	10	8	7
43	19	18	11	10	8	7
44	19	18	11	10	8	7
45	20	19	11	10	8	8
46	20	19	12	11	8	8
47	20	20	12	11	9	8
48	21	20	12	11	9	8
49	21	20	12	11	9	8
50	22	21	12	12	9	8
51	22	21	13	12	9	8
52	23	22	13	12	10	9
53	23	22	13	12	10	9
54	23	22	14	13	10	9
55	24	23	14	13	10	9
56	24	23	14	13	10	9
57	25	24	14	13	10	10
58	25	24	14	14	11	10
59	26	25	15	14	11	10
60	26	25	15	14	11	10
"	26	25	15	14	11	10
Proportional Parts						

130°

49°

41°

TABLE II

138°

	<i>l</i> sin	<i>d</i>	<i>l</i> csc	<i>l</i> tan	<i>d</i>	<i>l</i> cot	<i>l</i> sec	<i>d</i>	<i>l</i> cos	<i>d</i>
9.	10.	9.	10.	10.	10.	10.	10.	10.	9.	10.
0	81694	15	18306	93916	26	06084	12222	11	87778	60
1	709	15	291	942	26	058	233	11	767	59
2	723	14	277	967	25	033	244	11	756	58
3	738	15	262	993	26	007	255	11	745	57
4	752	14	248	94018	25	05982	266	11	734	56
5	767	14	233	044	25	956	277	11	723	55
6	781	14	219	069	25	931	288	11	712	54
7	796	14	204	095	25	905	299	11	701	53
8	810	14	190	120	25	880	310	11	690	52
9	825	15	175	146	25	854	321	11	679	51
10	839	15	161	171	26	829	332	11	668	50
11	854	15	146	197	26	803	343	11	657	49
12	868	14	132	222	25	778	354	11	646	48
13	882	14	118	248	26	752	365	11	635	47
14	897	15	103	273	26	727	376	11	624	46
15	911	15	089	299	25	701	387	12	613	45
16	926	15	074	324	25	676	399	11	601	44
17	940	15	060	350	25	650	410	11	590	43
18	955	15	045	375	25	625	421	11	579	42
19	969	14	031	401	25	599	432	11	568	41
20	983	15	017	426	26	574	443	11	557	40
21	998	14	002	452	25	548	454	11	546	39
22	82012	15	17988	477	26	523	465	11	535	38
23	026	15	974	503	25	497	476	11	524	37
24	041	15	959	528	26	472	487	12	513	36
25	055	14	945	554	25	446	499	11	501	35
26	069	15	931	579	25	421	510	11	490	34
27	084	15	916	604	26	396	521	11	479	33
28	098	14	902	630	25	370	532	11	468	32
29	112	14	888	655	25	345	543	11	457	31
30	82126	15	17874	94681	25	05319	12554	12	87446	30
31	141	14	859	706	26	294	566	11	434	29
32	155	14	845	732	26	268	577	11	423	28
33	169	14	831	757	25	243	588	11	412	27
34	184	15	816	783	26	217	599	11	401	26
35	198	14	802	808	26	192	610	11	390	25
36	212	14	788	834	25	166	622	12	378	24
37	226	14	774	859	25	141	633	11	367	23
38	240	14	760	884	25	116	644	11	356	22
39	255	15	745	910	25	090	655	11	345	21
40	269	14	731	935	25	065	666	11	334	20
41	283	14	717	961	26	039	678	12	322	19
42	297	14	703	986	25	014	689	11	311	18
43	311	15	689	95012	25	04988	700	11	300	17
44	326	14	674	037	25	963	712	11	288	16
45	340	14	660	062	26	938	723	11	277	15
46	354	14	646	088	25	912	734	11	266	14
47	368	14	632	113	26	887	745	11	255	13
48	382	14	618	139	25	861	757	12	243	12
49	396	14	604	164	26	836	768	11	232	11
50	410	14	590	190	25	810	779	12	221	10
51	424	15	576	215	25	785	791	11	209	9
52	439	14	561	240	26	760	802	11	198	8
53	453	14	547	266	25	734	813	11	187	7
54	467	14	533	291	25	709	825	12	175	6
55	481	14	519	317	25	683	836	11	164	5
56	495	14	505	342	26	658	847	12	153	4
57	509	14	491	368	25	632	859	11	141	3
58	523	14	477	393	25	607	870	11	130	2
59	537	14	463	418	26	582	881	12	119	1
60	82551	15	17449	95444	25	04556	12893	12	87107	0
9.	d	9.	d	9.	d	10.	d	9.	d	9.
<i>l</i> cos	<i>l</i> sec	<i>l</i> cot	<i>l</i> tan	<i>l</i> csc	<i>l</i> sin					

	26	25	15	14	12	11
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	0	0	0	0
3	1	1	1	1	1	1
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	2	2	1	1	1
7	3	3	2	2	1	1
8	3	3	2	2	2	1
9	4	4	2	2	2	2
10	4	4	2	2	2	2
11	5	5	3	3	2	2
12	5	5	3	3	2	2
13	6	5	3	3	3	2
14	6	6	4	3	3	3
15	6	6	4	4	3	3
16	7	7	4	4	3	3
17	7	7	4	4	3	3
18	8	8	4	4	4	3
19	8	8	5	4	4	3
20	9	8	5	5	4	4
21	9	9	5	5	4	4
22	10	9	6	5	4	4
23	10	10	6	5	5	4
24	10	10	6	6	5	4
25	11	10	6	6	5	5
26	11	11	6	6	5	5
27	12	11	7	6	5	5
28	12	12	7	7	6	5
29	13	12	7	7	6	5
30	13	12	8	7	6	6
31	13	13	8	7	6	6
32	14	13	8	7	6	6
33	14	14	8	8	7	6
34	15	14	8	8	7	6
35	15	15	9	8	7	6
36	16	15	9	8	7	7
37	16	15	9	9	7	7
38	16	16	10	9	8	7
39	17	16	10	9	8	7
40	17	17	10	9	8	7
41	18	17	10	10	8	8
42	18	18	10	10	8	8
43	19	18	11	10	9	8
44	19	18	11	10	9	8
45	20	19	11	10	9	8
46	20	19	12	11	9	8
47	20	20	12	11	9	9
48	21	20	12	11	10	9
49	21	20	12	11	10	9
50	22	21	12	12	10	9
51	22	21	13	12	10	9
52	23	22	13	12	10	10
53	23	22	13	12	11	10
54	23	22	14	13	11	10
55	24	23	14	13	11	10
56	24	23	14	13	11	10
57	25	24	14	13	11	10
58	25	24	14	14	12	11
59	26	25	15	14	12	11
60	26	25	15	14	12	11
"	26	25	15	14	12	11

Proportional Parts

131°

48°

$\angle$	$\angle$ sin	$\angle$ d	$\angle$ csc	$\angle$ tan	$\angle$ d	$\angle$ cot	$\angle$ sec	$\angle$ d	$\angle$ cos	$\angle$
9.	10.	9.	10.	10.	10.	10.	10.	10.	9.	9.
0	82551	17449	95444	04556	12893	87107	60			
1	565	435	469	531	904	096	59			
2	579	421	495	505	915	085	58			
3	593	407	520	480	927	073	57			
4	607	393	545	455	938	062	56			
5	621	379	571	429	950	050	55			
6	635	365	596	404	961	039	54			
7	649	351	622	378	972	028	53			
8	663	337	647	353	984	016	52			
9	677	323	672	328	995	005	51			
10	691	309	698	302	13007	86993	50			
11	705	295	723	277	018	982	49			
12	719	281	748	252	030	970	48			
13	733	267	774	226	041	959	47			
14	747	253	799	201	053	947	46			
15	761	239	825	175	064	936	45			
16	775	225	850	150	076	924	44			
17	788	212	875	125	087	913	43			
18	802	198	901	099	098	902	42			
19	816	184	926	074	110	890	41			
20	830	170	952	048	121	879	40			
21	844	156	977	023	133	867	39			
22	858	142	96002	03998	145	855	38			
23	872	128	028	972	156	844	37			
24	885	115	053	947	168	832	36			
25	899	101	078	922	179	821	35			
26	913	087	104	896	191	809	34			
27	927	073	129	871	202	798	33			
28	941	059	155	845	214	786	32			
29	955	045	180	820	225	775	31			
30	82968	17032	96205	03795	13237	86763	30			
31	982	018	231	769	248	752	29			
32	996	004	256	744	260	740	28			
33	83010	16990	281	719	272	728	27			
34	023	977	307	693	283	717	26			
35	037	963	332	668	295	705	25			
36	051	949	357	643	306	694	24			
37	065	935	383	617	318	682	23			
38	078	922	408	592	330	670	22			
39	092	908	433	567	341	659	21			
40	106	894	459	541	353	647	20			
41	120	880	484	516	365	635	19			
42	133	867	510	490	376	624	18			
43	147	853	535	465	388	612	17			
44	161	839	560	440	400	600	16			
45	174	826	586	414	411	589	15			
46	188	812	611	389	423	577	14			
47	202	798	636	364	435	565	13			
48	215	785	662	338	446	554	12			
49	229	771	687	313	458	542	11			
50	242	758	712	288	470	530	10			
51	256	744	738	262	482	518	9			
52	270	730	763	237	493	507	8			
53	283	717	788	212	505	495	7			
54	297	703	814	186	517	483	6			
55	310	690	839	161	528	472	5			
56	324	676	864	136	540	460	4			
57	338	662	890	110	552	448	3			
58	351	649	915	085	564	436	2			
59	365	635	940	060	575	425	1			
60	83373	16222	96966	03034	13587	86413	0			
9.	10.	9.	10.	10.	10.	9.	9.			
$\angle$ cos	$\angle$ sec	$\angle$ cot	$\angle$ tan	$\angle$ csc	$\angle$ sin					

Proportional Parts						
"	26	25	14	13	12	11
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	0	0	0	0
3	1	1	1	1	1	1
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	2	1	1	1	1
7	3	3	2	2	1	1
8	3	3	2	2	2	1
9	4	4	2	2	2	2
10	4	4	2	2	2	2
11	5	5	3	3	2	2
12	5	5	3	3	2	2
13	6	5	3	3	3	2
14	6	6	3	3	3	3
15	6	6	4	3	3	3
16	7	7	4	3	3	3
17	7	7	4	4	3	3
18	8	8	4	4	4	3
19	8	8	4	4	4	3
20	9	8	5	4	4	4
21	9	9	5	5	4	4
22	10	9	5	5	4	4
23	10	10	5	5	5	4
24	10	10	6	5	5	4
25	11	10	6	5	5	5
26	11	11	6	6	5	5
27	12	11	6	6	5	5
28	12	12	7	6	6	5
29	13	12	7	6	6	5
30	13	12	7	6	6	6
31	13	13	7	7	6	6
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33	14	14	8	7	7	6
34	15	14	8	7	7	6
35	15	15	8	8	7	6
36	16	15	8	8	7	7
37	16	15	9	8	7	7
38	16	16	9	8	8	7
39	17	16	9	8	8	7
40	17	17	9	9	8	7
41	18	17	10	9	8	8
42	18	18	10	9	8	8
43	19	18	10	9	8	8
44	19	18	10	10	9	8
45	20	19	10	10	9	8
46	20	19	11	10	9	8
47	20	20	11	10	9	9
48	21	20	11	10	9	9
49	21	20	11	11	10	9
50	22	21	12	11	10	9
51	22	21	12	11	10	9
52	23	22	12	11	10	10
53	23	22	12	11	11	10
54	23	22	13	12	11	10
55	24	23	13	12	11	10
56	24	23	13	12	11	10
57	25	24	13	12	11	10
58	25	24	14	13	12	11
59	26	25	14	13	12	11
60	26	25	14	13	12	11
"	26	25	14	13	12	11
Proportional Parts						

43°

TABLE II

136°

$\circ$	$\sin$	$d$	$\sec$	$\tan$	$d$	$\cot$	$\sec$	$d$	$\cos$	$\circ$
9.	10.	9.	10.	10.	10.	10.	10.	10.	9.	10.
0	83378	16622	96966	03034	13587	86413	60			
1	302	608	991	009	599	401	59			
2	405	595	97016	02984	611	389	58			
3	419	581	042	958	623	377	57			
4	432	568	067	933	634	366	56			
5	446	554	092	908	646	354	55			
6	459	541	118	882	658	342	54			
7	473	527	143	857	670	330	53			
8	486	514	168	832	682	318	52			
9	500	500	193	807	694	306	51			
10	513	487	219	781	705	295	50			
11	527	473	244	756	717	283	49			
12	540	460	269	731	729	271	48			
13	554	446	295	705	741	259	47			
14	567	433	320	680	753	247	46			
15	581	419	345	655	765	235	45			
16	594	406	371	629	777	223	44			
17	608	392	396	604	789	211	43			
18	621	379	421	579	800	200	42			
19	634	366	447	553	812	188	41			
20	648	352	472	528	824	176	40			
21	661	339	497	503	836	164	39			
22	674	326	523	477	848	152	38			
23	688	312	548	452	860	140	37			
24	701	299	573	427	872	128	36			
25	715	285	598	402	884	116	35			
26	728	272	624	376	896	104	34			
27	741	259	649	351	908	92	33			
28	755	245	674	326	920	80	32			
29	768	232	700	300	932	68	31			
30	83781	16219	97725	02275	13944	86056	30			
31	795	205	750	250	956	044	29			
32	808	192	776	224	968	032	28			
33	821	179	801	199	980	020	27			
34	834	166	826	174	992	008	26			
35	848	152	851	149	14004	85996	25			
36	861	139	877	123	016	984	24			
37	874	126	902	098	028	972	23			
38	887	113	927	073	040	960	22			
39	901	099	953	047	052	948	21			
40	914	086	978	022	064	936	20			
41	927	073	99003	01997	076	924	19			
42	940	060	029	971	088	912	18			
43	954	046	054	946	100	900	17			
44	967	033	079	921	112	888	16			
45	980	020	104	896	124	876	15			
46	993	007	130	870	136	864	14			
47	84006	15994	155	845	149	851	13			
48	020	980	180	820	161	839	12			
49	033	967	206	794	173	827	11			
50	046	954	231	769	185	815	10			
51	059	941	256	744	197	803	9			
52	072	928	281	719	209	791	8			
53	085	915	307	693	221	779	7			
54	098	902	332	668	234	766	6			
55	112	888	357	643	246	754	5			
56	125	875	383	617	258	742	4			
57	138	862	408	592	270	730	3			
58	151	849	433	567	282	718	2			
59	164	836	458	542	294	706	1			
60	84177	15823	98484	01516	14307	85693	0			
9.	10.	9.	10.	10.	10.	9.	10.			
$\cos$	$\sec$	$\cot$	$\tan$	$\sec$	$\sin$					

Proportional Parts						
"	26	25	14	13	12	11
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	0	0	0	0
3	1	1	1	1	1	1
4	2	2	1	1	1	1
5	2	2	1	1	1	1
6	3	2	1	1	1	1
7	3	3	2	2	1	1
8	3	3	2	2	2	1
9	4	4	2	2	2	2
10	4	4	2	2	2	2
11	5	5	3	2	2	2
12	5	5	3	3	2	2
13	6	5	3	3	3	2
14	6	6	3	3	3	3
15	6	6	4	3	3	3
16	7	7	4	3	3	3
17	7	7	4	4	3	3
18	8	8	4	4	4	3
19	8	8	4	4	4	3
20	9	8	5	4	4	4
21	9	9	5	5	4	4
22	10	9	5	5	4	4
23	10	10	5	5	5	4
24	10	10	6	5	5	4
25	11	10	6	5	5	5
26	11	11	6	6	5	5
27	12	11	6	6	5	5
28	12	12	7	6	6	5
29	13	12	7	6	6	5
30	13	12	7	6	6	6
31	13	13	7	7	6	6
32	14	13	7	7	6	6
33	14	14	8	7	7	6
34	15	14	8	7	7	6
35	15	15	8	8	7	6
36	16	15	8	8	7	7
37	16	15	9	8	7	7
38	16	16	9	8	8	7
39	17	16	9	8	8	7
40	17	17	9	9	8	7
41	18	17	10	9	8	8
42	18	18	10	9	8	8
43	19	18	10	9	9	8
44	19	18	10	10	9	8
45	20	19	10	10	9	8
46	20	19	11	10	9	8
47	20	20	11	10	9	9
48	21	20	11	10	10	9
49	21	20	11	11	10	9
50	22	21	12	11	10	9
51	22	21	12	11	10	9
52	23	22	12	11	10	10
53	23	22	12	11	11	10
54	23	22	13	12	11	10
55	24	23	13	12	11	10
56	24	23	13	12	11	10
57	25	24	13	12	11	10
58	25	24	14	13	12	11
59	26	25	14	13	12	11
60	26	25	14	13	12	11
"	26	25	14	13	12	11
Proportional Parts						

133°

46°



44°

TABLE II

135°

$\angle$	$\sin$	$d$	$\angle$	$\tan$	$d$	$\angle$	$\sec$	$d$	$\angle$	$\cos$	$\angle$
9.	10.	10.	9.	10.	10.	10.	10.	10.	9.	10.	9.
0	84177	15823	98484	01516	14307	85693	60				
1	190	810	509	25	491	319	681	59			
2	203	797	534	25	466	331	669	58			
3	216	784	560	25	440	343	657	57			
4	229	771	585	25	415	355	645	56			
5	242	758	610	390	368		632	55			
6	255	745	635	25	365	380	620	54			
7	269	731	661	26	339	392	608	53			
8	282	718	686	25	314	404	596	52			
9	295	705	711	25	289	417	583	51			
10	308	692	737	26	263	429	571	50			
11	321	679	762	25	238	441	559	49			
12	334	666	787	25	213	453	547	48			
13	347	653	812	25	188	466	534	47			
14	360	640	838	26	162	478	522	46			
15	373	627	863	25	137	490	510	45			
16	385	615	888	25	112	503	497	44			
17	398	602	913	25	87	515	485	43			
18	411	589	939	26	60	527	473	42			
19	424	576	964	25	36	540	460	41			
20	437	563	989	25	11	552	448	40			
21	450	550	99015	26	00985	564	436	39			
22	463	537	040	25	960	577	423	38			
23	476	524	065	25	935	589	411	37			
24	489	511	090	25	910	601	399	36			
25	502	498	116	26	884	614	386	35			
26	515	485	141	25	859	626	374	34			
27	528	472	166	25	834	639	361	33			
28	540	460	191	25	809	651	349	32			
29	553	447	217	26	783	663	337	31			
30	566	434	242	25	758	676	324	30			
31	579	421	267	25	733	688	312	29			
32	592	408	293	26	707	701	299	28			
33	605	395	318	25	682	713	287	27			
34	618	382	343	25	657	726	274	26			
35	630	370	368	25	632	738	262	25			
36	643	357	394	26	606	750	250	24			
37	656	344	419	25	581	763	237	23			
38	669	331	444	25	556	775	225	22			
39	682	318	469	25	531	788	212	21			
40	694	306	495	26	505	800	200	20			
41	707	293	520	25	480	813	187	19			
42	720	280	545	25	455	825	175	18			
43	733	267	570	25	430	838	162	17			
44	745	255	596	26	404	850	150	16			
45	758	242	621	25	379	863	137	15			
46	771	229	646	25	354	875	125	14			
47	784	216	672	26	328	888	112	13			
48	796	204	697	25	303	900	100	12			
49	809	191	722	25	278	913	87	11			
50	825	178	747	25	253	926	74	10			
51	835	165	773	26	227	938	62	9			
52	847	153	798	25	202	951	49	8			
53	860	140	823	25	177	963	37	7			
54	873	127	848	25	152	976	24	6			
55	885	115	874	26	126	988	12	5			
56	898	102	899	25	101	1001	0	4			
57	911	089	924	25	076	014	986	3			
58	923	077	949	25	051	026	974	2			
59	936	064	975	26	025	039	961	1			
60	84949	15051	00000	00000	15051	84949	0				
9.	d	10.	d	10.	d	10.	d	9.			
$\angle$	$\cos$	$\angle$	$\sec$	$\angle$	$\cot$	$\angle$	$\tan$	$\angle$	$\cos$	$\angle$	$\sin$

Proportional Parts					
"	26	25	14	13	12
0	0	0	0	0	0
1	0	0	0	0	0
2	1	1	0	0	0
3	1	1	1	1	1
4	2	2	1	1	1
5	2	2	1	1	1
6	3	2	1	1	1
7	3	3	2	2	2
8	3	3	2	2	2
9	4	4	2	2	2
10	4	4	2	2	2
11	5	5	3	3	3
12	5	5	3	3	3
13	6	5	3	3	3
14	6	6	3	3	3
15	6	6	4	3	3
16	7	7	4	3	3
17	7	7	4	4	3
18	8	8	4	4	4
19	8	8	4	4	4
20	9	8	5	4	4
21	9	9	5	5	4
22	10	9	5	5	4
23	10	10	5	5	5
24	10	10	6	5	5
25	11	10	6	5	5
26	11	11	6	6	5
27	12	11	6	6	5
28	12	12	7	6	6
29	13	12	7	6	6
30	13	12	7	6	6
31	13	13	7	7	6
32	14	13	7	7	6
33	14	14	8	7	7
34	15	14	8	7	7
35	15	15	8	8	7
36	16	15	8	8	7
37	16	15	8	8	7
38	16	16	9	8	8
39	17	16	9	8	8
40	17	17	9	9	8
41	18	17	10	9	8
42	18	18	10	9	8
43	19	18	10	9	9
44	19	18	10	10	9
45	20	19	10	10	9
46	20	19	11	10	9
47	20	20	11	10	9
48	21	20	11	10	10
49	21	20	11	11	10
50	22	21	12	11	10
51	22	21	12	11	10
52	23	22	12	11	10
53	23	22	12	11	11
54	23	22	13	12	11
55	24	23	13	12	11
56	24	23	13	12	11
57	25	24	13	12	11
58	25	24	14	13	12
59	26	25	14	13	12
60	26	25	14	13	12
"	26	25	14	13	12
Proportional Parts					

134°

45°



**TABLE III**  
**NATURAL TRIGONOMETRIC FUNCTIONS**

Of angles for each minute from  $0^\circ$  to  $90^\circ$ , correct  
to five significant figures

0°

TABLE III

1°

	sin	tan	cot	cos			sin	tan	cot	cos	
0	.00000	.00000	∞	1.0000	60	0	.01745	.01746	57.290	.99985	60
1	.029	.029	3437.7	.000	59	1	.774	.775	56.351	.984	59
2	.058	.058	1718.9	.000	58	2	.803	.804	55.442	.984	58
3	.087	.087	1145.9	.000	57	3	.832	.833	54.561	.983	57
4	.116	.116	859.44	.000	56	4	.862	.862	53.709	.983	56
5	.00145	.00145	687.55	1.0000	55	5	.01891	.01891	52.882	.99982	55
6	.175	.175	572.96	.000	54	6	.920	.920	52.081	.982	54
7	.204	.204	491.11	.000	53	7	.949	.949	51.303	.981	53
8	.233	.233	429.72	.000	52	8	.01978	.01978	50.549	.980	52
9	.262	.262	381.97	.000	51	9	.02007	.02007	49.816	.980	51
10	.00291	.00291	343.77	1.0000	50	10	.02036	.02036	49.104	.99979	50
11	.320	.320	312.52	.99999	49	11	.065	.066	48.412	.979	49
12	.349	.349	286.48	.999	48	12	.094	.095	47.740	.978	48
13	.378	.378	264.44	.999	47	13	.123	.124	47.085	.977	47
14	.407	.407	245.55	.999	46	14	.152	.153	46.449	.977	46
15	.00436	.00436	229.18	.99999	45	15	.02181	.02182	45.829	.99976	45
16	.465	.465	214.86	.999	44	16	.211	.211	45.226	.976	44
17	.495	.495	202.22	.999	43	17	.240	.240	44.639	.975	43
18	.524	.524	190.98	.999	42	18	.269	.269	44.066	.974	42
19	.553	.553	180.93	.998	41	19	.298	.298	43.508	.974	41
20	.00582	.00582	171.89	.99998	40	20	.02327	.02328	42.964	.99973	40
21	.611	.611	153.70	.998	39	21	.356	.357	42.433	.972	39
22	.640	.640	136.26	.998	38	22	.385	.386	41.916	.972	38
23	.669	.669	119.47	.998	37	23	.414	.415	41.411	.971	37
24	.698	.698	103.24	.998	36	24	.443	.444	40.917	.970	36
25	.00727	.00727	137.51	.99997	35	25	.02473	.02473	40.436	.99969	35
26	.756	.756	132.22	.997	34	26	.501	.502	39.965	.969	34
27	.785	.785	127.32	.997	33	27	.530	.531	39.506	.968	33
28	.814	.815	122.77	.997	32	28	.560	.560	39.057	.967	32
29	.844	.844	118.54	.996	31	29	.589	.589	38.618	.966	31
30	.00873	.00873	114.59	.99996	30	30	.02618	.02619	38.188	.99966	30
31	.902	.902	110.89	.996	29	31	.647	.648	37.769	.965	29
32	.931	.931	107.43	.996	28	32	.676	.677	37.358	.964	28
33	.960	.960	104.17	.995	27	33	.705	.706	36.956	.963	27
34	.00989	.00989	101.11	.995	26	34	.734	.735	36.563	.963	26
35	.01018	.01018	98.218	.99995	25	35	.02763	.02764	36.178	.99962	25
36	.047	.047	95.489	.995	24	36	.792	.793	35.801	.961	24
37	.076	.076	92.908	.994	23	37	.821	.822	35.431	.960	23
38	.105	.105	90.463	.994	22	38	.850	.851	35.070	.959	22
39	.134	.135	88.144	.994	21	39	.879	.881	34.715	.959	21
40	.01164	.01164	85.940	.99993	20	40	.02908	.02910	34.368	.99958	20
41	.193	.193	83.844	.993	19	41	.938	.939	34.027	.957	19
42	.222	.222	81.847	.993	18	42	.967	.968	33.694	.956	18
43	.251	.251	79.943	.992	17	43	.02996	.02997	33.366	.955	17
44	.280	.280	78.126	.992	16	44	.03025	.03026	33.045	.954	16
45	.01309	.01309	76.390	.99991	15	45	.03054	.03055	32.730	.99953	15
46	.338	.338	74.729	.991	14	46	.083	.084	32.421	.952	14
47	.367	.367	73.139	.991	13	47	.112	.114	32.118	.952	13
48	.396	.396	71.615	.990	12	48	.141	.143	31.821	.951	12
49	.425	.425	70.153	.990	11	49	.170	.172	31.528	.950	11
50	.01454	.01455	68.750	.99989	10	50	.03199	.03201	31.242	.99949	10
51	.483	.484	67.402	.989	9	51	.228	.230	30.960	.948	9
52	.513	.513	66.105	.989	8	52	.257	.259	30.683	.947	8
53	.542	.542	64.858	.988	7	53	.286	.288	30.412	.946	7
54	.571	.571	63.657	.988	6	54	.316	.317	30.145	.945	6
55	.01600	.01600	62.499	.99987	5	55	.03345	.03346	29.882	.99944	5
56	.629	.629	61.383	.987	4	56	.374	.376	29.624	.943	4
57	.658	.658	60.306	.986	3	57	.403	.405	29.371	.942	3
58	.687	.687	59.266	.986	2	58	.432	.434	29.122	.941	2
59	.716	.716	58.261	.985	1	59	.461	.463	28.877	.940	1
60	.01745	.01746	57.290	.99985	0	60	.03490	.03492	28.636	.99939	0
	cos	cot	tan	sin	'		cos	cot	tan	sin	'

89°

92

88°

2°

TABLE III

3°

'	sin	tan	cot	cos	'
0	.03490	.03492	28.636	.99939	60
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	55
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	50
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	45
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	40
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	35
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	30
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	25
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	20
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	15
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	5
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	0
	cos	cot	tan	sin	'

87°

'	sin	tan	cot	cos	'
0	.05234	.05241	19.081	.99863	60
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	55
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	50
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	45
16	698	703	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	40
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	35
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	30
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	25
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	20
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	15
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	10
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	5
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	0
	cos	cot	tan	sin	'

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4°

TABLE III

5°

'	sin	tan	cot	cos	'
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	.241	.754	59
2	.034	.051	.182	.752	58
3	.063	.080	.124	.750	57
4	.092	.110	.065	.748	56
5	.07121	.07139	14.008	.99746	55
6	.150	.168	13.951	.744	54
7	.179	.197	.894	.742	53
8	.208	.227	.838	.740	52
9	.237	.256	.782	.738	51
10	.07266	.07285	13.727	.99736	50
11	.295	.314	.672	.734	49
12	.324	.344	.617	.731	48
13	.353	.373	.563	.729	47
14	.382	.402	.510	.727	46
15	.07411	.07431	13.457	.99725	45
16	.440	.461	.404	.723	44
17	.469	.490	.352	.721	43
18	.498	.519	.300	.719	42
19	.527	.548	.248	.716	41
20	.07556	.07578	13.197	.99714	40
21	.585	.607	.146	.712	39
22	.614	.636	.096	.710	38
23	.643	.665	13.046	.708	37
24	.672	.695	12.996	.705	36
25	.07701	.07724	12.947	.99703	35
26	.730	.753	.898	.701	34
27	.759	.782	.850	.699	33
28	.788	.812	.801	.696	32
29	.817	.841	.754	.694	31
30	.07846	.07870	12.706	.99692	30
31	.875	.899	.659	.689	29
32	.904	.929	.612	.687	28
33	.933	.958	.566	.685	27
34	.962	.07987	.520	.683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	.046	.429	.678	24
37	.049	.075	.384	.676	23
38	.078	.104	.339	.673	22
39	.107	.134	.295	.671	21
40	.08136	.08163	12.251	.99668	20
41	.165	.192	.207	.666	19
42	.194	.221	.163	.664	18
43	.223	.251	.120	.661	17
44	.252	.280	.077	.659	16
45	.08281	.08309	12.035	.99657	15
46	.310	.339	11.992	.654	14
47	.339	.368	.950	.652	13
48	.368	.397	.909	.649	12
49	.397	.427	.867	.647	11
50	.08426	.08456	11.826	.99644	10
51	.455	.485	.785	.642	9
52	.484	.514	.745	.639	8
53	.513	.544	.705	.637	7
54	.542	.573	.664	.635	6
55	.08571	.08602	11.625	.99632	5
56	.600	.632	.585	.630	4
57	.629	.661	.546	.627	3
58	.658	.690	.507	.625	2
59	.687	.720	.468	.622	1
60	.08716	.08749	11.430	.99619	0
	cos	cot	tan	sin	'

85°

'	sin	tan	cot	cos	'
0	.08716	.08749	11.430	.99619	60
1	.745	.778	.392	.617	59
2	.774	.807	.354	.614	58
3	.803	.837	.316	.612	57
4	.831	.866	.279	.609	56
5	.08860	.08895	11.242	.99607	55
6	.889	.925	.205	.604	54
7	.918	.954	.168	.602	53
8	.947	.08983	.132	.599	52
9	.08976	.09013	.095	.596	51
10	.09005	.09042	11.059	.99594	50
11	.034	.071	11.024	.591	49
12	.063	.101	10.988	.588	48
13	.092	.130	.953	.586	47
14	.121	.159	.918	.583	46
15	.09150	.09189	10.883	.99580	45
16	.179	.218	.848	.578	44
17	.208	.247	.814	.575	43
18	.237	.277	.780	.572	42
19	.266	.306	.746	.570	41
20	.09295	.09335	10.712	.99567	40
21	.324	.365	.678	.564	39
22	.353	.394	.645	.562	38
23	.382	.423	.612	.559	37
24	.411	.453	.579	.556	36
25	.09440	.09482	10.546	.99553	35
26	.469	.511	.514	.551	34
27	.498	.541	.481	.548	33
28	.527	.570	.449	.545	32
29	.556	.600	.417	.542	31
30	.09585	.09629	10.385	.99540	30
31	.614	.658	.354	.537	29
32	.642	.688	.322	.534	28
33	.671	.717	.291	.531	27
34	.700	.746	.260	.528	26
35	.09729	.09776	10.229	.99526	25
36	.758	.805	.199	.523	24
37	.787	.834	.168	.520	23
38	.816	.864	.138	.517	22
39	.845	.893	.108	.514	21
40	.09874	.09923	10.078	.99511	20
41	.903	.952	.048	.508	19
42	.932	.09981	10.019	.506	18
43	.961	.10011	9.9893	.503	17
44	.09990	.040	.9601	.500	16
45	.10019	.10069	9.9310	.99497	15
46	.048	.099	.9021	.494	14
47	.077	.128	.8734	.491	13
48	.106	.158	.8448	.488	12
49	.135	.187	.8164	.485	11
50	.10164	.10216	9.7882	.99482	10
51	.192	.246	.7601	.479	9
52	.221	.275	.7322	.476	8
53	.250	.305	.7044	.473	7
54	.279	.334	.6768	.470	6
55	.10308	.10363	9.6493	.99467	5
56	.337	.393	.6220	.464	4
57	.366	.422	.5949	.461	3
58	.395	.452	.5679	.458	2
59	.424	.481	.5411	.455	1
60	.10453	.10510	9.5144	.99452	0
	cos	cot	tan	sin	'

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6°

TABLE III

7°

'	sin	tan	cot	cos	'
0	.10453	.10510	9.5144	.99452	60
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	55
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	50
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	45
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	40
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	35
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	30
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	25
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	20
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	15
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	10
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	5
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	0
	cos	cot	tan	sin	'

83°

95

'	sin	tan	cot	cos	'
0	.12187	.12278	8.1443	.99255	60
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	55
6	360	456	.0283	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	50
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	45
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	40
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	35
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	30
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	25
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	20
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	15
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	10
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	5
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	0
	cos	cot	tan	sin	'

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8°

TABLE III

9°

'	sin	tan	cot	cos	'
0	.13917	.14354	7.1154	.99027	60
1	.13946	.14384	7.1004	.99023	59
2	.13975	.14413	.8855	.99019	58
3	.14004	.14443	.8706	.99015	57
4	.14033	.14473	.8558	.99011	56
5	.14061	.14502	7.0410	.99006	55
6	.14090	.14532	.8264	.99002	54
7	.14119	.14562	7.0117	.98998	53
8	.14148	.14591	.8092	.98994	52
9	.14177	.14621	.7947	.98990	51
10	.14205	.14651	6.9682	.98986	50
11	.14234	.14681	.9538	.98982	49
12	.14263	.14710	.9395	.98978	48
13	.14292	.14740	.9252	.98973	47
14	.14320	.14770	.9110	.98969	46
15	.14349	.14800	6.8969	.98965	45
16	.14378	.14829	.8828	.98961	44
17	.14407	.14859	.8687	.98957	43
18	.14436	.14888	.8548	.98953	42
19	.14464	.14918	.8408	.98948	41
20	.14493	.14948	6.8269	.98944	40
21	.14522	.14978	.8131	.98940	39
22	.14551	.15007	.7994	.98936	38
23	.14580	.15037	.7856	.98931	37
24	.14608	.15067	.7720	.98927	36
25	.14637	.15096	6.7584	.98923	35
26	.14666	.15126	.7448	.98919	34
27	.14695	.15156	.7313	.98914	33
28	.14723	.15186	.7179	.98910	32
29	.14752	.15216	.7045	.98906	31
30	.14781	.15245	6.6912	.98902	30
31	.14810	.15275	.6779	.98897	29
32	.14838	.15305	.6646	.98893	28
33	.14867	.15335	.6514	.98889	27
34	.14896	.15365	.6383	.98884	26
35	.14925	.15394	6.6252	.98880	25
36	.14954	.15424	.6122	.98876	24
37	.14982	.15454	.5992	.98871	23
38	.15011	.15484	.5863	.98867	22
39	.15040	.15514	.5734	.98863	21
40	.15069	.15543	6.5606	.98858	20
41	.15097	.15573	.5478	.98854	19
42	.15126	.15603	.5350	.98849	18
43	.15155	.15633	.5223	.98845	17
44	.15184	.15663	.5097	.98841	16
45	.15212	.15693	6.4971	.98836	15
46	.15241	.15723	.4846	.98832	14
47	.15270	.15753	.4721	.98827	13
48	.15299	.15783	.4596	.98823	12
49	.15327	.15813	.4472	.98818	11
50	.15356	.15843	6.4348	.98814	10
51	.15385	.15873	.4225	.98809	9
52	.15414	.15903	.4103	.98805	8
53	.15442	.15933	.3980	.98800	7
54	.15471	.15963	.3859	.98796	6
55	.15500	.15993	6.3737	.98791	5
56	.15529	.16023	.3617	.98787	4
57	.15557	.16053	.3496	.98782	3
58	.15586	.16083	.3376	.98778	2
59	.15615	.16113	.3257	.98773	1
60	.15643	.15838	6.3138	.98769	0
	cos	cot	tan	sin	'

81°

'	sin	tan	cot	cos	'
0	.15643	.15838	6.3138	.98769	60
1	.15672	.15868	.3019	.98764	59
2	.15701	.15898	.2901	.98760	58
3	.15730	.15928	.2783	.98755	57
4	.15758	.15958	.2666	.98751	56
5	.15787	.15988	6.2549	.98746	55
6	.15816	.16017	.2432	.98741	54
7	.15845	.16047	.2316	.98737	53
8	.15873	.16077	.2200	.98732	52
9	.15902	.16107	.2085	.98728	51
10	.15931	.16137	6.1970	.98723	50
11	.15959	.16167	.1856	.98718	49
12	.15988	.16196	.1742	.98714	48
13	.16017	.16226	.1628	.98709	47
14	.16046	.16256	.1515	.98704	46
15	.16074	.16286	6.1402	.98700	45
16	.16103	.16316	.1290	.98695	44
17	.16132	.16346	.1178	.98690	43
18	.16160	.16376	.1066	.98686	42
19	.16189	.16405	.0955	.98681	41
20	.16218	.16435	6.0844	.98676	40
21	.16246	.16465	.0734	.98671	39
22	.16275	.16495	.0624	.98667	38
23	.16304	.16525	.0514	.98662	37
24	.16333	.16555	.0405	.98657	36
25	.16361	.16585	6.0296	.98652	35
26	.16390	.16615	.0188	.98648	34
27	.16419	.16645	.0080	.98643	33
28	.16447	.16674	5.9972	.98638	32
29	.16476	.16704	.9865	.98633	31
30	.16505	.16734	5.9758	.98629	30
31	.16533	.16764	.9651	.98624	29
32	.16562	.16794	.9545	.98619	28
33	.16591	.16824	.9439	.98614	27
34	.16620	.16854	.9333	.98609	26
35	.16648	.16884	5.9228	.98604	25
36	.16677	.16914	.9124	.98600	24
37	.16706	.16944	.9019	.98595	23
38	.16734	.16974	.8915	.98590	22
39	.16763	.17004	.8811	.98585	21
40	.16792	.17033	5.8708	.98580	20
41	.16820	.17063	.8605	.98575	19
42	.16849	.17093	.8502	.98570	18
43	.16878	.17123	.8400	.98565	17
44	.16906	.17153	.8298	.98561	16
45	.16935	.17183	5.8197	.98556	15
46	.16964	.17213	.8095	.98551	14
47	.16992	.17243	.7994	.98546	13
48	.17021	.17273	.7894	.98541	12
49	.17050	.17303	.7794	.98536	11
50	.17078	.17333	5.7694	.98531	10
51	.17107	.17363	.7594	.98526	9
52	.17136	.17393	.7495	.98521	8
53	.17164	.17423	.7396	.98516	7
54	.17193	.17453	.7297	.98511	6
55	.17222	.17483	5.7199	.98506	5
56	.17250	.17513	.7101	.98501	4
57	.17279	.17543	.7004	.98496	3
58	.17308	.17573	.6906	.98491	2
59	.17336	.17603	.6809	.98486	1
60	.17365	.17633	5.6713	.98481	0
	cos	cot	tan	sin	'

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10°

TABLE III

11°

'	sin	tan	cot	cos	'
0	.17365	.17633	5.6713	.98481	60
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	55
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	50
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	45
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	40
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	35
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	30
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	25
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	20
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	15
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	10
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	5
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	0
	cos	cot	tan	sin	'

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'	sin	tan	cot	cos	'
0	.19081	.19438	5.1446	.98163	60
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	55
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	50
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	45
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	40
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	35
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	25
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	20
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	15
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	10
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	5
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	0
	cos	cot	tan	sin	'

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	sin	tan	cot	cos			sin	tan	cot	cos	
0	.20791	.21256	4.7046	.97815	60	0	.22495	.23087	4.3315	.97437	60
1	820	286	.6979	809	59	1	523	117	.3257	430	59
2	848	316	.6912	803	58	2	552	148	.3200	424	58
3	877	347	.6845	797	57	3	580	179	.3143	417	57
4	905	377	.6779	791	56	4	608	209	.3086	411	56
5	.20933	.21408	4.6712	.97784	55	5	.22637	.23240	4.3029	.97404	55
6	962	438	.6646	778	54	6	665	271	.2972	398	54
7	.20990	469	.6580	772	53	7	693	301	.2916	391	53
8	.21019	499	.6514	766	52	8	722	332	.2859	384	52
9	047	529	.6448	760	51	9	750	363	.2803	378	51
10	.21076	.21560	4.6382	.97754	50	10	.22778	.23393	4.2747	.97371	50
11	104	590	.6317	748	49	11	807	424	.2691	365	49
12	132	621	.6252	742	48	12	835	455	.2635	358	48
13	161	651	.6187	735	47	13	863	485	.2580	351	47
14	189	682	.6122	729	46	14	892	516	.2524	345	46
15	.21218	.21712	4.6057	.97723	45	15	.22920	.23547	4.2468	.97338	45
16	246	743	.5993	717	44	16	948	578	.2413	331	44
17	275	773	.5928	711	43	17	.22977	608	.2358	325	43
18	303	804	.5864	705	42	18	.23005	639	.2303	318	42
19	331	834	.5800	698	41	19	033	670	.2248	311	41
20	.21360	.21864	4.5736	.97692	40	20	.23062	.23700	4.2193	.97304	40
21	388	895	.5673	686	39	21	090	731	.2139	298	39
22	417	925	.5609	680	38	22	118	762	.2084	291	38
23	445	956	.5546	673	37	23	146	793	.2030	284	37
24	474	.21986	.5483	667	36	24	175	823	.1976	278	36
25	.21502	.22017	4.5420	.97661	35	25	.23203	.23854	4.1922	.97271	35
26	530	047	.5357	655	34	26	231	885	.1868	264	34
27	559	078	.5294	648	33	27	260	916	.1814	257	33
28	587	108	.5232	642	32	28	288	946	.1760	251	32
29	616	139	.5169	636	31	29	316	.23977	.1706	244	31
30	.21644	.22169	4.5107	.97630	30	30	.23345	.24008	4.1653	.97237	30
31	672	200	.5045	623	29	31	373	039	.1600	230	29
32	701	231	.4983	617	28	32	401	069	.1547	223	28
33	729	261	.4922	611	27	33	429	100	.1493	217	27
34	758	292	.4860	604	26	34	458	131	.1441	210	26
35	.21786	.22322	4.4799	.97598	25	35	.23486	.24162	4.1388	.97203	25
36	814	353	.4737	592	24	36	514	193	.1335	196	24
37	843	383	.4676	585	23	37	542	223	.1282	189	23
38	871	414	.4615	579	22	38	571	254	.1230	182	22
39	899	444	.4555	573	21	39	599	285	.1178	176	21
40	.21928	.22475	4.4494	.97566	20	40	.23627	.24316	4.1126	.97169	20
41	956	505	.4434	560	19	41	656	347	.1074	162	19
42	.21985	536	.4373	553	18	42	684	377	.1022	155	18
43	.22013	567	.4313	547	17	43	712	408	.0970	148	17
44	041	597	.4253	541	16	44	740	439	.0918	141	16
45	.22070	.22628	4.4194	.97534	15	45	.23769	.24470	4.0867	.97134	15
46	098	658	.4134	528	14	46	797	501	.0815	127	14
47	126	689	.4075	521	13	47	825	532	.0764	120	13
48	155	719	.4015	515	12	48	853	562	.0713	113	12
49	183	750	.3956	508	11	49	882	593	.0662	106	11
50	.22212	.22781	4.3897	.97502	10	50	.23910	.24624	4.0611	.97100	10
51	240	811	.3838	496	9	51	938	655	.0560	093	9
52	268	842	.3779	489	8	52	966	686	.0509	086	8
53	297	872	.3721	483	7	53	.23995	717	.0459	079	7
54	325	903	.3662	476	6	54	.24023	747	.0408	072	6
55	.22353	.22934	4.3604	.97470	5	55	.24051	.24778	4.0358	.97065	5
56	382	964	.3546	463	4	56	079	809	.0308	058	4
57	410	.22995	.3488	457	3	57	108	840	.0257	051	3
58	438	.23026	.3430	450	2	58	136	871	.0207	044	2
59	467	056	.3372	444	1	59	164	902	.0158	037	1
60	.22495	.23087	4.3315	.97437	0	60	.24192	.24933	4.0108	.97030	0
	cos	cot	tan	sin			cos	cot	tan	sin	

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TABLE III

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'	sin	tan	cot	cos	'
0	.24192	.24933	4.0108	.97030	60
1	220	.964	.0058	.023	59
2	249	.24995	4.0009	.015	58
3	277	.25026	3.9959	.008	57
4	305	.056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	.118	.9812	.987	54
7	390	.149	.9763	.980	53
8	418	.180	.9714	.973	52
9	446	.211	.9665	.966	51
10	.24474	.25242	3.9617	.96959	50
11	503	.273	.9568	.952	49
12	531	.304	.9520	.945	48
13	559	.335	.9471	.937	47
14	587	.366	.9423	.930	46
15	.24615	.25397	3.9375	.96923	45
16	644	.428	.9327	.916	44
17	672	.459	.9279	.909	43
18	700	.490	.9232	.902	42
19	728	.521	.9184	.894	41
20	.24756	.25552	3.9136	.96887	40
21	784	.583	.9089	.880	39
22	813	.614	.9042	.873	38
23	841	.645	.8995	.866	37
24	869	.676	.8947	.858	36
25	.24897	.25707	3.8900	.96851	35
26	925	.738	.8854	.844	34
27	954	.769	.8807	.837	33
28	.24982	.800	.8760	.829	32
29	.25010	.831	.8714	.822	31
30	.25038	.25862	3.8667	.96815	30
31	066	.893	.8621	.807	29
32	094	.924	.8575	.800	28
33	122	.955	.8528	.793	27
34	151	.25986	.8482	.786	26
35	.25179	.26017	3.8436	.96778	25
36	207	.048	.8391	.771	24
37	235	.079	.8345	.764	23
38	263	.110	.8299	.756	22
39	291	.141	.8254	.749	21
40	.25320	.26172	3.8208	.96742	20
41	348	.203	.8163	.734	19
42	376	.235	.8118	.727	18
43	404	.266	.8073	.719	17
44	432	.297	.8028	.712	16
45	.25460	.26328	3.7983	.96705	15
46	488	.359	.7938	.697	14
47	516	.390	.7893	.690	13
48	545	.421	.7848	.682	12
49	573	.452	.7804	.675	11
50	.25601	.26483	3.7760	.96667	10
51	629	.515	.7715	.660	9
52	657	.546	.7671	.653	8
53	685	.577	.7627	.645	7
54	713	.608	.7583	.638	6
55	.25741	.26639	3.7539	.96630	5
56	769	.670	.7495	.623	4
57	798	.701	.7451	.615	3
58	826	.733	.7408	.608	2
59	854	.764	.7364	.600	1
60	.25882	.26795	3.7321	.96593	0
	cos	cot	tan	sin	'

75°

'	sin	tan	cot	cos	'
0	.25882	.26795	3.7321	.96593	60
1	910	.826	.7277	.585	59
2	938	.857	.7234	.578	58
3	966	.888	.7191	.570	57
4	.25994	.920	.7148	.562	56
5	.26022	.26951	3.7105	.96555	55
6	.050	.26982	.7062	.547	54
7	.079	.27013	.7019	.540	53
8	.107	.044	.6976	.532	52
9	.135	.076	.6933	.524	51
10	.26163	.27107	3.6891	.96517	50
11	.191	.138	.6848	.509	49
12	.219	.169	.6806	.502	48
13	.247	.201	.6764	.494	47
14	.275	.232	.6722	.486	46
15	.26303	.27263	3.6680	.96479	45
16	.331	.294	.6638	.471	44
17	.359	.326	.6596	.463	43
18	.387	.357	.6554	.456	42
19	.415	.388	.6512	.448	41
20	.26443	.27419	3.6470	.96440	40
21	.471	.451	.6429	.433	39
22	.500	.482	.6387	.425	38
23	.528	.513	.6346	.417	37
24	.556	.545	.6305	.410	36
25	.26584	.27576	3.6264	.96402	35
26	.612	.607	.6222	.394	34
27	.640	.638	.6181	.386	33
28	.668	.670	.6140	.379	32
29	.696	.701	.6100	.371	31
30	.26724	.27732	3.6059	.96363	30
31	.752	.764	.6018	.355	29
32	.780	.795	.5978	.347	28
33	.808	.826	.5937	.340	27
34	.836	.858	.5897	.332	26
35	.26864	.27889	3.5856	.96324	25
36	.892	.921	.5816	.316	24
37	.920	.952	.5776	.308	23
38	.948	.27983	.5736	.301	22
39	.26976	.28015	.5696	.293	21
40	.27004	.28046	3.5656	.96285	20
41	.032	.077	.5616	.277	19
42	.060	.109	.5576	.269	18
43	.088	.140	.5536	.261	17
44	.116	.172	.5497	.253	16
45	.27144	.28203	3.5457	.96246	15
46	.172	.234	.5418	.238	14
47	.200	.266	.5379	.230	13
48	.228	.297	.5339	.222	12
49	.256	.329	.5300	.214	11
50	.27284	.28360	3.5261	.96206	10
51	.312	.391	.5222	.198	9
52	.340	.423	.5183	.190	8
53	.368	.454	.5144	.182	7
54	.396	.486	.5105	.174	6
55	.27424	.28517	3.5067	.96166	5
56	.452	.549	.5028	.158	4
57	.480	.580	.4989	.150	3
58	.508	.612	.4951	.142	2
59	.536	.643	.4912	.134	1
60	.27564	.28675	3.4874	.96126	0
	cos	cot	tan	sin	'

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TABLE III

17°

	sin	tan	cot	cos	
0	.27564	.28675	3.4874	.96126	60
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	55
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	50
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	45
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	40
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	35
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	30
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	25
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	20
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	15
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	5
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	0
	cos	cot	tan	sin	

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100

	sin	tan	cot	cos	
0	.29237	.30573	3.2709	.95630	60
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	55
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	50
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	45
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	40
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	35
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	30
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	25
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	20
41	376	882	.1366	275	19
42	403	914	.1334	260	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	15
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	10
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	5
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	0
	cos	cot	tan	sin	

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TABLE III

19°

	sin	tan	cot	cos	
0	.30902	.32492	3.0777	.95106	60
1	929	524	.0746	.097	59
2	957	556	.0716	.088	58
3	.30985	588	.0686	.079	57
4	.31012	621	.0655	.070	56
5	.31040	.32653	3.0625	.95061	55
6	068	685	.0595	.052	54
7	095	717	.0565	.043	53
8	123	749	.0535	.033	52
9	151	782	.0505	.024	51
10	.31178	.32814	3.0475	.95015	50
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	.988	47
14	289	943	.0356	.979	46
15	.31316	.32975	3.0326	.94970	45
16	344	.33007	.0296	.961	44
17	372	040	.0267	.952	43
18	399	072	.0237	.943	42
19	427	104	.0208	.933	41
20	.31454	.33136	3.0178	.94924	40
21	482	169	.0149	.915	39
22	510	201	.0120	.906	38
23	537	233	.0090	.897	37
24	565	266	.0061	.888	36
25	.31593	.33298	3.0032	.94878	35
26	620	330	3.0003	.869	34
27	648	363	2.9974	.860	33
28	675	395	.9945	.851	32
29	703	427	.9916	.842	31
30	.31730	.33460	2.9887	.94832	30
31	758	492	.9858	.823	29
32	786	524	.9829	.814	28
33	813	557	.9800	.805	27
34	841	589	.9772	.795	26
35	.31868	.33621	2.9743	.94786	25
36	896	654	.9714	.777	24
37	923	686	.9686	.768	23
38	951	718	.9657	.758	22
39	.31979	751	.9629	.749	21
40	.32006	.33783	2.9600	.94740	20
41	034	816	.9572	.730	19
42	061	848	.9544	.721	18
43	089	881	.9515	.712	17
44	116	913	.9487	.702	16
45	.32144	.33945	2.9459	.94693	15
46	171	.33978	.9431	.684	14
47	199	.34010	.9403	.674	13
48	227	043	.9375	.665	12
49	254	075	.9347	.656	11
50	.32282	.34108	2.9319	.94646	10
51	309	140	.9291	.637	9
52	337	173	.9263	.627	8
53	364	205	.9235	.618	7
54	392	238	.9208	.609	6
55	.32419	.34270	2.9180	.94599	5
56	447	303	.9152	.590	4
57	474	335	.9125	.580	3
58	502	368	.9097	.571	2
59	529	400	.9070	.561	1
60	.32557	.34433	2.9042	.94552	0
	cos	cot	tan	sin	

	sin	tan	cot	cos	
0	.32557	.34433	2.9042	.94552	60
1	584	465	.9015	.542	59
2	612	498	.8987	.533	58
3	639	530	.8960	.523	57
4	667	563	.8933	.514	56
5	.32694	.34596	2.8905	.94504	55
6	722	628	.8878	.495	54
7	749	661	.8851	.485	53
8	777	693	.8824	.476	52
9	804	726	.8797	.466	51
10	.32832	.34758	2.8770	.94457	50
11	859	791	.8743	.447	49
12	887	824	.8716	.438	48
13	914	856	.8689	.428	47
14	942	889	.8662	.418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	954	.8609	.399	44
17	.33024	.34987	.8582	.390	43
18	051	.35020	.8556	.380	42
19	079	052	.8529	.370	41
20	.33106	.35085	2.8502	.94361	40
21	134	118	.8476	.351	39
22	161	150	.8449	.342	38
23	189	183	.8423	.332	37
24	216	216	.8397	.322	36
25	.33244	.35248	2.8370	.94313	35
26	271	281	.8344	.303	34
27	298	314	.8318	.293	33
28	326	346	.8291	.284	32
29	353	379	.8265	.274	31
30	.33381	.35412	2.8239	.94264	30
31	408	445	.8213	.254	29
32	436	477	.8187	.245	28
33	463	510	.8161	.235	27
34	490	543	.8135	.225	26
35	.33518	.35576	2.8109	.94215	25
36	545	608	.8083	.206	24
37	573	641	.8057	.196	23
38	600	674	.8032	.186	22
39	627	707	.8006	.176	21
40	.33655	.35740	2.7980	.94167	20
41	682	772	.7955	.157	19
42	710	805	.7929	.147	18
43	737	838	.7903	.137	17
44	764	871	.7878	.127	16
45	.33792	.35904	2.7852	.94118	15
46	819	937	.7827	.108	14
47	846	.35969	.7801	.098	13
48	874	.36002	.7776	.088	12
49	901	035	.7751	.078	11
50	.33929	.36068	2.7725	.94068	10
51	956	101	.7700	.058	9
52	.33983	134	.7675	.049	8
53	.34011	167	.7650	.039	7
54	038	199	.7625	.029	6
55	.34065	.36232	2.7600	.94019	5
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	.93989	2
59	175	364	.7500	.93979	1
60	.34202	.36397	2.7475	.93969	0
	cos	cot	tan	sin	

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20°

TABLE III

21°

	sin	tan	cot	cos	
0	.34202	.36397	2.7475	.93969	60
1	.229	.430	.7450	.959	59
2	.257	.463	.7425	.949	58
3	.284	.496	.7400	.939	57
4	.311	.529	.7376	.929	56
5	.34339	.36562	2.7351	.93919	55
6	.366	.595	.7326	.909	54
7	.393	.628	.7302	.899	53
8	.421	.661	.7277	.889	52
9	.448	.694	.7253	.879	51
10	.34475	.36727	2.7228	.93869	60
11	.503	.760	.7204	.859	49
12	.530	.793	.7179	.849	48
13	.557	.826	.7155	.839	47
14	.584	.859	.7130	.829	46
15	.34612	.36892	2.7106	.93819	45
16	.639	.925	.7082	.809	44
17	.666	.958	.7058	.799	43
18	.694	.36991	.7034	.789	42
19	.721	.37024	.7009	.779	41
20	.34748	.37057	2.6985	.93769	40
21	.775	.090	.6961	.759	39
22	.803	.123	.6937	.748	38
23	.830	.157	.6913	.738	37
24	.857	.190	.6889	.728	36
25	.34884	.37223	2.6865	.93718	35
26	.912	.256	.6841	.708	34
27	.939	.289	.6818	.698	33
28	.966	.322	.6794	.688	32
29	.34993	.355	.6770	.677	31
30	.35021	.37388	2.6746	.93667	30
31	.048	.422	.6723	.657	29
32	.075	.455	.6699	.647	28
33	.102	.488	.6675	.637	27
34	.130	.521	.6652	.626	26
35	.35157	.37554	2.6628	.93616	25
36	.184	.588	.6605	.606	24
37	.211	.621	.6581	.596	23
38	.239	.654	.6558	.585	22
39	.266	.687	.6534	.575	21
40	.35293	.37720	2.6511	.93565	20
41	.320	.754	.6488	.555	19
42	.347	.787	.6464	.544	18
43	.375	.820	.6441	.534	17
44	.402	.853	.6418	.524	16
45	.35429	.37887	2.6395	.93514	15
46	.456	.920	.6371	.503	14
47	.484	.953	.6348	.493	13
48	.511	.37986	.6325	.483	12
49	.538	.38020	.6302	.472	11
50	.35565	.38053	2.6279	.93462	10
51	.592	.086	.6256	.452	9
52	.619	.120	.6233	.441	8
53	.647	.153	.6210	.431	7
54	.674	.186	.6187	.420	6
55	.35701	.38220	2.6165	.93410	5
56	.728	.253	.6142	.400	4
57	.755	.286	.6119	.389	3
58	.782	.320	.6096	.379	2
59	.810	.353	.6074	.368	1
60	.35837	.38386	2.6051	.93358	0
	cos	cot	tan	sin	'

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68°

	sin	tan	cot	cos	
0	.35837	.38386	2.6051	.93358	60
1	.864	.420	.6028	.348	59
2	.891	.453	.6006	.337	58
3	.918	.487	.5983	.327	57
4	.945	.520	.5961	.316	56
5	.35973	.38553	2.5938	.93306	55
6	.36000	.587	.5916	.295	54
7	.027	.620	.5893	.285	53
8	.054	.654	.5871	.274	52
9	.081	.687	.5848	.264	51
10	.36108	.38721	2.5826	.93253	50
11	.135	.754	.5804	.243	49
12	.162	.787	.5782	.232	48
13	.190	.821	.5759	.222	47
14	.217	.854	.5737	.211	46
15	.36244	.38888	2.5715	.93201	45
16	.271	.921	.5693	.190	44
17	.298	.955	.5671	.180	43
18	.325	.38988	.5649	.169	42
19	.352	.39022	.5627	.159	41
20	.36379	.39055	2.5605	.93148	40
21	.406	.089	.5583	.137	39
22	.434	.122	.5561	.127	38
23	.461	.156	.5539	.116	37
24	.488	.190	.5517	.106	36
25	.36515	.39223	2.5495	.93095	35
26	.542	.257	.5473	.084	34
27	.569	.290	.5452	.074	33
28	.596	.324	.5430	.063	32
29	.623	.357	.5408	.052	31
30	.36650	.39391	2.5386	.93042	30
31	.677	.425	.5365	.031	29
32	.704	.458	.5343	.020	28
33	.731	.492	.5322	.010	27
34	.758	.526	.5300	.000	26
35	.36785	.39559	2.5279	.92988	25
36	.812	.593	.5257	.978	24
37	.839	.626	.5236	.967	23
38	.867	.660	.5214	.956	22
39	.894	.694	.5193	.945	21
40	.36921	.39727	2.5172	.92935	20
41	.948	.761	.5150	.924	19
42	.36975	.795	.5129	.913	18
43	.37002	.829	.5108	.902	17
44	.029	.862	.5086	.892	16
45	.37056	.39896	2.5065	.92881	15
46	.083	.930	.5044	.870	14
47	.110	.963	.5023	.859	13
48	.137	.39997	.5002	.849	12
49	.164	.40031	.4981	.838	11
50	.37191	.40065	2.4960	.92827	10
51	.218	.098	.4939	.816	9
52	.245	.132	.4918	.805	8
53	.272	.166	.4897	.794	7
54	.299	.200	.4876	.784	6
55	.37326	.40234	2.4855	.92773	5
56	.353	.267	.4834	.762	4
57	.380	.301	.4813	.751	3
58	.407	.335	.4792	.740	2
59	.434	.369	.4772	.729	1
60	.37461	.40403	2.4751	.92718	0
	cos	cot	tan	sin	'

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TABLE III

23°

	sin	tan	cot	cos	
0	.37461	.40403	2.4751	.92718	60
1	.488	.436	.4730	.707	59
2	.515	.470	.4709	.697	58
3	.542	.504	.4689	.686	57
4	.569	.538	.4668	.675	56
5	.37595	.40572	2.4648	.92664	55
6	.622	.606	.4627	.653	54
7	.649	.640	.4606	.642	53
8	.676	.674	.4586	.631	52
9	.703	.707	.4566	.620	51
10	.37730	.40741	2.4545	.92609	50
11	.757	.775	.4525	.598	49
12	.784	.809	.4504	.587	48
13	.811	.843	.4484	.576	47
14	.838	.877	.4464	.565	46
15	.37865	.40911	2.4443	.92554	45
16	.892	.945	.4423	.543	44
17	.919	.40979	.4403	.532	43
18	.946	.41013	.4383	.521	42
19	.973	.047	.4362	.510	41
20	.37999	.41081	2.4342	.92499	40
21	.38026	.115	.4322	.488	39
22	.053	.149	.4302	.477	38
23	.080	.183	.4282	.466	37
24	.107	.217	.4262	.455	36
25	.38134	.41251	2.4242	.92444	35
26	.161	.285	.4222	.432	34
27	.188	.319	.4202	.421	33
28	.215	.353	.4182	.410	32
29	.241	.387	.4162	.399	31
30	.38268	.41421	2.4142	.92388	30
31	.295	.455	.4122	.377	29
32	.322	.490	.4102	.366	28
33	.349	.524	.4083	.355	27
34	.376	.558	.4063	.343	26
35	.38403	.41592	2.4043	.92332	25
36	.430	.626	.4023	.321	24
37	.456	.660	.4004	.310	23
38	.483	.694	.3984	.299	22
39	.510	.728	.3964	.287	21
40	.38537	.41763	2.3945	.92276	20
41	.564	.797	.3925	.265	19
42	.591	.831	.3906	.254	18
43	.617	.865	.3886	.243	17
44	.644	.899	.3867	.231	16
45	.38671	.41933	2.3847	.92220	15
46	.698	.41968	.3828	.209	14
47	.725	.42002	.3808	.198	13
48	.752	.036	.3789	.186	12
49	.778	.070	.3770	.175	11
50	.38805	.42105	2.3750	.92164	10
51	.832	.139	.3731	.152	9
52	.859	.173	.3712	.141	8
53	.886	.207	.3693	.130	7
54	.912	.242	.3673	.119	6
55	.38939	.42276	2.3654	.92107	5
56	.966	.310	.3635	.096	4
57	.38993	.345	.3616	.085	3
58	.39020	.379	.3597	.073	2
59	.046	.413	.3578	.062	1
60	.39073	.42447	2.3559	.92050	0
	cos	cot	tan	sin	'

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	sin	tan	cot	cos	
0	.39073	.42447	2.3559	.92050	60
1	.100	.482	.3539	.039	59
2	.127	.516	.3520	.028	58
3	.153	.551	.3501	.016	57
4	.180	.585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	55
6	.234	.654	.3445	.982	54
7	.260	.688	.3426	.971	53
8	.287	.722	.3407	.959	52
9	.314	.757	.3388	.948	51
10	.39341	.42791	2.3369	.91936	50
11	.367	.826	.3351	.925	49
12	.394	.860	.3332	.914	48
13	.421	.894	.3313	.902	47
14	.448	.929	.3294	.891	46
15	.39474	.42963	2.3276	.91879	45
16	.501	.42998	.3257	.868	44
17	.528	.43032	.3238	.856	43
18	.555	.067	.3220	.845	42
19	.581	.101	.3201	.833	41
20	.39608	.43136	2.3183	.91822	40
21	.635	.170	.3164	.810	39
22	.661	.205	.3146	.799	38
23	.688	.239	.3127	.787	37
24	.715	.274	.3109	.775	36
25	.39741	.43308	2.3090	.91764	35
26	.768	.343	.3072	.752	34
27	.795	.378	.3053	.741	33
28	.822	.412	.3035	.729	32
29	.848	.447	.3017	.718	31
30	.39875	.43481	2.2998	.91706	30
31	.902	.516	.2980	.694	29
32	.928	.550	.2962	.683	28
33	.955	.585	.2944	.671	27
34	.39982	.620	.2925	.660	26
35	.40008	.43654	2.2907	.91648	25
36	.035	.689	.2889	.636	24
37	.062	.724	.2871	.625	23
38	.088	.758	.2853	.613	22
39	.115	.793	.2835	.601	21
40	.40141	.43828	2.2817	.91590	20
41	.168	.862	.2799	.578	19
42	.195	.897	.2781	.566	18
43	.221	.932	.2763	.555	17
44	.248	.43966	.2745	.543	16
45	.40275	.44001	2.2727	.91531	15
46	.301	.036	.2709	.519	14
47	.328	.071	.2691	.508	13
48	.355	.105	.2673	.496	12
49	.381	.140	.2655	.484	11
50	.40408	.44175	2.2637	.91472	10
51	.434	.210	.2620	.461	9
52	.461	.244	.2602	.449	8
53	.488	.279	.2584	.437	7
54	.514	.314	.2566	.425	6
55	.40541	.44349	2.2549	.91414	5
56	.567	.384	.2531	.402	4
57	.594	.418	.2513	.390	3
58	.621	.453	.2496	.378	2
59	.647	.488	.2478	.366	1
60	.40674	.44523	2.2460	.91355	0
	cos	cot	tan	sin	'

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24°

TABLE III

25°

	sin	tan	cot	cos	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	865	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	cos	cot	tan	sin	

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	sin	tan	cot	cos	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	.287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	0
	cos	cot	tan	sin	

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26°

TABLE III

27°

'	sin	tan	cot	cos	'
0	.43837	.48773	2.0503	.89879	60
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	50
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	45
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	40
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	35
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	30
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	25
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	20
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	15
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	10
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	5
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	0
	cos	cot	tan	sin	'

63°

'	sin	tan	cot	cos	'
0	.45399	.50953	1.9626	.89101	60
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	.099	.9570	048	56
5	.45529	.51136	1.9556	.89035	55
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	50
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	45
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	40
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	35
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	30
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	25
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	20
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	15
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	10
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	5
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	0
	cos	cot	tan	sin	'

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TABLE III

29°

	sin	tan	cot	cos			sin	tan	cot	cos	
0	.46947	.53171	1.8807	.88295	60	0	.48481	.55431	1.8040	.87462	60
1	.973	.208	.8794	.281	59	1	.506	.469	.8028	.448	59
2	.46999	.246	.8781	.267	58	2	.532	.507	.8016	.434	58
3	.47024	.283	.8768	.254	57	3	.557	.545	.8003	.420	57
4	.050	.320	.8755	.240	56	4	.583	.583	.7991	.406	56
5	.47076	.53358	1.8741	.88226	55	5	.48608	.55621	1.7979	.87391	55
6	.101	.395	.8728	.213	54	6	.634	.659	.7966	.377	54
7	.127	.432	.8715	.199	53	7	.659	.697	.7954	.363	53
8	.153	.470	.8702	.185	52	8	.684	.736	.7942	.349	52
9	.178	.507	.8689	.172	51	9	.710	.774	.7930	.335	51
10	.47204	.53545	1.8676	.88158	50	10	.48735	.55812	1.7917	.87321	50
11	.229	.582	.8663	.144	49	11	.761	.850	.7905	.306	49
12	.255	.620	.8650	.130	48	12	.786	.888	.7893	.292	48
13	.281	.657	.8637	.117	47	13	.811	.926	.7881	.278	47
14	.306	.694	.8624	.103	46	14	.837	.55964	.7868	.264	46
15	.47332	.53732	1.8611	.88089	45	15	.48862	.56003	1.7856	.87250	45
16	.358	.769	.8598	.075	44	16	.888	.041	.7844	.235	44
17	.383	.807	.8585	.062	43	17	.913	.079	.7832	.221	43
18	.409	.844	.8572	.048	42	18	.938	.117	.7820	.207	42
19	.434	.882	.8559	.034	41	19	.964	.156	.7808	.193	41
20	.47460	.53920	1.8546	.88020	40	20	.48989	.56194	1.7796	.87178	40
21	.486	.957	.8533	.88006	39	21	.49014	.232	.7783	.164	39
22	.511	.53995	.8520	.87993	38	22	.040	.270	.7771	.150	38
23	.537	.54032	.8507	.87979	37	23	.065	.309	.7759	.136	37
24	.562	.070	.8495	.965	36	24	.090	.347	.7747	.121	36
25	.47588	.54107	1.8482	.87951	35	25	.49116	.56385	1.7735	.87107	35
26	.614	.145	.8469	.937	34	26	.141	.424	.7723	.093	34
27	.639	.183	.8456	.923	33	27	.166	.462	.7711	.079	33
28	.665	.220	.8443	.909	32	28	.192	.501	.7699	.064	32
29	.690	.258	.8430	.896	31	29	.217	.539	.7687	.050	31
30	.47716	.54296	1.8418	.87882	30	30	.49242	.56577	1.7675	.87036	30
31	.741	.333	.8405	.868	29	31	.268	.616	.7663	.021	29
32	.767	.371	.8392	.854	28	32	.293	.654	.7651	.87007	28
33	.793	.409	.8379	.840	27	33	.318	.693	.7639	.86993	27
34	.818	.446	.8367	.826	26	34	.344	.731	.7627	.978	26
35	.47844	.54484	1.8354	.87812	25	35	.49369	.56769	1.7615	.86964	25
36	.869	.522	.8341	.798	24	36	.394	.808	.7603	.949	24
37	.895	.560	.8329	.784	23	37	.419	.846	.7591	.935	23
38	.920	.597	.8316	.770	22	38	.445	.885	.7579	.921	22
39	.946	.635	.8303	.756	21	39	.470	.923	.7567	.906	21
40	.47971	.54673	1.8291	.87743	20	40	.49495	.56962	1.7556	.86892	20
41	.47997	.711	.8278	.729	19	41	.521	.57000	.7544	.878	19
42	.48022	.748	.8265	.715	18	42	.546	.039	.7532	.863	18
43	.048	.786	.8253	.701	17	43	.571	.078	.7520	.849	17
44	.073	.824	.8240	.687	16	44	.596	.116	.7508	.834	16
45	.48099	.54862	1.8228	.87673	15	45	.49622	.57155	1.7496	.86820	15
46	.124	.900	.8215	.659	14	46	.647	.193	.7485	.805	14
47	.150	.938	.8202	.645	13	47	.672	.232	.7473	.791	13
48	.175	.54975	.8190	.631	12	48	.697	.271	.7461	.777	12
49	.201	.55013	.8177	.617	11	49	.723	.309	.7449	.762	11
50	.48226	.55051	1.8165	.87603	10	50	.49748	.57348	1.7437	.86748	10
51	.252	.089	.8152	.589	9	51	.773	.386	.7426	.733	9
52	.277	.127	.8140	.575	8	52	.798	.425	.7414	.719	8
53	.303	.165	.8127	.561	7	53	.824	.464	.7402	.704	7
54	.328	.203	.8115	.546	6	54	.849	.503	.7391	.690	6
55	.48354	.55241	1.8103	.87532	5	55	.49874	.57541	1.7379	.86675	5
56	.379	.279	.8090	.518	4	56	.899	.580	.7367	.661	4
57	.405	.317	.8078	.504	3	57	.924	.619	.7355	.646	3
58	.430	.355	.8065	.490	2	58	.950	.657	.7344	.632	2
59	.456	.393	.8053	.476	1	59	.49975	.696	.7332	.617	1
60	.48481	.55431	1.8040	.87462	0	60	.50000	.57735	1.7321	.86603	0
	cos	cot	tan	sin			cos	cot	tan	sin	

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60°

30°

TABLE III

31°

	sin	tan	cot	cos	
0	.50000	.57735	1.7321	.86603	60
1	.025	.774	.7309	.588	59
2	.050	.813	.7297	.573	58
3	.076	.851	.7286	.559	57
4	.101	.890	.7274	.544	56
5	.50126	.57929	1.7262	.86530	55
6	.151	.57968	.7251	.515	54
7	.176	.58007	.7239	.501	53
8	.201	.046	.7228	.486	52
9	.227	.085	.7216	.471	51
10	.50252	.58124	1.7205	.86457	50
11	.277	.162	.7193	.442	49
12	.302	.201	.7182	.427	48
13	.327	.240	.7170	.413	47
14	.352	.279	.7159	.398	46
15	.50377	.58318	1.7147	.86384	45
16	.403	.357	.7136	.369	44
17	.428	.396	.7124	.354	43
18	.453	.435	.7113	.340	42
19	.478	.474	.7102	.325	41
20	.50503	.58513	1.7090	.86310	40
21	.528	.552	.7079	.295	39
22	.553	.591	.7067	.281	38
23	.578	.631	.7056	.266	37
24	.603	.670	.7045	.251	36
25	.50628	.58709	1.7033	.86237	35
26	.654	.748	.7022	.222	34
27	.679	.787	.7011	.207	33
28	.704	.826	.6999	.192	32
29	.729	.865	.6988	.178	31
30	.50754	.58905	1.6977	.86163	30
31	.779	.944	.6965	.148	29
32	.804	.58983	.6954	.133	28
33	.829	.59022	.6943	.119	27
34	.854	.061	.6932	.104	26
35	.50879	.59101	1.6920	.86089	25
36	.904	.140	.6909	.074	24
37	.929	.179	.6898	.059	23
38	.954	.218	.6887	.045	22
39	.50979	.258	.6875	.030	21
40	.51004	.59297	1.6864	.86015	20
41	.029	.336	.6853	.86000	19
42	.054	.376	.6842	.85985	18
43	.079	.415	.6831	.85970	17
44	.104	.454	.6820	.85956	16
45	.51129	.59494	1.6808	.85941	15
46	.154	.533	.6797	.926	14
47	.179	.573	.6786	.911	13
48	.204	.612	.6775	.896	12
49	.229	.651	.6764	.881	11
50	.51254	.59691	1.6753	.85866	10
51	.279	.730	.6742	.851	9
52	.304	.770	.6731	.836	8
53	.329	.809	.6720	.821	7
54	.354	.849	.6709	.806	6
55	.51379	.59888	1.6698	.85792	5
56	.404	.928	.6687	.777	4
57	.429	.59967	.6676	.762	3
58	.454	.60007	.6665	.747	2
59	.479	.046	.6654	.732	1
60	.51504	.60086	1.6643	.85717	0
'	cos	cot	tan	sin	

	sin	tan	cot	cos	'
0	.51504	.60086	1.6643	.85717	60
1	.529	.126	.6632	.702	59
2	.554	.165	.6621	.687	58
3	.579	.205	.6610	.672	57
4	.604	.245	.6599	.657	56
5	.51628	.60284	1.6588	.85642	55
6	.653	.324	.6577	.627	54
7	.678	.364	.6566	.612	53
8	.703	.403	.6555	.597	52
9	.728	.443	.6545	.582	51
10	.51753	.60483	1.6534	.85567	50
11	.778	.522	.6523	.551	49
12	.803	.562	.6512	.536	48
13	.828	.602	.6501	.521	47
14	.852	.642	.6490	.506	46
15	.51877	.60681	1.6479	.85491	45
16	.902	.721	.6469	.476	44
17	.927	.761	.6458	.461	43
18	.952	.801	.6447	.446	42
19	.51977	.841	.6436	.431	41
20	.52002	.60881	1.6426	.85416	40
21	.026	.921	.6415	.401	39
22	.051	.60960	.6404	.385	38
23	.076	.61000	.6393	.370	37
24	.101	.040	.6383	.355	36
25	.52126	.61080	1.6372	.85340	35
26	.151	.120	.6361	.325	34
27	.175	.160	.6351	.310	33
28	.200	.200	.6340	.294	32
29	.225	.240	.6329	.279	31
30	.52250	.61280	1.6319	.85264	30
31	.275	.320	.6308	.249	29
32	.299	.360	.6297	.234	28
33	.324	.400	.6287	.218	27
34	.349	.440	.6276	.203	26
35	.52374	.61480	1.6265	.85188	25
36	.399	.520	.6255	.173	24
37	.423	.561	.6244	.157	23
38	.448	.601	.6234	.142	22
39	.473	.641	.6223	.127	21
40	.52498	.61681	1.6212	.85112	20
41	.522	.721	.6202	.096	19
42	.547	.761	.6191	.081	18
43	.572	.801	.6181	.066	17
44	.597	.842	.6170	.051	16
45	.52621	.61882	1.6160	.85035	15
46	.646	.922	.6149	.020	14
47	.671	.61962	.6139	.85005	13
48	.696	.62003	.6128	.84989	12
49	.720	.043	.6118	.974	11
50	.52745	.62083	1.6107	.84959	10
51	.770	.124	.6097	.943	9
52	.794	.164	.6087	.928	8
53	.819	.204	.6076	.913	7
54	.844	.245	.6066	.897	6
55	.52869	.62285	1.6055	.84882	5
56	.893	.325	.6045	.866	4
57	.918	.366	.6034	.851	3
58	.943	.406	.6024	.836	2
59	.967	.446	.6014	.820	1
60	.52992	.62487	1.6003	.84805	0
'	cos	cot	tan	sin	

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32°

TABLE III

33°

	sin	tan	cot	cos	
0	.52992	.62487	1.6003	.84805	60
1	.53017	.62527	.5993	.789	59
2	.041	.568	.5983	.774	58
3	.066	.608	.5972	.759	57
4	.091	.649	.5962	.743	56
5	.53115	.62689	1.5952	.84728	55
6	.140	.730	.5941	.712	54
7	.164	.770	.5931	.697	53
8	.189	.811	.5921	.681	52
9	.214	.852	.5911	.666	51
10	.53238	.62892	1.5900	.84650	50
11	.263	.933	.5890	.635	49
12	.288	.62973	.5880	.619	48
13	.312	.63014	.5869	.604	47
14	.337	.055	.5859	.588	46
15	.53361	.63095	1.5849	.84573	45
16	.386	.136	.5839	.557	44
17	.411	.177	.5829	.542	43
18	.435	.217	.5818	.526	42
19	.460	.258	.5808	.511	41
20	.53484	.63299	1.5798	.84495	40
21	.509	.340	.5788	.480	39
22	.534	.380	.5778	.464	38
23	.558	.421	.5768	.448	37
24	.583	.462	.5757	.433	36
25	.53607	.63503	1.5747	.84417	35
26	.632	.544	.5737	.402	34
27	.656	.584	.5727	.386	33
28	.681	.625	.5717	.370	32
29	.705	.666	.5707	.355	31
30	.53730	.63707	1.5697	.84339	30
31	.754	.748	.5687	.324	29
32	.779	.789	.5677	.308	28
33	.804	.830	.5667	.292	27
34	.828	.871	.5657	.277	26
35	.53853	.63912	1.5647	.84261	25
36	.877	.953	.5637	.245	24
37	.902	.63994	.5627	.230	23
38	.926	.64035	.5617	.214	22
39	.951	.076	.5607	.198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	.158	.5587	.167	19
42	.024	.199	.5577	.151	18
43	.049	.240	.5567	.135	17
44	.073	.281	.5557	.120	16
45	.54097	.64322	1.5547	.84104	15
46	.122	.363	.5537	.088	14
47	.146	.404	.5527	.072	13
48	.171	.446	.5517	.057	12
49	.195	.487	.5507	.041	11
50	.54220	.64528	1.5497	.84025	10
51	.244	.569	.5487	.84009	9
52	.269	.610	.5477	.83994	8
53	.293	.652	.5468	.978	7
54	.317	.693	.5458	.962	6
55	.54342	.64734	1.5448	.83946	5
56	.366	.775	.5438	.930	4
57	.391	.817	.5428	.915	3
58	.415	.858	.5418	.899	2
59	.440	.899	.5408	.883	1
60	.54464	.64941	1.5399	.83867	0
	cos	cot	tan	sin	

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	sin	tan	cot	cos	
0	.54464	.64941	1.5399	.83867	60
1	.488	.64982	.5389	.851	59
2	.513	.65024	.5379	.835	58
3	.537	.065	.5369	.819	57
4	.561	.106	.5359	.804	56
5	.54586	.65148	1.5350	.83788	55
6	.610	.189	.5340	.772	54
7	.635	.231	.5330	.756	53
8	.659	.272	.5320	.740	52
9	.683	.314	.5311	.724	51
10	.54708	.65355	1.5301	.83708	50
11	.732	.397	.5291	.692	49
12	.756	.438	.5282	.676	48
13	.781	.480	.5272	.660	47
14	.805	.521	.5262	.645	46
15	.54829	.65563	1.5253	.83629	45
16	.854	.604	.5243	.613	44
17	.878	.646	.5233	.597	43
18	.902	.688	.5224	.581	42
19	.927	.729	.5214	.565	41
20	.54951	.65771	1.5204	.83549	40
21	.975	.813	.5195	.533	39
22	.54999	.854	.5185	.517	38
23	.55024	.896	.5175	.501	37
24	.048	.938	.5166	.485	36
25	.55072	.65980	1.5156	.83469	35
26	.097	.66021	.5147	.453	34
27	.121	.063	.5137	.437	33
28	.145	.105	.5127	.421	32
29	.169	.147	.5118	.405	31
30	.55194	.66189	1.5108	.83389	30
31	.218	.230	.5099	.373	29
32	.242	.272	.5089	.356	28
33	.266	.314	.5080	.340	27
34	.291	.356	.5070	.324	26
35	.55315	.66398	1.5061	.83308	25
36	.339	.440	.5051	.292	24
37	.363	.482	.5042	.276	23
38	.388	.524	.5032	.260	22
39	.412	.566	.5023	.244	21
40	.55436	.66608	1.5013	.83228	20
41	.460	.650	.5004	.212	19
42	.484	.692	.4994	.195	18
43	.509	.734	.4985	.179	17
44	.533	.776	.4975	.163	16
45	.55557	.66818	1.4966	.83147	15
46	.581	.860	.4957	.131	14
47	.605	.902	.4947	.115	13
48	.630	.944	.4938	.098	12
49	.654	.66986	.4928	.082	11
50	.55678	.67028	1.4919	.83066	10
51	.702	.071	.4910	.050	9
52	.726	.113	.4900	.034	8
53	.750	.155	.4891	.017	7
54	.775	.197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	.823	.282	.4863	.969	4
57	.847	.324	.4854	.953	3
58	.871	.366	.4844	.936	2
59	.895	.409	.4835	.920	1
60	.55919	.67451	1.4826	.82904	0
	cos	cot	tan	sin	

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TABLE III

35°

	sin	tan	cot	cos	
0	.55919	.67451	1.4826	.82904	60
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	55
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	50
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	45
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	40
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	35
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	30
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	25
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	20
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	15
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	10
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	0
	cos	cot	tan	sin	

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	sin	tan	cot	cos	
0	.57358	.70021	1.4281	.81915	60
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	55
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	50
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	45
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	40
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	35
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	30
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	25
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	20
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	15
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	10
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	0
	cos	cot	tan	sin	

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'	sin	tan	cot	cos	'
0	.58779	.72654	1.3764	.80902	60
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	55
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	50
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	45
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	40
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	35
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	30
31	506	74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	25
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	20
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	15
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	10
51	972	946	.3343	021	9
52	.59995	.74991	1.3335	.80003	8
53	.60019	.75037	1.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	5
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	0
	cos	cot	tan	sin	'

'	sin	tan	cot	cos	'
0	.60182	.75355	1.3270	.79864	60
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	55
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	50
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	45
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	40
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	35
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	30
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	25
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	20
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	15
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	10
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	5
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	0
	cos	cot	tan	sin	'

'	sin	tan	cot	cos	'
0	.61566	.78129	1.2799	.78801	60
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	55
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	50
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	45
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	40
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	35
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	30
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	25
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	20
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	10
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	5
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	0
	cos	cot	tan	sin	'

'	sin	tan	cot	cos	'
0	.62932	.80978	1.2349	.77715	60
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	55
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	50
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	45
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	40
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	35
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	30
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	25
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	15
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	10
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	5
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	0
	cos	cot	tan	sin	'

40°

TABLE III

41°

'	sin	tan	cot	cos	'
0	.64279	.83910	1.1918	.76604	60
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	.059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	55
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	50
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	45
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	40
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	35
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945	.85408	1.1708	.76041	30
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	25
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	20
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	15
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	10
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	5
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	0
	cos	cot	tan	sin	'

49°

'	sin	tan	cot	cos	'
0	.65606	.86929	1.1504	.75471	60
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	55
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	50
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	45
16	956	749	.1396	165	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	40
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	35
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	30
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	25
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	20
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	15
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	10
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	5
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	0
	cos	cot	tan	sin	'

48°



42°

TABLE III

43°

'	sin	tan	cot	cos	'
0	.66913	.90040	1.1106	.74314	60
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	55
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	50
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	45
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	40
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	35
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	30
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	25
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	20
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	15
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	10
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	5
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	0
	cos	cot	tan	sin	'

'	sin	tan	cot	cos	'
0	.68200	.93252	1.0724	.73135	60
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	55
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	50
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	45
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	40
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	35
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	30
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	25
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	20
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	15
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	10
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	5
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	0
	cos	cot	tan	sin	'

47°

46°

TABLE III

44°

'	sin	tan	cot	cos	'
0	.69466	.96569	1.0355	.71934	60
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	55
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	50
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	45
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	40
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	35
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	30
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	25
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	20
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	15
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	10
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	5
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	0
	cos	cot	tan	sin	'

45°

TABLE IV. RADIAN MEASURE, 0° TO 180°, RADIUS = 1.

Degrees				Minutes				Seconds			
0°	0.00000 00	60°	1.04719 76	120°	2.09439 51	0'	0.00000 00	0"	0.00000 00		
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48		
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97		
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45		
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94		
5	0.08726 65	65	1.13446 40	125	2.18166 16	5	0.00145 44	5	0.00002 42		
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91		
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39		
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88		
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36		
10	0.17453 29	70	1.22173 05	130	2.26892 80	10	0.00290 89	10	0.00004 85		
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33		
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82		
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30		
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79		
15	0.26179 94	75	1.30899 69	135	2.35619 45	15	0.00436 33	15	0.00007 27		
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76		
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24		
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73		
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21		
20	0.34906 59	80	1.39626 34	140	2.44346 10	20	0.00581 78	20	0.00009 70		
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18		
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67		
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15		
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64		
25	0.43633 23	85	1.48352 99	145	2.53072 74	25	0.00727 22	25	0.00012 12		
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61		
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09		
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57		
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06		
30	0.52359 88	90	1.57079 63	150	2.61799 39	30	0.00872 66	30	0.00014 54		
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03		
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51		
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00		
34	0.59341 19	94	1.64060 95	154	2.68780 70	34	0.00989 02	34	0.00016 48		
35	0.61086 52	95	1.65806 28	155	2.70526 03	35	0.01018 11	35	0.00016 97		
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45		
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94		
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42		
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91		
40	0.69813 17	100	1.74532 93	160	2.79252 68	40	0.01163 55	40	0.00019 39		
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88		
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36		
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85		
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33		
45	0.78539 82	105	1.83259 57	165	2.87979 33	45	0.01309 00	45	0.00021 82		
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30		
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79		
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27		
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76		
50	0.87266 46	110	1.91986 22	170	2.96705 97	50	0.01454 44	50	0.00024 24		
51	0.89011 79	111	1.93731 55	171	2.98451 30	51	0.01483 53	51	0.00024 73		
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21		
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70		
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18		
55	0.95993 11	115	2.00712 86	175	3.05432 62	55	0.01599 89	55	0.00026 66		
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15		
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63		
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12		
59	1.02974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60		
60	1.04719 76	120	2.09439 51	180	3.14159 27	60	0.01745 33	60	0.00029 09		
Degrees				Minutes				Seconds			

TABLE V. HAVERSINES

D E G R E E S	0'		10'		20'		30'		40'		50'	
	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>
Obtain characteristic of <i>l hav</i> by inspection of <i>n hav</i>												
0	—	0000	3254	0000	9275	0000	2796	0000	5295	0000	7233	0001
1	8817	0001	0156	0001	1316	0001	2339	0002	3254	0002	4081	0003
2	4837	0003	5532	0004	6176	0004	6775	0005	7336	0005	7862	0006
3	8358	0007	8828	0008	9273	0008	9697	0009	0101	0010	0487	0011
4	0856	0012	1211	0013	1551	0014	1879	0015	2195	0017	2499	0018
5	2794	0019	3078	0020	3354	0022	3621	0023	3880	0024	4132	0026
6	4376	0027	4614	0029	4845	0031	5071	0032	5290	0034	5504	0036
7	5714	0037	5918	0039	6117	0041	6312	0043	6503	0045	6689	0047
8	6872	0049	7051	0051	7226	0053	7397	0055	7566	0057	7731	0059
9	7893	0062	8052	0064	8208	0066	8361	0069	8512	0071	8660	0073
10	8806	0076	8949	0079	9090	0081	9229	0084	9365	0086	9499	0089
11	9631	0092	9762	0095	9890	0097	0016	0100	0141	0103	0264	0106
12	0385	0109	0504	0112	0622	0115	0738	0119	0852	0122	0966	0125
13	1077	0128	1187	0131	1296	0135	1404	0138	1510	0142	1614	0145
14	1718	0149	1820	0152	1921	0156	2021	0159	2120	0163	2217	0167
15	2314	0170	2409	0174	2504	0178	2597	0182	2689	0186	2781	0190
16	2871	0194	2961	0198	3049	0202	3137	0206	3223	0210	3309	0214
17	3394	0218	3478	0223	3561	0227	3644	0231	3726	0236	3807	0240
18	3887	0245	3966	0249	4045	0254	4123	0258	4200	0263	4276	0268
19	4352	0272	4427	0277	4502	0282	4576	0287	4649	0292	4721	0297
20	4793	0302	4865	0307	4935	0312	5006	0317	5075	0322	5144	0327
21	5213	0332	5281	0337	5348	0343	5415	0348	5481	0353	5547	0359
22	5612	0364	5677	0370	5741	0375	5805	0381	5868	0386	5931	0392
23	5993	0397	6055	0403	6116	0409	6177	0415	6238	0421	6298	0426
24	6358	0432	6417	0438	6476	0444	6534	0450	6592	0456	6650	0462
25	6707	0468	6764	0475	6820	0481	6876	0487	6932	0493	6987	0500
26	7042	0506	7096	0512	7150	0519	7204	0525	7258	0532	7311	0538
27	7364	0545	7416	0552	7468	0558	7520	0565	7572	0572	7623	0578
28	7674	0585	7724	0592	7774	0599	7824	0606	7874	0613	7923	0620
29	7972	0627	8021	0634	8069	0641	8117	0648	8165	0655	8213	0663
30	8260	0670	8307	0677	8354	0684	8400	0692	8446	0699	8492	0707
31	8538	0714	8583	0722	8629	0729	8673	0737	8718	0744	8763	0752
32	8807	0760	8851	0767	8894	0775	8938	0783	8981	0791	9024	0799
33	9067	0807	9109	0815	9152	0823	9194	0831	9236	0839	9277	0847
34	9319	0855	9360	0863	9401	0871	9442	0879	9482	0888	9523	0896
35	9563	0904	9603	0913	9643	0921	9682	0929	9721	0938	9761	0946
36	9800	0955	9838	0963	9877	0972	9915	0981	9954	0989	9992	0998
37	0030	1007	0067	1016	0105	1024	0142	1033	0179	1042	0216	1051
38	0253	1060	0289	1069	0326	1078	0362	1087	0398	1096	0434	1105
39	0470	1114	0505	1123	0541	1133	0576	1142	0611	1151	0646	1160
40	0681	1170	0716	1179	0750	1189	0784	1198	0819	1207	0853	1217
41	0887	1226	0920	1236	0954	1246	0987	1255	1020	1265	1054	1275
42	1087	1284	1119	1294	1152	1304	1185	1314	1217	1323	1249	1333
43	1282	1343	1314	1363	1345	1363	1377	1373	1409	1383	1440	1393
44	1472	1403	1503	1413	1534	1424	1565	1434	1596	1444	1626	1454
45	1657	1464	1687	1475	1718	1485	1748	1495	1778	1506	1808	1516
46	1838	1527	1867	1537	1897	1548	1926	1558	1956	1569	1985	1579
47	2014	1590	2043	1601	2072	1611	2101	1622	2129	1633	2158	1644
48	2186	1654	2215	1665	2243	1676	2271	1687	2299	1698	2327	1709
49	2355	1720	2382	1731	2410	1742	2437	1753	2465	1764	2492	1775
50	2519	1786	2546	1797	2573	1808	2600	1820	2627	1831	2653	1842
51	2680	1853	2706	1865	2732	1876	2759	1887	2785	1899	2811	1910
52	2837	1922	2863	1933	2888	1945	2914	1956	2940	1968	2965	1979
53	2991	1991	3016	2003	3041	2014	3066	2026	3091	2038	3116	2049
54	3141	2061	3166	2073	3190	2085	3215	2096	3239	2108	3264	2120
55	3288	2132	3312	2144	3336	2156	3361	2168	3384	2180	3408	2192
56	3432	2204	3456	2216	3480	2228	3503	2240	3527	2252	3550	2265
57	3573	2277	3596	2289	3620	2301	3643	2314	3666	2326	3689	2338
58	3711	2350	3734	2363	3757	2375	3779	2388	3802	2400	3824	2412
59	3847	2426	3869	2437	3891	2450	3913	2462	3935	2475	3957	2487

TABLE V. HAVERSINES

D E G R E E S	0'		10'		20'		30'		40'		50'	
	<i>l hav</i> 9.	<i>n hav</i>	<i>l hav</i> 9.	<i>n hav</i>	<i>l hav</i> 9.	<i>n hav</i>	<i>l hav</i> 9.	<i>n hav</i>	<i>l hav</i> 9.	<i>n hav</i>	<i>l hav</i> 9.	<i>n hav</i>
60	3979	2500	4001	2513	4023	2525	4045	2538	4066	2551	4088	2563
61	4109	2576	4131	2589	4152	2601	4173	2614	4195	2627	4216	2640
62	4237	2653	4258	2665	4279	2678	4300	2691	4320	2704	4341	2717
63	4362	2730	4382	2743	4403	2766	4423	2769	4444	2782	4464	2795
64	4484	2808	4504	2821	4524	2834	4545	2847	4565	2861	4584	2874
65	4604	2887	4624	2900	4644	2913	4664	2927	4683	2940	4703	2953
66	4722	2966	4742	2980	4761	2993	4780	3006	4799	3020	4819	3033
67	4838	3046	4857	3060	4876	3073	4895	3087	4914	3100	4932	3113
68	4951	3127	4970	3140	4989	3154	5007	3167	5026	3181	5044	3195
69	5063	3208	5081	3222	5099	3235	5117	3249	5136	3263	5154	3276
70	5172	3290	5190	3304	5208	3317	5226	3331	5244	3345	5261	3358
71	5279	3372	5297	3386	5314	3400	5332	3413	5349	3427	5367	3441
72	5384	3455	5402	3469	5419	3483	5436	3496	5454	3510	5471	3524
73	5488	3538	5505	3552	5522	3566	5539	3580	5556	3594	5572	3608
74	5589	3622	5606	3636	5623	3650	5639	3664	5656	3678	5672	3692
75	5689	3706	5705	3720	5722	3734	5738	3748	5754	3762	5771	3776
76	5787	3790	5803	3805	5819	3819	5835	3833	5851	3847	5867	3861
77	5883	3875	5899	3889	5915	3904	5930	3918	5946	3932	5962	3946
78	5977	3960	5993	3975	6009	3989	6024	4003	6039	4017	6055	4032
79	6070	4046	6086	4060	6101	4075	6116	4089	6131	4103	6146	4117
80	6161	4132	6176	4146	6191	4160	6206	4175	6221	4189	6236	4203
81	6251	4218	6266	4232	6280	4247	6295	4261	6310	4275	6324	4290
82	6339	4304	6353	4319	6368	4333	6382	4347	6397	4362	6411	4376
83	6425	4391	6440	4405	6454	4420	6468	4434	6482	4448	6496	4463
84	6510	4477	6524	4492	6538	4506	6552	4521	6566	4535	6580	4550
85	6594	4564	6607	4579	6621	4593	6635	4608	6648	4622	6662	4637
86	6676	4651	6689	4666	6703	4680	6716	4695	6730	4709	6743	4724
87	6756	4738	6770	4753	6783	4767	6796	4782	6809	4796	6822	4811
88	6835	4826	6848	4840	6862	4855	6875	4869	6887	4884	6900	4898
89	6913	4913	6926	4927	6939	4942	6952	4956	6964	4971	6977	4985
90	6990	5000	7002	5015	7015	5029	7027	5044	7040	5058	7052	5073
91	7065	5087	7077	5102	7090	5116	7102	5131	7114	5145	7126	5160
92	7139	5174	7151	5189	7163	5204	7175	5218	7187	5233	7199	5247
93	7211	5262	7223	5276	7235	5291	7247	5305	7259	5320	7271	5334
94	7283	5349	7294	5363	7306	5378	7318	5392	7329	5407	7341	5421
95	7353	5436	7364	5450	7376	5465	7387	5479	7399	5494	7410	5508
96	7421	5523	7433	5537	7444	5552	7455	5566	7467	5580	7478	5595
97	7489	5609	7500	5624	7511	5638	7523	5653	7534	5667	7545	5681
98	7556	5696	7567	5710	7577	5725	7588	5739	7599	5753	7610	5768
99	7621	5782	7632	5797	7642	5811	7653	5825	7664	5840	7674	5854
100	7685	5868	7696	5883	7706	5897	7717	5911	7727	5925	7738	5940
101	7748	5954	7759	5968	7769	5983	7779	5997	7790	6011	7800	6025
102	7810	6040	7820	6054	7830	6068	7841	6082	7851	6096	7861	6111
103	7871	6125	7881	6139	7891	6153	7901	6167	7911	6181	7921	6195
104	7931	6210	7940	6224	7950	6238	7960	6252	7970	6266	7980	6280
105	7989	6294	7999	6308	8009	6322	8018	6336	8028	6350	8037	6364
106	8047	6378	8056	6392	8066	6406	8075	6420	8085	6434	8094	6448
107	8104	6462	8113	6476	8122	6490	8131	6504	8141	6517	8150	6531
108	8159	6545	8168	6559	8177	6573	8187	6587	8196	6600	8205	6614
109	8214	6628	8223	6642	8232	6655	8241	6669	8250	6683	8258	6696
110	8267	6710	8276	6724	8285	6737	8294	6751	8302	6765	8311	6778
111	8320	6792	8329	6806	8337	6819	8346	6833	8354	6846	8363	6860
112	8371	6873	8380	6887	8388	6900	8397	6913	8405	6927	8414	6940
113	8422	6954	8430	6967	8439	6980	8447	6994	8455	7007	8464	7020
114	8472	7034	8480	7047	8488	7060	8496	7073	8504	7087	8513	7100
115	8521	7113	8529	7126	8537	7139	8545	7153	8553	7166	8561	7179
116	8568	7192	8576	7205	8584	7218	8592	7231	8600	7244	8608	7257
117	8615	7270	8623	7283	8631	7296	8638	7309	8646	7322	8654	7335
118	8661	7347	8669	7360	8676	7373	8684	7386	8691	7399	8699	7411
119	8706	7424	8714	7437	8721	7449	8729	7462	8736	7475	8743	7487

TABLE V. HAVERSINES

D E G R E S	0'		10'		20'		30'		40'		50'	
	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>	<i>l hav</i>	<i>n hav</i>
	9.		9.		9.		9.		9.		9.	
120	8751	7500	8758	7513	8765	7525	8772	7538	8780	7550	8787	7563
121	8794	7575	8801	7588	8808	7600	8815	7612	8822	7625	8829	7637
122	8836	7650	8843	7662	8850	7674	8857	7686	8864	7699	8871	7711
123	8878	7723	8885	7735	8892	7748	8898	7760	8905	7772	8912	7784
124	8919	7796	8925	7808	8932	7820	8939	7832	8945	7844	8952	7856
125	8959	7868	8965	7880	8972	7892	8978	7904	8985	7915	8991	7927
126	8998	7939	9004	7951	9010	7962	9017	7974	9023	7986	9030	7997
127	9036	8009	9042	8021	9048	8032	9055	8044	9061	8055	9067	8067
128	9073	8078	9079	8090	9085	8101	9092	8113	9098	8124	9104	8135
129	9110	8147	9116	8158	9122	8169	9128	8180	9134	8192	9140	8203
130	9146	8214	9151	8225	9157	8236	9163	8247	9169	8258	9175	8269
131	9180	8280	9186	8291	9192	8302	9198	8313	9203	8324	9209	8335
132	9215	8346	9220	8356	9226	8367	9231	8378	9237	8389	9242	8399
133	9248	8410	9253	8421	9259	8431	9264	8442	9270	8452	9275	8463
134	9281	8473	9286	8484	9291	8494	9297	8505	9302	8515	9307	8525
135	9312	8536	9318	8546	9323	8556	9328	8566	9333	8576	9338	8587
136	9343	8597	9348	8607	9353	8617	9359	8627	9364	8637	9369	8647
137	9374	8657	9379	8667	9383	8677	9388	8686	9393	8696	9398	8706
138	9403	8716	9408	8726	9413	8735	9417	8745	9422	8754	9427	8764
139	9432	8774	9436	8783	9441	8793	9446	8802	9450	8811	9455	8821
140	9460	8830	9464	8840	9469	8849	9473	8858	9478	8867	9482	8877
141	9487	8886	9491	8895	9496	8904	9500	8913	9505	8922	9509	8931
142	9513	8940	9518	8949	9522	8958	9526	8967	9531	8976	9535	8984
143	9539	8993	9543	9002	9548	9011	9552	9019	9556	9028	9560	9037
144	9564	9045	9568	9054	9572	9062	9576	9071	9580	9079	9584	9087
145	9588	9096	9592	9104	9596	9112	9600	9121	9604	9129	9608	9137
146	9612	9145	9616	9153	9620	9161	9623	9169	9627	9177	9631	9185
147	9635	9193	9638	9201	9642	9209	9646	9217	9650	9225	9653	9233
148	9657	9240	9660	9248	9664	9256	9668	9263	9671	9271	9675	9278
149	9678	9286	9682	9293	9685	9301	9689	9308	9692	9316	9695	9323
150	9699	9330	9702	9337	9706	9345	9709	9352	9712	9359	9716	9366
151	9719	9373	9722	9380	9725	9387	9729	9394	9732	9401	9735	9408
152	9738	9415	9741	9422	9744	9428	9747	9435	9751	9442	9754	9448
153	9757	9455	9760	9462	9763	9468	9766	9475	9769	9481	9772	9488
154	9774	9494	9777	9500	9780	9507	9783	9513	9786	9519	9789	9525
155	9792	9532	9794	9538	9797	9544	9800	9550	9803	9556	9805	9562
156	9808	9568	9811	9574	9813	9579	9816	9585	9819	9591	9821	9597
157	9824	9603	9826	9608	9829	9614	9831	9619	9834	9625	9836	9630
158	9839	9636	9841	9641	9844	9647	9846	9652	9849	9657	9851	9663
159	9853	9668	9856	9673	9858	9678	9860	9683	9863	9688	9865	9693
160	9867	9698	9869	9703	9871	9708	9874	9713	9876	9718	9878	9723
161	9880	9728	9882	9732	9884	9737	9886	9742	9888	9746	9890	9751
162	9892	9755	9894	9750	9896	9764	9898	9769	9900	9773	9902	9777
163	9904	9782	9906	9786	9908	9790	9910	9794	9911	9798	9913	9802
164	9915	9806	9917	9810	9919	9814	9920	9818	9922	9822	9924	9826
165	9925	9830	9927	9833	9929	9837	9930	9841	9932	9844	9933	9848
166	9935	9851	9937	9855	9938	9858	9940	9862	9941	9865	9943	9869
167	9944	9872	9945	9876	9947	9878	9948	9881	9950	9885	9951	9888
168	9952	9891	9954	9894	9955	9897	9956	9900	9957	9903	9959	9905
169	9960	9908	9961	9911	9962	9914	9963	9916	9965	9919	9966	9921
170	9967	9924	9968	9927	9969	9929	9970	9931	9971	9934	9972	9936
171	9973	9938	9974	9941	9975	9943	9976	9945	9977	9947	9978	9949
172	9979	9951	9980	9953	9981	9955	9981	9957	9982	9959	9983	9961
173	9984	9963	9985	9964	9985	9966	9986	9968	9987	9969	9987	9971
174	9988	9973	9989	9974	9989	9975	9990	9977	9991	9978	9991	9980
175	9992	9981	9992	9982	9993	9983	9993	9985	9994	9986	9994	9987
176	9995	9988	9995	9989	9996	9990	9996	9991	9996	9992	9997	9992
177	9997	9993	9997	9994	9998	9995	9998	9995	9998	9996	9998	9996
178	9999	9997	9999	9997	9999	9998	9999	9998	9999	9999	9999	9999
179	9999	9999	9999	9999	0000	0000	0000	0000	0000	0000	0000	0000
180	0000	0000										





















